

GRE Mathematics Practice Test 60

Standard-Level Target

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A mixed standard-level target, intended to approximate public GRE Mathematics practice in scope and overall demand. This is cycle 11 of 19, position 5 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

zarimion.com/gre

100 practice tests, complete answer keys, and future updates.

Please retain the source attribution when sharing this material.



This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Evaluate $\int_0^1 \int_0^x x^{12} y^2 \, dy \, dx$.

- (A) $\frac{1}{45}$
- (B) $\frac{1}{16}$
- (C) $\frac{1}{64}$
- (D) $\frac{1}{48}$
- (E) $\frac{1}{39}$

2. Subspaces U, V of $\mathbb{R}^{64}$ have dimensions 3 and 4. What is the largest possible value of $\dim(U \cap V)$?

- (A) 2
- (B) 3
- (C) 5
- (D) 1
- (E) 4

3. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(12j)!}{(j!)^{12}} x^{2j}$?

- (A) 12^6
- (B) 12^{-12}
- (C) $\frac{1}{12}$
- (D) 12^{-6}
- (E) $12^{-\frac{1}{6}}$

Questions 4–6

4. Independent trials have success probability $1/27$. Let T count trials up to and including the first success. Find $E[T]$.

- (A) 9
- (B) 36
- (C) 18
- (D) 45
- (E) 27

5. Evaluate $\iiint_{x^2+y^2+z^2 \leq 12} (x^2 + y^2 + z^2)^1 dV$.

- (A) $\frac{4\pi 12^{\frac{5}{2}}}{5}$
- (B) $\frac{5}{36} \left(\frac{4\pi 12^{\frac{5}{2}}}{5} \right)$
- (C) $5 \left(\frac{4\pi 12^{\frac{5}{2}}}{5} \right)$
- (D) $\frac{1}{2} \left(\frac{4\pi 12^{\frac{5}{2}}}{5} \right)$
- (E) $\frac{1}{3} \left(\frac{4\pi 12^{\frac{5}{2}}}{5} \right)$

6. Let V consist of real polynomials of degree at most 9 that vanish at 56, 57, and 59. Find $\dim V$.

- (A) 6
- (B) 8
- (C) 9
- (D) 7
- (E) 5

Questions 7–9

7. Evaluate $\int_0^1 x^{11}(\log x)^1 dx$.

- (A) $\frac{1}{2} \left(\frac{(-1)^1 1!}{12^2} \right)$
- (B) $2 \left(\frac{(-1)^1 1!}{12^2} \right)$
- (C) $-\frac{(-1)^1 1!}{12^2}$
- (D) $\frac{(-1)^1 1!}{12^2}$
- (E) $12 \left(\frac{(-1)^1 1!}{12^2} \right)$

8. A uniformly random subset of 27 objects is selected from a collection with 28 red and 29 blue objects. If X is the number of red objects selected, find $E[X]$.

- (A) $\frac{290}{19}$
- (B) $\frac{271}{19}$
- (C) $\frac{126}{19}$
- (D) $\frac{504}{19}$
- (E) $\frac{252}{19}$

9. Let $\mathbf{F} = (12y^2, z^2, x^2)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (0, 0, 1)$.

- (A) -24
- (B) -12
- (C) -23
- (D) 24
- (E) -48

Questions 10–12

10. The roots r, s of $x^2 - 63x + 392 = 0$ are counted with multiplicity. Find $r^2 + s^2$.

- (A) 3185
- (B) 3183
- (C) 3184
- (D) 3187
- (E) 3186

11. Let $f(x) = \log(1 + 12x)$. Find $f^{(2)}(0)$.

- (A) $\frac{1}{12} \left((-1)^1 1! 12^2 \right)$
- (B) $2 \left((-1)^1 1! 12^2 \right)$
- (C) $-(-1)^1 1! 12^2$
- (D) $\frac{1}{2} \left((-1)^1 1! 12^2 \right)$
- (E) $(-1)^1 1! 12^2$

12. X, Y are independent Poisson variables with means 27 and 28. Given $X + Y = 29$, find $E[X \mid X + Y]$.

- (A) $\frac{783}{110}$
- (B) $\frac{1566}{55}$
- (C) $\frac{893}{55}$
- (D) $\frac{783}{55}$
- (E) $\frac{838}{55}$

Questions 13–15

13. Solve $y' + 12y = x^2e^{-12x}$, $y(0) = 0$, and find $y(1)$.

- (A) e^{-12}
- (B) $\frac{1}{2e^{12}}$
- (C) $\frac{1}{3e^{12}}$
- (D) $\frac{e^{12}}{3}$
- (E) $\frac{4}{e^{12}}$

14. For real t , let $M(t) = \begin{pmatrix} t-56 & 7 & 0 \\ 0 & t-56 & 1 \\ 0 & 0 & t-56 \end{pmatrix}$. At $t = 56$, what is $\text{rank } M(t)$?

- (A) 3
- (B) 4
- (C) 0
- (D) 1
- (E) 2

15. Let C be $x^2/4 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C (-y) dx + (12x) dy$.

- (A) 26π
- (B) $1 + 26\pi$
- (C) -26π
- (D) 13π
- (E) 52π

Questions 16–18

16. X, Y are independent, with variances 27 and 28. Find $\text{Cov}(X + 2Y, 3X - Y)$.

- (A) 9
- (B) 25
- (C) 41
- (D) 17
- (E) 33

17. Find the total outward flux of $\mathbf{F} = (12x^2, y^2, z^2)$ through the boundary of $[0, 1]^3$.

- (A) 14
- (B) -14
- (C) 15
- (D) 7
- (E) 28

18. Let $u = (11, 2)^T$ and $v = (1, 0)^T$. Find the squared norm of the orthogonal projection of v onto $\text{span}(u)$.

- (A) $\frac{117}{250}$
- (B) $\frac{617}{250}$
- (C) $\frac{367}{250}$
- (D) $\frac{246}{125}$
- (E) $\frac{121}{125}$

Questions 19–21

19. Evaluate $\int_0^1 \frac{x^1}{1+12x^2} dx$.

- (A) $\frac{\log(13)}{2}$
- (B) $\log(13)$
- (C) $\frac{\log(13)}{24}$
- (D) $\frac{\log(13)}{12}$
- (E) $-\frac{\log(13)}{24}$

20. X has density cx^{27} on $0 < x < 1$ and zero elsewhere. Find the normalizing constant c .

- (A) 37
- (B) 10
- (C) 28
- (D) 19
- (E) 46

21. What is the maximum of x^2y^{12} over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\left(\frac{2}{14}\right)^{12} \left(\frac{12}{14}\right)^2$
- (B) $\frac{6}{49}$
- (C) $\frac{1}{2^{14}}$
- (D) $\left(\frac{2}{14}\right)^2 \left(\frac{12}{14}\right)^{12}$
- (E) $14 \left(\left(\frac{2}{14}\right)^2 \left(\frac{12}{14}\right)^{12} \right)$

Questions 22–24

22. An idempotent 62×62 real matrix P has rank 1. What is $\text{tr } P$?

- (A) 1
- (B) 0
- (C) 3
- (D) 4
- (E) 2

23. Evaluate $\int_C (2x + 12y) dx + (12x + 2y) dy$, where C is the line segment from $(0, 0)$ to $(6, 1)$.

- (A) -109
- (B) 109
- (C) 110
- (D) 218
- (E) $\frac{109}{2}$

24. Which subset of $\mathbb{R}$ is compact, with 22 fixed as displayed?

- (A) $\mathbb{Q} \cap [0, 22]$
- (B) $\{0\} \cup \{22/k : k \geq 1\}$
- (C) $\{22/k : k \geq 1\}$
- (D) $22\mathbb{Z}$
- (E) $(0, 22)$

Questions 25–27

25. Solve $y'' - 26y' + 168y = 0$ with $y(0) = 1$, $y'(0) = 0$. Find $y(1)$.

- (A) $(7 - 6e^2)e^{12} + 1$
- (B) $\frac{(7 - 6e^2)e^{12}}{2}$
- (C) $(-7 + 6e^2)e^{12}$
- (D) $(7 - 6e^2)e^{12}$
- (E) $(14 - 12e^2)e^{12}$

26. For $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $b = \begin{bmatrix} 56 \\ 7 \\ 63 \end{bmatrix}$, let x minimize $\|Ax - b\|^2$. Find x_2 .

- (A) 8
- (B) 9
- (C) 5
- (D) 6
- (E) 7

27. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(12x) - \sum_{j=0}^0 (-1)^j (12x)^{2j+1} / (2j+1)!}{x^3}$.

- (A) $\frac{(-1)^1 12^3}{3!}$
- (B) $12 \left(\frac{(-1)^1 12^3}{3!} \right)$
- (C) $-\frac{(-1)^1 12^3}{3!}$
- (D) $\frac{1}{12} \left(\frac{(-1)^1 12^3}{3!} \right)$
- (E) $3 \left(\frac{(-1)^1 12^3}{3!} \right)$

Questions 28–30

28. For $a_k = (-1)^k + 22/k$, find $\limsup_{k \rightarrow \infty} a_k - \liminf_{k \rightarrow \infty} a_k$.

- (A) 3
- (B) 2
- (C) 0
- (D) 4
- (E) 1

29. For $D = \{(x, y) : x^2 + y^2 \leq 12\}$, evaluate $\iint_D (x^2 + y^2)^1 dA$.

- (A) 36π
- (B) 48π
- (C) 144π
- (D) 72π
- (E) 6π

30. What is the coefficient of x^2 in $(x + 11)^4$?

- (A) 727
- (B) 728
- (C) 726
- (D) 725
- (E) 724

Questions 31–33

31. Find the directional derivative of $f(x, y, z) = x^2 + 12y^2 + 2z^2$ at $(1, 1, 1)$ in the direction $(1, 6, 0)$.

- (A) $1 + \frac{146\sqrt{37}}{37}$
- (B) $\frac{292\sqrt{37}}{37}$
- (C) $-\frac{146\sqrt{37}}{37}$
- (D) $\frac{73\sqrt{37}}{37}$
- (E) $\frac{146\sqrt{37}}{37}$

32. For which real a is $f(x) = x^a$ in $L^{22}(0, 1)$?

- (A) $a < 1/22$
- (B) All real a
- (C) $a > -1/22$
- (D) $a > 0$
- (E) $a \geq -1/22$

33. For $f(x) = x^2 e^{12x}$, find $f^{(4)}(0)$.

- (A) $\frac{1}{2} \left(\frac{4! 12^2}{2} \right)$
- (B) $\frac{1}{6} \left(\frac{4! 12^2}{2} \right)$
- (C) $\frac{1}{12} \left(\frac{4! 12^2}{2} \right)$
- (D) $\frac{4! 12^2}{2}$
- (E) $2 \left(\frac{4! 12^2}{2} \right)$

Questions 34–36

34. A matrix A satisfies $(A - 57I)(A - 59I) = 0$, and its minimal polynomial has degree two. If $A^2 = uA + vI$, find u .

- (A) 114
- (B) 118
- (C) 115
- (D) 116
- (E) 117

35. Evaluate $\int_0^1 x^{12}(1-x)^1 dx$.

- (A) $\frac{12! 1!}{14!}$
- (B) $14 \left(\frac{12! 1!}{14!} \right)$
- (C) $7 \left(\frac{12! 1!}{14!} \right)$
- (D) $\frac{1}{2} \left(\frac{12! 1!}{14!} \right)$
- (E) $13 \left(\frac{12! 1!}{14!} \right)$

36. A map $T : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $|T(x) - T(y)| \leq |x - y|/22$. If $T(x_*) = x_*$ and $|x_0 - x_*| \leq 1$, what is the smallest guaranteed upper bound for $|T^3(x_0) - x_*|$?

- (A) $\frac{10647}{10648}$
- (B) $\frac{1}{10648}$
- (C) $\frac{10649}{21296}$
- (D) $\frac{1}{21296}$
- (E) $\frac{1}{31944}$

Questions 37–39

37. For $x > 0$, $xy' = 12y$ and $y(1) = 1$. Find $y(e^2)$.

- (A) $-e^{24}$
- (B) $2e^{24}$
- (C) e^{24}
- (D) $1 + e^{24}$
- (E) $\frac{e^{24}}{2}$

38. Let $V = \begin{bmatrix} 1 & 56 & 3136 \\ 1 & 58 & 3364 \\ 1 & 61 & 3721 \end{bmatrix}$. Find $\det V$.

- (A) 31
- (B) 29
- (C) 32
- (D) 28
- (E) 30

39. Find the coefficient of x^4 in $\cos(12x)/(1 - 1x^2)$.

- (A) -793
- (B) 793
- (C) 19032
- (D) 792
- (E) 864

Questions 40–42

40. Let $\zeta = e^{2\pi i/19}$. Find $\sum_{k=0}^{18} \zeta^k$.

- (A) 1
- (B) 4
- (C) 0
- (D) 2
- (E) 3

41. Let $F(x) = \int_0^{x^2} e^{12t} dt$. Find $F'(1)$.

- (A) e^{24}
- (B) $\frac{e^{12}}{2}$
- (C) $2e^{12}$
- (D) $24e^{12}$
- (E) e^{12}

42. Let $u = (1, 2, \dots, 62)^T$, $v = (1, 1, \dots, 1)^T$, and $B = I + uv^T$. Find $\det B$.

- (A) 1955
- (B) 1954
- (C) 1953
- (D) 1952
- (E) 1956

Questions 43–45

43. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 12z^2 = x + 2y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.
- (A) -24
(B) 48
(C) -96
(D) -2
(E) -48
44. An analytic function has real part $u(x, y) = x^2 - y^2 + 17x$ and satisfies $f(0) = 0$. Find $\text{Im } f(1 + i)$.
- (A) 25
(B) 13
(C) 7
(D) 31
(E) 19
45. Find the upward flux of $\mathbf{F} = (0, 0, z)$ through $z = 12x^2 + 2y^2$, $0 \leq x, y \leq 1$.
- (A) $\frac{7}{3}$
(B) $\frac{28}{3}$
(C) $\frac{14}{3}$
(D) $-\frac{14}{3}$
(E) $\frac{17}{3}$

Questions 46–48

46. Let D be a diagonal 62×62 real matrix with distinct diagonal entries. What is the dimension of the space of matrices X satisfying $XD = DX$?

- (A) 64
- (B) 62
- (C) 60
- (D) 63
- (E) 61

47. Find the arc length of $r(t) = (12t, 2t)$ for $0 \leq t \leq 1$.

- (A) $\sqrt{37}$
- (B) $-2\sqrt{37}$
- (C) $1 + 2\sqrt{37}$
- (D) $4\sqrt{37}$
- (E) $2\sqrt{37}$

48. A tree has 31 vertices. What is the sum of its vertex degrees?

- (A) 60
- (B) 100
- (C) 20
- (D) 40
- (E) 80

Questions 49–51

49. Evaluate $\int_0^{\log 2} e^x(e^x + 12)^2 dx$.

- (A) $\frac{14^3 - 13^3}{3}$
- (B) $14^2 - 13^2$
- (C) $\frac{14^3}{3}$
- (D) $3 \left(\frac{14^3 - 13^3}{3} \right)$
- (E) $-\frac{14^3 - 13^3}{3}$

50. The symmetric group S_{62} acts on $\{1, \dots, 62\}$. What is the index of the stabilizer of 1?

- (A) 61
- (B) 62
- (C) 60
- (D) 64
- (E) 63

51. If $y'' + 4y = 0$, $y(0) = 12$, and $y'(0) = 6$, find $y(\pi/4)$.

- (A) 3
- (B) 4
- (C) 6
- (D) $\frac{3}{2}$
- (E) -3

Questions 52–54

52. A connected planar graph has 30 vertices and 55 edges. How many faces, including the unbounded face, are in a planar embedding?

- (A) 9
- (B) 18
- (C) 27
- (D) 36
- (E) 45

53. Evaluate $\int_0^\infty x^2 e^{-12x} dx$.

- (A) $\frac{1}{2} \left(\frac{2!}{12^3} \right)$
- (B) $12 \left(\frac{2!}{12^3} \right)$
- (C) $\frac{2!}{12^3}$
- (D) $\frac{1}{12} \left(\frac{2!}{12^3} \right)$
- (E) $1728 \left(\frac{2!}{12^3} \right)$

54. In $\mathbb{Z}/1698\mathbb{Z}$, how many residues are idempotent ($x^2 = x$)?

- (A) 7
- (B) 6
- (C) 10
- (D) 8
- (E) 9

Questions 55–57

55. The solution of $y' = 12x^2y^2$ has $y(0) = 1/13$. Find $y(1)$.

- (A) $\frac{1}{9}$
- (B) $-\frac{1}{9}$
- (C) $\frac{2}{9}$
- (D) $\frac{1}{18}$
- (E) $\frac{10}{9}$

56. How many integers in $\{1, \dots, 162\}$ are divisible by 2 or 3?

- (A) 180
- (B) 108
- (C) 36
- (D) 72
- (E) 144

57. The region $0 \leq x \leq 1$, $0 \leq y \leq 12x^2$ is revolved about the x -axis. Find its volume.

- (A) $1 + \frac{144\pi}{5}$
- (B) $\frac{288\pi}{5}$
- (C) $\frac{144\pi}{5}$
- (D) $-\frac{144\pi}{5}$
- (E) $\frac{72\pi}{5}$

Questions 58–60

58. In the additive group $\mathbb{Z}/72\mathbb{Z}$, find the order of $[3]$.

- (A) 22
- (B) 25
- (C) 24
- (D) 23
- (E) 26

59. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(12+2j)(14+2j)}$.

- (A) $\frac{1}{12}$
- (B) $\frac{1}{14}$
- (C) $\frac{1}{2}$
- (D) $\frac{1}{24}$
- (E) $\frac{1}{6}$

60. Let $F_0 = 0$, $F_1 = 1$, and $F_{k+2} = F_{k+1} + F_k$. How many binary strings of length 27 have no adjacent 1s?

- (A) F_{28}
- (B) F_{29}
- (C) F_{30}
- (D) F_{31}
- (E) F_{27}

Questions 61–63

61. Evaluate $\iiint_B 12(x^2 + y^2 + z^2)^1 dV$, where B is the unit ball.

- (A) $\frac{96\pi}{5}$
- (B) $-\frac{48\pi}{5}$
- (C) $\frac{24\pi}{5}$
- (D) $1 + \frac{48\pi}{5}$
- (E) $\frac{48\pi}{5}$

62. How many invertible 2×2 matrices over $\mathbb{F}_{31}$ are there?

- (A) $(31^2 - 31^0)(31^2 - 31^0)$
- (B) $(31^2 - 31^0)$
- (C) $31^2 31^2$
- (D) $(31^2 - 31^0)(31^2 - 31^1)$
- (E) $(31^2 - 31^1)(31^2 - 31^2)$

63. Evaluate $\int_0^{\pi/2} \sin^5 t \cos^{25} t dt$.

- (A) $\frac{2! 12!}{2 \cdot 15!}$
- (B) $15 \left(\frac{2! 12!}{2 \cdot 15!} \right)$
- (C) $91 \left(\frac{2! 12!}{2 \cdot 15!} \right)$
- (D) $2 \left(\frac{2! 12!}{2 \cdot 15!} \right)$
- (E) $\frac{1}{2} \left(\frac{2! 12!}{2 \cdot 15!} \right)$

Questions 64–66

- 64.** Apply one Newton iteration to solve $x^2 - 27 = 0$, starting at $x_0 = 1$. What is x_1 ?
- (A) 10
 - (B) 14
 - (C) 22
 - (D) 18
 - (E) 6
- 65.** Let $f(x, y) = x^2 + 24xy + (146)y^2$. What is the product of the eigenvalues of the Hessian of f ?
- (A) 584
 - (B) 8
 - (C) 146
 - (D) 2
 - (E) 4
- 66.** How many permutations in S_{62} consist of one 3-cycle and 59 fixed points?
- (A) 75641
 - (B) 75642
 - (C) 75639
 - (D) 75638
 - (E) 75640

Answer Key

Practice Test 60 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	D	23	B	45	C
2	B	24	B	46	B
3	D	25	D	47	E
4	E	26	E	48	A
5	A	27	A	49	A
6	D	28	B	50	B
7	D	29	D	51	A
8	E	30	C	52	C
9	A	31	E	53	C
10	A	32	C	54	D
11	E	33	D	55	A
12	D	34	D	56	B
13	C	35	A	57	C
14	E	36	B	58	C
15	A	37	C	59	D
16	B	38	E	60	B
17	A	39	B	61	E
18	E	40	C	62	D
19	C	41	C	63	A
20	C	42	B	64	B
21	D	43	E	65	B
22	A	44	E	66	E

zarimion.com/gre

100 practice tests, complete answer keys, and future updates.
Please retain the source attribution when sharing this material.

