

GRE Mathematics Practice Test 58

Upper-Intermediate Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

Intermediate reinforcement, combining standard applications with selected more demanding arguments. This is cycle 11 of 19, position 3 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Evaluate $\int_0^{\pi/2} \sin^9 t \cos^{25} t \, dt$.

- (A) $1820 \left(\frac{4! 12!}{2 \cdot 17!} \right)$
- (B) $2 \left(\frac{4! 12!}{2 \cdot 17!} \right)$
- (C) $\frac{4! 12!}{2 \cdot 17!}$
- (D) $17 \left(\frac{4! 12!}{2 \cdot 17!} \right)$
- (E) $\frac{1}{2} \left(\frac{4! 12!}{2 \cdot 17!} \right)$

2. A 3×3 matrix A has characteristic polynomial $(t - 54)(t + 1)(t - 2)$. What is $\text{tr}(A^3)$?

- (A) 157470
- (B) 157472
- (C) 157473
- (D) 157471
- (E) 157469

3. The solution of $y' = 12x^3y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\frac{\sqrt{2} \cdot 3^{\frac{3}{4}}}{6}$
- (B) $\sqrt[4]{3}$
- (C) $\frac{1}{3}$
- (D) $\frac{3^{\frac{3}{4}}}{3}$
- (E) $\frac{6^{\frac{3}{4}}}{6}$

Questions 4–6

4. Independent trials have success probability $1/25$. Let T count trials up to and including the first success. Find $E[T]$.

- (A) 25
- (B) 41
- (C) 9
- (D) 33
- (E) 17

5. For $f(x) = x^4 e^{12x}$, find $f^{(6)}(0)$.

- (A) $\frac{1}{360} \left(\frac{6! 12^2}{2} \right)$
- (B) $\frac{1}{6} \left(\frac{6! 12^2}{2} \right)$
- (C) $2 \left(\frac{6! 12^2}{2} \right)$
- (D) $\frac{6! 12^2}{2}$
- (E) $\frac{1}{30} \left(\frac{6! 12^2}{2} \right)$

6. Let N be the 60×60 matrix with ones immediately above the diagonal and zeros elsewhere. Find $\dim(\ker N^{18} / \ker N^9)$.

- (A) 8
- (B) 9
- (C) 7
- (D) 10
- (E) 11

Questions 7–9

7. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(12+4j)(16+4j)}$.

- (A) $\frac{1}{12}$
- (B) $\frac{1}{4}$
- (C) $\frac{1}{3}$
- (D) $\frac{1}{16}$
- (E) $\frac{1}{48}$

8. A uniformly random subset of 25 objects is selected from a collection with 26 red and 27 blue objects. If X is the number of red objects selected, find $E[X]$.

- (A) $\frac{756}{53}$
- (B) $\frac{650}{53}$
- (C) $\frac{1300}{53}$
- (D) $\frac{325}{53}$
- (E) $\frac{703}{53}$

9. Two solutions of $y'' + 12x^4y' + 4y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $\frac{2}{e^{\frac{12}{5}}}$
- (B) $e^{-\frac{12}{5}} + 1$
- (C) $e^{-\frac{12}{5}}$
- (D) $\frac{1}{2e^{\frac{12}{5}}}$
- (E) $-\frac{1}{e^{\frac{12}{5}}}$

Questions 10–12

10. Let A be the 60×60 matrix whose diagonal entries are 57 and whose off-diagonal entries are 1. What is its largest eigenvalue?

- (A) 117
- (B) 115
- (C) 118
- (D) 114
- (E) 116

11. Evaluate $\int_0^1 x^{11}(\log x)^3 dx$.

- (A) $\frac{1}{4} \left(\frac{(-1)^3 3!}{12^4} \right)$
- (B) $12 \left(\frac{(-1)^3 3!}{12^4} \right)$
- (C) $288 \left(\frac{(-1)^3 3!}{12^4} \right)$
- (D) $-\frac{(-1)^3 3!}{12^4}$
- (E) $\frac{(-1)^3 3!}{12^4}$

12. X, Y are independent Poisson variables with means 25 and 26. Given $X + Y = 27$, find $\text{Var}(X | X + Y)$.

- (A) $\frac{3900}{289}$
- (B) $\frac{975}{289}$
- (C) $\frac{2239}{289}$
- (D) $\frac{2528}{289}$
- (E) $\frac{1950}{289}$

Questions 13–15

13. Let $F(x) = \int_0^x (x-t)^3 e^{12t} dt$. Find $F^{(4)}(0)$.

- (A) $\frac{1}{6}$
- (B) 72
- (C) 0
- (D) 24
- (E) 6

14. Positive real numbers x, y satisfy $\log_2 x + \log_2 y = \log_2 594$ and $x + y = 65$. Find $x^2 + y^2$.

- (A) 3037
- (B) 3035
- (C) 3039
- (D) 3036
- (E) 3038

15. Find the upward flux of $\mathbf{F} = (x, y, z)$ through $z = 12x^2 + 4y^2$, $0 \leq x, y \leq 1$.

- (A) $-\frac{13}{3}$
- (B) $\frac{16}{3}$
- (C) $-\frac{16}{3}$
- (D) $-\frac{8}{3}$
- (E) $-\frac{32}{3}$

Questions 16–18

16. X, Y are independent, with variances 25 and 26. Find $\text{Cov}(X + 2Y, 3X - Y)$.

- (A) 16
- (B) 9
- (C) 23
- (D) 30
- (E) 37

17. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^{12}y^3 dV$.

- (A) $\frac{1}{52}$
- (B) $6 \left(\frac{12! 3!}{18!} \right)$
- (C) $\frac{12! 3!}{18!}$
- (D) $\frac{1}{6} \left(\frac{12! 3!}{18!} \right)$
- (E) $18 \left(\frac{12! 3!}{18!} \right)$

18. A sequence satisfies $u_0 = 54$ and $u_{k+1} = 2u_k + 11$ for all $k \geq 0$. Find u_6 .

- (A) 4151
- (B) 4147
- (C) 4149
- (D) 4148
- (E) 4150

Questions 19–21

19. Let C be the unit circle $x^2 + y^2 = 1$, $z = 4$, counterclockwise viewed from above. Evaluate $\int_C -12y(x^2 + y^2)^3 dx + (12x(x^2 + y^2)^3) dy + 4z dz$.

- (A) 24π
- (B) 48π
- (C) 12π
- (D) -24π
- (E) $1 + 24\pi$

20. X has density cx^{25} on $0 < x < 1$ and zero elsewhere. Find the normalizing constant c .

- (A) 18
- (B) 26
- (C) 34
- (D) 10
- (E) 42

21. For $x > 0$, $x^2 y'' + (1 - 16)xy' + 48y = 0$, $y(1) = 0$, $y'(1) = 12$. Find $y(e)$.

- (A) $-e^4 + e^{12}$
- (B) $e^4 + e^{12}$
- (C) $-\frac{3(1 - e^8)e^4}{2}$
- (D) $\frac{3(1 - e^8)e^4}{2}$
- (E) $\frac{e^{12}}{16}$

Questions 22–24

22. Linear maps $T : \mathbb{R}^{61} \rightarrow \mathbb{R}^{61}$ and $U : \mathbb{R}^{61} \rightarrow \mathbb{R}^{61}$ have ranks 3 and 2. What is the largest possible rank of UT ?

- (A) 2
- (B) 0
- (C) 3
- (D) 4
- (E) 1

23. Find the curvature of the helix $r(t) = (12 \cos t, 12 \sin t, 4t)$.

- (A) $-\frac{3}{40}$
- (B) $\frac{43}{40}$
- (C) $\frac{3}{20}$
- (D) $\frac{3}{40}$
- (E) $\frac{3}{80}$

24. Which subset of $\mathbb{R}$ is compact, with 20 fixed as displayed?

- (A) $\{0\} \cup \{20/k : k \geq 1\}$
- (B) $\{20/k : k \geq 1\}$
- (C) $\mathbb{Q} \cap [0, 20]$
- (D) $(0, 20)$
- (E) $20\mathbb{Z}$

Questions 25–27

25. For $x > 0$, $x^2y'' + (-27)xy' + 192y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

- (A) $-\frac{(1 - e^4)e^{12}}{8}$
(B) $\frac{(1 - e^4)e^{12}}{4}$
(C) $-\frac{e^{12}}{4} + 1 + \frac{e^{16}}{4}$
(D) $-\frac{(1 - e^4)e^{12}}{2}$
(E) $-\frac{(1 - e^4)e^{12}}{4}$

26. Fix a nonzero vector $u \in \mathbb{R}^{60}$. What is the dimension of the space of skew-symmetric 60×60 matrices A satisfying $Au = 0$?

- (A) 1711
(B) 1712
(C) 1713
(D) 1710
(E) 1709

27. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(12x) - \sum_{j=0}^2 (-1)^j (12x)^{2j+1} / (2j+1)!}{x^7}$.

- (A) $\frac{1}{12} \left(\frac{(-1)^3 12^7}{7!} \right)$
(B) $12 \left(\frac{(-1)^3 12^7}{7!} \right)$
(C) $-\frac{(-1)^3 12^7}{7!}$
(D) $\frac{(-1)^3 12^7}{7!}$
(E) $7 \left(\frac{(-1)^3 12^7}{7!} \right)$

Questions 28–30

28. For $a_k = (-1)^k + 20/k$, find $\limsup_{k \rightarrow \infty} a_k - \liminf_{k \rightarrow \infty} a_k$.

- (A) 4
- (B) 0
- (C) 2
- (D) 1
- (E) 3

29. Let $f(x, y) = e^{12x+4y}$. Find $\partial^4 f / \partial x^3 \partial y$ at $(0, 0)$.

- (A) $4 \cdot 12^3$
- (B) $\frac{1}{9} (4 \cdot 12^3)$
- (C) $\frac{1}{27} (4 \cdot 12^3)$
- (D) $3 (4 \cdot 12^3)$
- (E) $\frac{1}{4} (4 \cdot 12^3)$

30. Let $A = \begin{bmatrix} 1 & 54 & 0 \\ 0 & 1 & 11 \\ 0 & 0 & 1 \end{bmatrix}$. What is the $(1, 3)$ entry of A^5 ?

- (A) 5941
- (B) 5938
- (C) 5942
- (D) 5940
- (E) 5939

Questions 31–33

31. Evaluate $\int_0^1 \int_0^x x^{12} y^4 dy dx$.

- (A) $\frac{1}{90}$
- (B) $\frac{1}{108}$
- (C) $\frac{1}{65}$
- (D) $\frac{1}{85}$
- (E) $\frac{1}{18}$

32. For which real a is $f(x) = x^a$ in $L^{20}(0, 1)$?

- (A) $a > -1/20$
- (B) All real a
- (C) $a \geq -1/20$
- (D) $a < 1/20$
- (E) $a > 0$

33. Let $F(x) = \int_0^{x^4} e^{12t} dt$. Find $F'(1)$.

- (A) $48e^{12}$
- (B) $4e^{12}$
- (C) e^{12}
- (D) $\frac{e^{12}}{4}$
- (E) e^{48}

Questions 34–36

34. An invertible 60×60 matrix A has determinant 3. Find $\det(\text{adj } A)$.

- (A) $2 \cdot 3^{59}$
- (B) 3^{59}
- (C) 3^{58}
- (D) 3^{60}
- (E) $3^{59} \cdot 5$

35. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 12z^2 = x + 4y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.

- (A) -48
- (B) -96
- (C) 96
- (D) -4
- (E) -24

36. $T : \mathbb{R} \rightarrow \mathbb{R}$ has Lipschitz constant at most $1/20$. If $x_1 = T(x_0)$ and $|x_1 - x_0| \leq 1$, what is the sharp upper bound on $|T^3(x_0) - x_*|$, where x_* is its fixed point?

- (A) $\frac{1}{15200}$
- (B) $\frac{7599}{7600}$
- (C) $\frac{7601}{15200}$
- (D) $\frac{1}{22800}$
- (E) $\frac{1}{7600}$

Questions 37–39

37. If $y'' + 16y = 12$ and $y(0) = y'(0) = 0$, find $y(\pi/4)$.

- (A) $\frac{3}{4}$
- (B) $\frac{3}{2}$
- (C) $\frac{5}{2}$
- (D) $-\frac{3}{2}$
- (E) 3

38. On P_{61} , the real polynomials of degree at most 61, define $T(p) = x^2 p'' - 72p$. Find its rank.

- (A) 63
- (B) 61
- (C) 62
- (D) 60
- (E) 59

39. Evaluate $\iiint_{x^2+y^2+z^2 \leq 12} (x^2 + y^2 + z^2)^3 dV$.

- (A) $\frac{1}{2} \left(\frac{4\pi 12^{\frac{9}{2}}}{9} \right)$
- (B) $\frac{1}{3} \left(\frac{4\pi 12^{\frac{9}{2}}}{9} \right)$
- (C) $\frac{4\pi 12^{\frac{9}{2}}}{9}$
- (D) $\frac{3}{28} \left(\frac{4\pi 12^{\frac{9}{2}}}{9} \right)$
- (E) $9 \left(\frac{4\pi 12^{\frac{9}{2}}}{9} \right)$

Questions 40–42

40. Let $\zeta = e^{2\pi i/17}$. Find $\sum_{k=0}^{16} \zeta^k$.

- (A) 0
- (B) 3
- (C) 1
- (D) 4
- (E) 2

41. If $y' = 24xy$ and $y(0) = 1$, find the coefficient of x^8 in the Maclaurin series of y .

- (A) 865
- (B) 1728
- (C) 432
- (D) -864
- (E) 864

42. How many residues x modulo 84 satisfy $6x \equiv 36 \pmod{84}$ and $2x \equiv 0 \pmod{84}$?

- (A) 4
- (B) 1
- (C) 2
- (D) 0
- (E) 3

Questions 43–45

43. Evaluate $\int_C (4x^3 + 12y) dx + (12x + 4y^3) dy$, where C is $r(t) = (t, 4t^2)$, $0 \leq t \leq 1$.
- (A) 305
(B) 306
(C) -305
(D) 610
(E) $\frac{305}{2}$
44. An analytic function has real part $u(x, y) = x^2 - y^2 + 15x$ and satisfies $f(0) = 0$. Find $\text{Im } f(1 + i)$.
- (A) 12
(B) 22
(C) 7
(D) 17
(E) 27
45. Find the coefficient of x^4 in $e^{3x} \log(1 + 12x)$.
- (A) -89424
(B) -5184
(C) -3714
(D) -3726
(E) 3726

Questions 46–48

46. The field K has 5^{12} elements. Find the largest possible multiplicative order of a nonzero element of K .

- (A) $5^{13} - 1$
- (B) $5^{12} - 1$
- (C) $5^{12} + 1$
- (D) 5^{12}
- (E) $5^{11} - 1$

47. For $f(x, y) = x^2 + 12xy + 4y^2$, what is its maximum directional derivative at $(1, -1)$?

- (A) 4
- (B) 116
- (C) $2\sqrt{29}$
- (D) 14
- (E) -10

48. A tree has 29 vertices. What is the sum of its vertex degrees?

- (A) 74
- (B) 38
- (C) 92
- (D) 20
- (E) 56

Questions 49–51

49. Evaluate $\int_0^{\log 2} e^x (e^x + 12)^4 dx$.

- (A) $\frac{14^5}{5}$
- (B) $\frac{14^5 - 13^5}{5}$
- (C) $-\frac{14^5 - 13^5}{5}$
- (D) $14^4 - 13^4$
- (E) $5 \left(\frac{14^5 - 13^5}{5} \right)$

50. Let $G = \mathbb{Z}/354\mathbb{Z} \times \mathbb{Z}/240\mathbb{Z}$. Find the order of G/H , where $H = \{g \in G : 6g = 0\}$.

- (A) 2361
- (B) 2358
- (C) 2362
- (D) 2360
- (E) 2359

51. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 12x$ and has density $\rho(x, y) = x^2y$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{11}{6}$
- (B) $\frac{5}{12}$
- (C) $-\frac{5}{6}$
- (D) $\frac{5}{6}$
- (E) $\frac{5}{3}$

Questions 52–54

52. A connected planar graph has 28 vertices and 51 edges. How many faces, including the unbounded face, are in a planar embedding?

- (A) 33
- (B) 41
- (C) 17
- (D) 9
- (E) 25

53. Find the directional derivative of $f(x, y, z) = x^2y + 4xz^2 + 12y^2z$ at $(1, 1, 1)$ in the direction $(1, 4, 0)$.

- (A) $\frac{53\sqrt{17}}{17}$
- (B) $\frac{212\sqrt{17}}{17}$
- (C) $-\frac{106\sqrt{17}}{17}$
- (D) $1 + \frac{106\sqrt{17}}{17}$
- (E) $\frac{106\sqrt{17}}{17}$

54. How many group homomorphisms $\mathbb{Z}/354\mathbb{Z} \rightarrow \mathbb{Z}/240\mathbb{Z}$ are there?

- (A) 6
- (B) 8
- (C) 4
- (D) 5
- (E) 7

Questions 55–57

55. Find the arc length of $r(t) = (12(t - t^3/3), 12t^2)$ for $0 \leq t \leq 1/4$.

- (A) $\frac{49}{32}$
- (B) $\frac{49}{8}$
- (C) $\frac{65}{16}$
- (D) $\frac{49}{16}$
- (E) $-\frac{49}{16}$

56. How many integers in $\{1, \dots, 150\}$ are divisible by 2 or 3?

- (A) 166
- (B) 100
- (C) 34
- (D) 67
- (E) 133

57. The region $0 \leq x \leq 1$, $0 \leq y \leq 12x^4$ is revolved about the line $x = -4$. Find its volume.

- (A) $\frac{58\pi}{5}$
- (B) $\frac{232\pi}{5}$
- (C) $\frac{116\pi}{5}$
- (D) $-\frac{116\pi}{5}$
- (E) $1 + \frac{116\pi}{5}$

Questions 58–60

58. How many 1-dimensional subspaces does $\mathbb{F}_{31}^5$ have?

- (A) $\frac{31^8 - 1}{31 - 1}$
- (B) $\frac{31^4 - 1}{31 - 1}$
- (C) $\frac{31^5 - 1}{31 - 1}$
- (D) $\frac{31^7 - 1}{31 - 1}$
- (E) $\frac{31^6 - 1}{31 - 1}$

59. If $y'' - 24y' + 144y = 4e^{12x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) $4e^{12}$
- (B) $1 + 2e^{12}$
- (C) e^{12}
- (D) $-2e^{12}$
- (E) $2e^{12}$

60. Let $F_0 = 0$, $F_1 = 1$, and $F_{k+2} = F_{k+1} + F_k$. How many binary strings of length 25 have no adjacent 1s?

- (A) F_{26}
- (B) F_{27}
- (C) F_{29}
- (D) F_{28}
- (E) F_{25}

Questions 61–63

61. Evaluate $\lim_{x \rightarrow 0} \frac{(1 + 12x)^{1/(3x)} - e^{12/3}}{x}$.

- (A) $-24e^4$
- (B) e^4
- (C) $24e^4$
- (D) $-2e^4$
- (E) $-48e^4$

62. How many monic irreducible polynomials of degree 3 are there over $\mathbb{F}_{31}$?

- (A) 9922
- (B) 9921
- (C) 9919
- (D) 9918
- (E) 9920

63. For $D = \{(x, y) : x^2 + y^2 \leq 12\}$, evaluate $\iint_D (x^2 + y^2)^3 dA$.

- (A) 432π
- (B) 10368π
- (C) $\frac{20736\pi}{7}$
- (D) 20736π
- (E) 5184π

Questions 64–66

64. Apply one Newton iteration to solve $x^2 - 25 = 0$, starting at $x_0 = 1$. What is x_1 ?

- (A) 5
- (B) 13
- (C) 17
- (D) 9
- (E) 21

65. Evaluate $\iiint_B 12z^2(x^2 + y^2 + z^2)^4 dV$, where B is the unit ball.

- (A) $\frac{32\pi}{13}$
- (B) $1 + \frac{16\pi}{13}$
- (C) $-\frac{16\pi}{13}$
- (D) $\frac{16\pi}{13}$
- (E) $\frac{8\pi}{13}$

66. How many elements z of $\mathbb{F}_{5^{12}}$ satisfy $z^{25} = z$?

- (A) 27
- (B) 26
- (C) 25
- (D) 23
- (E) 24

Answer Key

Practice Test 58 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	C	23	D	45	D
2	D	24	A	46	B
3	D	25	E	47	C
4	A	26	A	48	E
5	D	27	D	49	B
6	B	28	C	50	D
7	E	29	A	51	D
8	B	30	D	52	E
9	C	31	A	53	E
10	E	32	A	54	A
11	E	33	B	55	D
12	E	34	B	56	B
13	E	35	B	57	C
14	A	36	E	58	C
15	C	37	B	59	E
16	C	38	B	60	B
17	C	39	C	61	A
18	C	40	A	62	E
19	A	41	E	63	E
20	B	42	D	64	B
21	C	43	A	65	D
22	A	44	D	66	C

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