

GRE Mathematics Practice Test 59

Bridge to Standard

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A bridge toward standard practice, with routine multi-step questions and a smaller number of challenging concepts. This is cycle 11 of 19, position 4 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Find the upward flux of $\mathbf{F} = (0, 0, z)$ through $z = 12x^2 + 3y^2$, $0 \leq x, y \leq 1$.

- (A) 6
- (B) $\frac{5}{2}$
- (C) 5
- (D) 10
- (E) -5

2. A linear map $T : \mathbb{R}^{62} \rightarrow \mathbb{R}^{64}$ has rank 4. What is its nullity?

- (A) 59
- (B) 57
- (C) 56
- (D) 58
- (E) 60

3. Evaluate $\int_0^1 \frac{x^2}{1 + 12x^3} dx$.

- (A) $\frac{\log(13)}{12}$
- (B) $\frac{\log(13)}{36}$
- (C) $\log(13)$
- (D) $\frac{\log(13)}{3}$
- (E) $-\frac{\log(13)}{36}$

Questions 4–6

4. Independent trials have success probability $1/26$. Let T count trials up to and including the first success. Find $E[T]$.

- (A) 42
- (B) 18
- (C) 34
- (D) 26
- (E) 10

5. Under $x = u + 12v$, $y = u - 3v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) $\frac{1}{15}$
- (B) 9
- (C) 15
- (D) 225
- (E) 36

6. What is the dimension of the real vector space of 61×61 symmetric matrices?

- (A) 1891
- (B) 1893
- (C) 1890
- (D) 1892
- (E) 1889

Questions 7–9

7. The region $0 \leq x \leq 1$, $0 \leq y \leq 12x^3$ is revolved about the x -axis. Find its volume.

- (A) $1 + \frac{144\pi}{7}$
- (B) $\frac{144\pi}{7}$
- (C) $\frac{288\pi}{7}$
- (D) $\frac{72\pi}{7}$
- (E) $-\frac{144\pi}{7}$

8. A uniformly random subset of 26 objects is selected from a collection with 27 red and 28 blue objects. If X is the number of red objects selected, find $E[X]$.

- (A) $\frac{702}{55}$
- (B) $\frac{1404}{55}$
- (C) $\frac{812}{55}$
- (D) $\frac{757}{55}$
- (E) $\frac{351}{55}$

9. Solve $y' + 12y = x^3e^{-12x}$, $y(0) = 0$, and find $y(1)$.

- (A) $\frac{1}{3e^{12}}$
- (B) $\frac{3}{e^{12}}$
- (C) $\frac{e^{12}}{4}$
- (D) $\frac{1}{4e^{12}}$
- (E) e^{-12}

Questions 10–12

10. Let $A = \begin{bmatrix} 1 & 55 & 0 \\ 0 & 1 & 9 \\ 0 & 0 & 1 \end{bmatrix}$. What is the $(1, 2)$ entry of A^6 ?

- (A) 329
- (B) 332
- (C) 331
- (D) 330
- (E) 328

11. For $x > 0$, $xy' = 12y$ and $y(1) = 1$. Find $y(e^3)$.

- (A) $1 + e^{36}$
- (B) $-e^{36}$
- (C) $2e^{36}$
- (D) e^{36}
- (E) $\frac{e^{36}}{2}$

12. X, Y are independent Poisson variables with means 26 and 27. Given $X + Y = 28$, find $\text{Var}(X \mid X + Y)$.

- (A) $\frac{19656}{2809}$
- (B) $\frac{22465}{2809}$
- (C) $\frac{9828}{2809}$
- (D) $\frac{39312}{2809}$
- (E) $\frac{25274}{2809}$

Questions 13–15

13. Evaluate $\int_0^1 \int_0^x x^{12} y^3 \, dy \, dx$.

- (A) $\frac{1}{85}$
- (B) $\frac{1}{17}$
- (C) $\frac{1}{52}$
- (D) $\frac{1}{64}$
- (E) $\frac{1}{68}$

14. An invertible 61×61 matrix A has determinant 4. Find $\det(A^T A)$.

- (A) 18
- (B) 16
- (C) 15
- (D) 14
- (E) 17

15. Find the total outward flux of $\mathbf{F} = (12x^3, y^3, z^3)$ through the boundary of $[0, 1]^3$.

- (A) 15
- (B) 14
- (C) -14
- (D) 7
- (E) 28

Questions 16–18

16. X, Y are independent, with variances 26 and 27. Find $\text{Cov}(X + 2Y, 3X - Y)$.

- (A) 24
- (B) 32
- (C) 8
- (D) 40
- (E) 16

17. Evaluate $\lim_{x \rightarrow 0} (1 + 12x)^{1/(2x)}$.

- (A) e^{144}
- (B) e^6
- (C) $e^{\frac{1}{6}}$
- (D) 12
- (E) e^{13}

18. On the space P_{62} of real polynomials of degree at most 62, define $T(p) = p'$. What is the rank of T ?

- (A) 63
- (B) 62
- (C) 64
- (D) 60
- (E) 61

Questions 19–21

19. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-12)^j}{j^{\frac{2}{3}} (\log j)^m}$ converge absolutely at $x = 11$? Give the least such m , or 0 if none exists.

- (A) 4
- (B) 3
- (C) 1
- (D) 0
- (E) 2

20. X has density cx^{26} on $0 < x < 1$ and zero elsewhere. Find the normalizing constant c .

- (A) 9
- (B) 27
- (C) 45
- (D) 18
- (E) 36

21. The solution of $y' = 12x^2y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\frac{1}{2}$
- (B) $\frac{1}{4}$
- (C) $\frac{\sqrt[3]{18}}{6}$
- (D) $2^{\frac{2}{3}}$
- (E) $\frac{\sqrt[3]{2}}{2}$

Questions 22–24

22. Subspaces U, V of $\mathbb{R}^{63}$ have dimensions 4 and 5. What is the largest possible value of $\dim(U \cap V)$?

- (A) 4
- (B) 5
- (C) 3
- (D) 2
- (E) 6

23. Find the coefficient of x^3 in the Taylor series of $(1 - 12x)^{-2}$ at zero.

- (A) $\frac{3}{4} (4 \cdot 12^3)$
- (B) $-4 \cdot 12^3$
- (C) $\frac{1}{4} (4 \cdot 12^3)$
- (D) $4 \cdot 12^3$
- (E) $\frac{1}{12} (4 \cdot 12^3)$

24. Which subset of $\mathbb{R}$ is compact, with 21 fixed as displayed?

- (A) $\{21/k : k \geq 1\}$
- (B) $\mathbb{Q} \cap [0, 21]$
- (C) $(0, 21)$
- (D) $\{0\} \cup \{21/k : k \geq 1\}$
- (E) $21\mathbb{Z}$

Questions 25–27

25. Two solutions of $y'' + 12x^1y' + 3y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $\frac{1}{2e^6}$
- (B) $e^{-6} + 1$
- (C) $\frac{2}{e^6}$
- (D) $-\frac{1}{e^6}$
- (E) e^{-6}

26. Let V consist of real polynomials of degree at most 10 that vanish at 55, 56, and 58. Find $\dim V$.

- (A) 10
- (B) 9
- (C) 8
- (D) 7
- (E) 6

27. If $y'' + 9y = 0$, $y(0) = 12$, and $y'(0) = 5$, find $y(\pi/6)$.

- (A) $-\frac{5}{3}$
- (B) $\frac{10}{3}$
- (C) $\frac{5}{3}$
- (D) $\frac{8}{3}$
- (E) $\frac{5}{6}$

Questions 28–30

28. For $a_k = (-1)^k + 21/k$, find $\limsup_{k \rightarrow \infty} a_k - \liminf_{k \rightarrow \infty} a_k$.

- (A) 4
- (B) 3
- (C) 1
- (D) 0
- (E) 2

29. The solution of $y' = 12x^3y^2$ has $y(0) = 1/13$. Find $y(1)$.

- (A) $\frac{1}{20}$
- (B) $\frac{1}{10}$
- (C) $\frac{1}{5}$
- (D) $-\frac{1}{10}$
- (E) $\frac{11}{10}$

30. The roots r, s of $x^2 - 64x + 495 = 0$ are counted with multiplicity. Find $r^2 + s^2$.

- (A) 3106
- (B) 3104
- (C) 3108
- (D) 3107
- (E) 3105

Questions 31–33

31. For $\mathbf{F} = (12x^3, y^3, z^3)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) $\frac{51}{2}$
- (B) 52
- (C) 51
- (D) 102
- (E) -51

32. For which real a is $f(x) = x^a$ in $L^{21}(0, 1)$?

- (A) All real a
- (B) $a > 0$
- (C) $a < 1/21$
- (D) $a > -1/21$
- (E) $a \geq -1/21$

33. Let C be the unit circle $x^2 + y^2 = 1$, $z = 3$, counterclockwise viewed from above. Evaluate $\int_C -12y(x^2 + y^2)^2 dx + (12x(x^2 + y^2)^2) dy$.

- (A) -24π
- (B) 24π
- (C) 12π
- (D) 48π
- (E) $1 + 24\pi$

Questions 34–36

34. For real t , let $M(t) = \begin{pmatrix} t-55 & 9 & 0 \\ 0 & t-55 & 1 \\ 0 & 0 & t-55 \end{pmatrix}$. At $t = 55$, what is $\text{rank } M(t)$?

- (A) 3
- (B) 2
- (C) 1
- (D) 4
- (E) 0

35. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 12z^2 = x + 3y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.

- (A) 72
- (B) -3
- (C) -72
- (D) -36
- (E) -24

36. A map $T : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $|T(x) - T(y)| \leq |x - y|/21$. If $T(x_*) = x_*$ and $|x_0 - x_*| \leq 1$, what is the smallest guaranteed upper bound for $|T^3(x_0) - x_*|$?

- (A) $\frac{1}{9261}$
- (B) $\frac{1}{27783}$
- (C) $\frac{1}{18522}$
- (D) $\frac{4631}{9261}$
- (E) $\frac{9261}{9260}$

Questions 37–39

37. The function $y(x)$ satisfies $y^3 + 12y = 2x$ and $y(0) = 0$. Find $y'(0)$.

- (A) $\frac{2}{15}$
- (B) $\frac{1}{6}$
- (C) $-\frac{1}{6}$
- (D) 0
- (E) 12

38. Let $u = (11, 3)^T$ and $v = (1, 0)^T$. Find the squared norm of the orthogonal projection of v onto $\text{span}(u)$.

- (A) $\frac{158}{65}$
- (B) $\frac{93}{65}$
- (C) $\frac{251}{130}$
- (D) $\frac{121}{130}$
- (E) $\frac{28}{65}$

39. Let $\mathbf{F} = (12y^3, z^3, x^3)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (0, 0, 1)$.

- (A) -72
- (B) -35
- (C) -36
- (D) 36
- (E) -18

Questions 40–42

40. Let $\zeta = e^{2\pi i/18}$. Find $\sum_{k=0}^{17} \zeta^k$.

- (A) 1
- (B) 0
- (C) 4
- (D) 2
- (E) 3

41. Evaluate $\int_0^1 x^{11}(\log x)^2 dx$.

- (A) $72 \left(\frac{(-1)^2 2!}{12^3} \right)$
- (B) $-\frac{(-1)^2 2!}{12^3}$
- (C) $\frac{(-1)^2 2!}{12^3}$
- (D) $\frac{1}{3} \left(\frac{(-1)^2 2!}{12^3} \right)$
- (E) $12 \left(\frac{(-1)^2 2!}{12^3} \right)$

42. An idempotent 61×61 real matrix P has rank 5. What is $\text{tr } P$?

- (A) 3
- (B) 6
- (C) 7
- (D) 4
- (E) 5

Questions 43–45

43. Let $F(x) = \int_0^{x^3} e^{12t} dt$. Find $F'(1)$.

- (A) $\frac{e^{12}}{3}$
- (B) e^{12}
- (C) $3e^{12}$
- (D) e^{36}
- (E) $36e^{12}$

44. An analytic function has real part $u(x, y) = x^2 - y^2 + 16x$ and satisfies $f(0) = 0$. Find $\text{Im } f(1 + i)$.

- (A) 6
- (B) 24
- (C) 18
- (D) 30
- (E) 12

45. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(12 + 3j)(15 + 3j)}$.

- (A) $\frac{1}{12}$
- (B) $\frac{1}{3}$
- (C) $\frac{1}{15}$
- (D) $\frac{1}{4}$
- (E) $\frac{1}{36}$

Questions 46–48

46. How many generators does the cyclic group of order 31 have?

- (A) 28
- (B) 32
- (C) 30
- (D) 31
- (E) 29

47. For $D = \{(x, y) : x^2 + y^2 \leq 12\}$, evaluate $\iint_D (x^2 + y^2)^2 dA$.

- (A) 576π
- (B) $\frac{1728\pi}{5}$
- (C) 1152π
- (D) 48π
- (E) 1728π

48. A tree has 30 vertices. What is the sum of its vertex degrees?

- (A) 96
- (B) 77
- (C) 39
- (D) 58
- (E) 20

Questions 49–51

49. Find the coefficient of x^3 in $e^{2x} \log(1 + 12x)$.

- (A) -456
- (B) 576
- (C) 468
- (D) 2736
- (E) 456

50. Let $R = (\mathbb{Z}/128\mathbb{Z})/([16])$, where $([16])$ is the ideal generated by $[16]$. How many elements does R have?

- (A) 18
- (B) 17
- (C) 16
- (D) 14
- (E) 15

51. For $f(x, y) = x^2 + 12xy + 3y^2$, what is its maximum directional derivative at $(1, -1)$?

- (A) $2\sqrt{34}$
- (B) 136
- (C) -10
- (D) 16
- (E) 6

Questions 52–54

52. A connected planar graph has 29 vertices and 53 edges. How many faces, including the unbounded face, are in a planar embedding?

- (A) 42
- (B) 18
- (C) 34
- (D) 26
- (E) 10

53. What is the maximum of x^3y^{12} over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\left(\frac{3}{15}\right)^3 \left(\frac{12}{15}\right)^{12}$
- (B) $\frac{1}{2^{15}}$
- (C) $\frac{4}{25}$
- (D) $15 \left(\left(\frac{3}{15}\right)^3 \left(\frac{12}{15}\right)^{12} \right)$
- (E) $\left(\frac{3}{15}\right)^{12} \left(\frac{12}{15}\right)^3$

54. What is the largest integer k such that 2^k divides $64!$?

- (A) 64
- (B) 61
- (C) 62
- (D) 63
- (E) 65

Questions 55–57

55. Solve $y'' - 27y' + 180y = 0$ with $y(0) = 1$, $y'(0) = 0$. Find $y(1)$.

- (A) $\frac{(5 - 4e^3)e^{12}}{2}$
- (B) $(-5 + 4e^3)e^{12}$
- (C) $(5 - 4e^3)e^{12} + 1$
- (D) $(10 - 8e^3)e^{12}$
- (E) $(5 - 4e^3)e^{12}$

56. How many integers in $\{1, \dots, 156\}$ are divisible by 2 or 3?

- (A) 104
- (B) 36
- (C) 70
- (D) 172
- (E) 138

57. Let C be $x^2/9 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C (-y) dx + (12x) dy$.

- (A) $1 + 39\pi$
- (B) -39π
- (C) 39π
- (D) $\frac{39\pi}{2}$
- (E) 78π

Questions 58–60

58. In the ring $\mathbb{Z}/354\mathbb{Z}$, how many elements are units?

- (A) 118
- (B) 117
- (C) 115
- (D) 114
- (E) 116

59. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(12x) - \sum_{j=0}^1 (-1)^j (12x)^{2j+1} / (2j+1)!}{x^5}$.

- (A) $-\frac{(-1)^2 12^5}{5!}$
- (B) $12 \left(\frac{(-1)^2 12^5}{5!} \right)$
- (C) $5 \left(\frac{(-1)^2 12^5}{5!} \right)$
- (D) $\frac{1}{12} \left(\frac{(-1)^2 12^5}{5!} \right)$
- (E) $\frac{(-1)^2 12^5}{5!}$

60. Let $F_0 = 0$, $F_1 = 1$, and $F_{k+2} = F_{k+1} + F_k$. How many binary strings of length 26 have no adjacent 1s?

- (A) F_{28}
- (B) F_{29}
- (C) F_{27}
- (D) F_{26}
- (E) F_{30}

Questions 61–63

61. Evaluate $\int_0^{\infty} x^3 e^{-12x} dx$.

- (A) $20736 \left(\frac{3!}{12^4} \right)$
- (B) $\frac{1}{3} \left(\frac{3!}{12^4} \right)$
- (C) $\frac{1}{12} \left(\frac{3!}{12^4} \right)$
- (D) $12 \left(\frac{3!}{12^4} \right)$
- (E) $\frac{3!}{12^4}$

62. A permutation consists of disjoint cycles of lengths 8 and 10 and fixes all other points. What is its order?

- (A) 39
- (B) 42
- (C) 41
- (D) 38
- (E) 40

63. Evaluate $\int_0^1 \int_x^1 12y^3 dy dx$.

- (A) $\frac{12}{5}$
- (B) 3
- (C) 6
- (D) $\frac{3}{5}$
- (E) 4

Questions 64–66

64. Apply one Newton iteration to solve $x^2 - 26 = 0$, starting at $x_0 = 1$. What is x_1 ?
- (A) $\frac{29}{2}$
 - (B) $\frac{27}{2}$
 - (C) 27
 - (D) $\frac{27}{4}$
 - (E) $\frac{31}{2}$
65. For $y' = 12y(1 - y/3)$, what is the derivative of the right-hand side with respect to y at the positive equilibrium?
- (A) -12
 - (B) -11
 - (C) 12
 - (D) -6
 - (E) -24
66. In $R = \mathbb{F}_3[x]/(x^{12})$, how many elements are there?
- (A) 3^{15}
 - (B) 3^{13}
 - (C) 3^{14}
 - (D) 3^{11}
 - (E) 3^{12}

Answer Key

Practice Test 59 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	C	23	D	45	E
2	D	24	D	46	C
3	B	25	E	47	A
4	D	26	C	48	D
5	C	27	C	49	E
6	A	28	E	50	C
7	B	29	B	51	A
8	A	30	A	52	D
9	D	31	C	53	A
10	D	32	D	54	D
11	D	33	B	55	E
12	A	34	B	56	A
13	E	35	C	57	C
14	B	36	A	58	E
15	B	37	B	59	E
16	A	38	D	60	A
17	B	39	C	61	E
18	B	40	B	62	E
19	D	41	C	63	A
20	B	42	E	64	B
21	E	43	C	65	A
22	A	44	C	66	E

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