

GRE Mathematics Practice Test 61

Advanced Challenge

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

The most demanding form in this five-test cycle. Emphasis on connected ideas, non-routine reasoning, and careful conceptual distinctions. This is cycle 12 of 19, position 1 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

zarimion.com/gre

100 practice tests, complete answer keys, and future updates.

Please retain the source attribution when sharing this material.



This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Let $f(x) = \log(1 + 13x)$. Find $f^{(6)}(0)$.

- (A) $\frac{1}{13} \left((-1)^5 5! 13^6 \right)$
- (B) $6 \left((-1)^5 5! 13^6 \right)$
- (C) $\frac{1}{6} \left((-1)^5 5! 13^6 \right)$
- (D) $-(-1)^5 5! 13^6$
- (E) $(-1)^5 5! 13^6$

2. Let N be the 63×63 matrix with ones immediately above the diagonal and zeros elsewhere. Find $\dim(\ker N^{20} / \ker N^{10})$.

- (A) 8
- (B) 11
- (C) 9
- (D) 10
- (E) 12

3. The solution of $y' = 13x^5 y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\frac{13^{\frac{5}{6}} \sqrt[6]{6}}{13}$
- (B) $\frac{6}{13}$
- (C) $\frac{13^{\frac{5}{6}}}{13}$
- (D) $\frac{13^{\frac{5}{6}} \sqrt[6]{3}}{13}$
- (E) $\frac{\sqrt[6]{13} \cdot 6^{\frac{5}{6}}}{6}$

Questions 4–6

4. Independent exponential waiting times X, Y have rates 23 and 25. Find $E[\min(X, Y)]$.

- (A) $\frac{1}{48}$
- (B) $\frac{47}{48}$
- (C) $\frac{49}{96}$
- (D) $\frac{1}{144}$
- (E) $\frac{1}{96}$

5. If $y'' - 26y' + 169y = 6e^{13x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) $-3e^{13}$
- (B) $6e^{13}$
- (C) $1 + 3e^{13}$
- (D) $3e^{13}$
- (E) $\frac{3e^{13}}{2}$

6. Let A be the 63×63 matrix whose diagonal entries are 60 and whose off-diagonal entries are 1. What is its largest eigenvalue?

- (A) 120
- (B) 123
- (C) 122
- (D) 121
- (E) 124

Questions 7–9

7. Evaluate $\int_0^1 x^{13}(1-x)^5 dx$.

- (A) $1938 \left(\frac{13! 5!}{19!} \right)$
- (B) $14 \left(\frac{13! 5!}{19!} \right)$
- (C) $\frac{13! 5!}{19!}$
- (D) $19 \left(\frac{13! 5!}{19!} \right)$
- (E) $\frac{1}{6} \left(\frac{13! 5!}{19!} \right)$

8. $X_1, \dots, X_{23}$ are independent with common mean and variance 25. With $\bar{X}$ their sample mean, find $E[\sum_i (X_i - \bar{X})^2]$.

- (A) 550
- (B) 184
- (C) 916
- (D) 367
- (E) 733

9. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{13t} \cos(6t)$, and $y = e^{13t} \sin(6t)$, find $d^2 f / dt^2$ at $t = 1$.

- (A) 26
- (B) 12
- (C) 0
- (D) 676
- (E) 410

Questions 10–12

10. Positive real numbers x, y satisfy $\log_2 x + \log_2 y = \log_2 570$ and $x + y = 67$. Find $x^2 + y^2$.

- (A) 3347
- (B) 3350
- (C) 3349
- (D) 3351
- (E) 3348

11. The region $0 \leq x \leq 1, 0 \leq y \leq 13x^6$ is revolved about the line $x = -2$. Find its volume.

- (A) $1 + \frac{299\pi}{28}$
- (B) $\frac{299\pi}{28}$
- (C) $\frac{299\pi}{14}$
- (D) $-\frac{299\pi}{28}$
- (E) $\frac{299\pi}{56}$

12. A permutation of 25 objects is chosen uniformly. Let Z be the number of fixed points. Find $E[Z^2]$.

- (A) 4
- (B) 0
- (C) 3
- (D) 1
- (E) 2

Questions 13–15

13. Evaluate $\int_0^\infty x^6 e^{-13x} dx$.

- (A) 720
- (B) $\frac{1}{6} \left(\frac{6!}{13^7} \right)$
- (C) $\frac{1}{13} \left(\frac{6!}{13^7} \right)$
- (D) $13 \left(\frac{6!}{13^7} \right)$
- (E) $\frac{6!}{13^7}$

14. A sequence satisfies $u_0 = 57$ and $u_{k+1} = 2u_k + 10$ for all $k \geq 0$. Find u_9 .

- (A) 34293
- (B) 34295
- (C) 34296
- (D) 34292
- (E) 34294

15. Let $\mathbf{F} = (13y^6, z^6, x^6)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (1, 1, 2)/\sqrt{6}$.

- (A) $-56\sqrt{6}$
- (B) $-28\sqrt{6}$
- (C) $1 - 28\sqrt{6}$
- (D) $-14\sqrt{6}$
- (E) $28\sqrt{6}$

Questions 16–18

16. A coin success probability p has uniform prior on $[0, 1]$. After observing 23 successes and 2 failures in independent tosses conditional on p , what is the predictive probability of success on the next toss?

- (A) $\frac{8}{27}$
- (B) $\frac{1}{9}$
- (C) $\frac{4}{9}$
- (D) $\frac{8}{9}$
- (E) $\frac{17}{18}$

17. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^{13}y^5 dV$.

- (A) $\frac{1}{84}$
- (B) $6 \left(\frac{13!5!}{21!} \right)$
- (C) $\frac{13!5!}{21!}$
- (D) $\frac{1}{6} \left(\frac{13!5!}{21!} \right)$
- (E) $21 \left(\frac{13!5!}{21!} \right)$

18. Linear maps $T : \mathbb{R}^{64} \rightarrow \mathbb{R}^{64}$ and $U : \mathbb{R}^{64} \rightarrow \mathbb{R}^{64}$ have ranks 5 and 4. What is the largest possible rank of UT ?

- (A) 3
- (B) 5
- (C) 6
- (D) 4
- (E) 2

Questions 19–21

19. Find the curvature of the helix $r(t) = (13 \cos t, 13 \sin t, 6t)$.

- (A) $\frac{13}{410}$
- (B) $-\frac{13}{205}$
- (C) $\frac{26}{205}$
- (D) $\frac{13}{205}$
- (E) $\frac{218}{205}$

20. A random variable has mean 0 and variance 23. What upper bound does the Chebyshev inequality give for $P(|X| \geq 46)$?

- (A) $\frac{93}{184}$
- (B) $\frac{1}{276}$
- (C) $\frac{1}{92}$
- (D) $\frac{1}{184}$
- (E) $\frac{91}{92}$

21. Evaluate $\iiint_B 13z^2(x^2 + y^2 + z^2)^6 dV$, where B is the unit ball.

- (A) $-\frac{52\pi}{51}$
- (B) $\frac{26\pi}{51}$
- (C) $1 + \frac{52\pi}{51}$
- (D) $\frac{104\pi}{51}$
- (E) $\frac{52\pi}{51}$

Questions 22–24

22. Fix a nonzero vector $u \in \mathbb{R}^{63}$. What is the dimension of the space of skew-symmetric 63×63 matrices A satisfying $Au = 0$?

- (A) 1890
- (B) 1891
- (C) 1892
- (D) 1889
- (E) 1893

23. For $\mathbf{F} = (13x^6y, y^6z, xz^6)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) 355
- (B) 354
- (C) -354
- (D) 177
- (E) 708

24. A continuous $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(q) = q^{18}$ for every rational q . Find $f(2)$.

- (A) 2^{21}
- (B) 2^{18}
- (C) 2^{19}
- (D) 2^{17}
- (E) 2^{20}

Questions 25–27

25. Let $F(x) = \int_0^{x^6} e^{13t} dt$. Find $F'(1)$.

- (A) $6e^{13}$
- (B) $\frac{e^{13}}{6}$
- (C) e^{78}
- (D) e^{13}
- (E) $78e^{13}$

26. Let $A = \begin{bmatrix} 1 & 57 & 0 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{bmatrix}$. What is the $(1, 3)$ entry of A^4 ?

- (A) 3422
- (B) 3419
- (C) 3418
- (D) 3421
- (E) 3420

27. Find the arc length of $r(t) = (13(t - t^3/3), 13t^2)$ for $0 \leq t \leq 1/6$.

- (A) $-\frac{1417}{648}$
- (B) $\frac{2065}{648}$
- (C) $\frac{1417}{1296}$
- (D) $\frac{1417}{324}$
- (E) $\frac{1417}{648}$

Questions 28–30

28. Find the radius of convergence of $\sum_{k \geq 1} k^2 z^{2k} / 18^{2k} + \sum_{k \geq 0} z^{2k+1} / (19)^{2k+1}$.

- (A) 18
- (B) 6
- (C) 24
- (D) 12
- (E) 30

29. For $D = \{(x, y) : x^2 + y^2 \leq 13\}$, evaluate $\iint_D (x^2 + y^2)^5 dA$.

- (A) $\frac{4826809\pi}{6}$
- (B) $\frac{371293\pi}{6}$
- (C) $\frac{4826809\pi}{3}$
- (D) $\frac{4826809\pi}{11}$
- (E) 4826809π

30. An invertible 63×63 matrix A has determinant 3. Find $\det(\text{adj } A)$.

- (A) 3^{62}
- (B) 3^{61}
- (C) $2 \cdot 3^{62}$
- (D) 3^{63}
- (E) $3^{62} \cdot 5$

Questions 31–33

31. What is the maximum of x^6y^{13} over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\frac{78}{361}$
- (B) $\left(\frac{6}{19}\right)^{13} \left(\frac{13}{19}\right)^6$
- (C) $\frac{1}{2^{19}}$
- (D) $\left(\frac{6}{19}\right)^6 \left(\frac{13}{19}\right)^{13}$
- (E) $19 \left(\left(\frac{6}{19}\right)^6 \left(\frac{13}{19}\right)^{13} \right)$

32. On $[0, 1]$, f is 18 on rationals and $1/18$ on irrationals. Find its upper Darboux integral minus its lower Darboux integral.

- (A) $\frac{341}{18}$
- (B) $\frac{359}{18}$
- (C) $\frac{323}{36}$
- (D) $\frac{323}{18}$
- (E) $\frac{18}{323}$

33. Find the coefficient of x^4 in $e^{5x} \log(1 + 13x)$.

- (A) -4264
- (B) $-\frac{28561}{4}$
- (C) -4251
- (D) -102336
- (E) 4264

Questions 34–36

34. On P_{64} , the real polynomials of degree at most 64, define $T(p) = x^2p'' - 20xp' + 110p$. Find its rank.

- (A) 64
- (B) 62
- (C) 63
- (D) 65
- (E) 61

35. For $x > 0$, $x^2y'' + (-31)xy' + 247y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

- (A) $\frac{(1 - e^6)e^{13}}{6}$
- (B) $-\frac{e^{13}}{6} + 1 + \frac{e^{19}}{6}$
- (C) $-\frac{(1 - e^6)e^{13}}{6}$
- (D) $-\frac{(1 - e^6)e^{13}}{12}$
- (E) $-\frac{(1 - e^6)e^{13}}{3}$

36. A set with 18 elements has the discrete topology. How many of its subsets are simultaneously open and closed?

- (A) 2^{17}
- (B) 2^{18}
- (C) 2^{20}
- (D) 2^{19}
- (E) 2^{21}

Questions 37–39

- 37.** Find the coefficient of x^6 in the Taylor series of $(1 - 13x)^{-2}$ at zero.
- (A) $7 \cdot 13^6$
 - (B) $\frac{6}{7} (7 \cdot 13^6)$
 - (C) $2 (7 \cdot 13^6)$
 - (D) $\frac{1}{13} (7 \cdot 13^6)$
 - (E) $\frac{1}{7} (7 \cdot 13^6)$
- 38.** Subspaces U, V of $\mathbb{R}^{65}$ have dimensions 6 and 7. What is the smallest possible value of $\dim(U \cap V)$?
- (A) 4
 - (B) 1
 - (C) 0
 - (D) 3
 - (E) 2
- 39.** Under $x = u + 13v$, $y = u - 6v$, what is the area of the image of $0 \leq u, v \leq 1$?
- (A) 78
 - (B) $\frac{1}{19}$
 - (C) 361
 - (D) 19
 - (E) 7

Questions 40–42

40. Find the coefficient of z^{-1} in $e^{13z}/[z^2(1-z)]$ for $0 < |z| < 1$.

- (A) 18
- (B) 10
- (C) 22
- (D) 6
- (E) 14

41. For $f(x) = x^6 e^{13x}$, find $f^{(8)}(0)$.

- (A) $2 \left(\frac{8! 13^2}{2} \right)$
- (B) $\frac{8! 13^2}{2}$
- (C) $\frac{2}{13} \left(\frac{8! 13^2}{2} \right)$
- (D) $\frac{1}{56} \left(\frac{8! 13^2}{2} \right)$
- (E) $\frac{1}{20160} \left(\frac{8! 13^2}{2} \right)$

42. What is the largest integer k such that 3^k divides $66!$?

- (A) 33
- (B) 31
- (C) 30
- (D) 32
- (E) 29

Questions 43–45

43. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(13+6j)(19+6j)}$.

- (A) $\frac{1}{13}$
- (B) $\frac{1}{78}$
- (C) $\frac{1}{6}$
- (D) $\frac{6}{13}$
- (E) $\frac{1}{19}$

44. An entire function satisfies $|f(z)| \leq C(1 + |z|^{13})$ for some $C > 0$. What is the largest possible degree of f ?

- (A) 9
- (B) 17
- (C) 5
- (D) 21
- (E) 13

45. Find the total outward flux of $\mathbf{F} = (13x^6y, y^6z, xz^6)$ through the boundary of $[0, 1]^3$.

- (A) 15
- (B) $\frac{17}{2}$
- (C) $-\frac{15}{2}$
- (D) $\frac{15}{4}$
- (E) $\frac{15}{2}$

Questions 46–48

46. In the ring $\mathbb{Z}/366\mathbb{Z}$, how many elements satisfy $x^2 = 1$?

- (A) 5
- (B) 6
- (C) 4
- (D) 2
- (E) 3

47. Evaluate $\int_0^{\pi/2} \sin^{13} t \cos^{27} t \, dt$.

- (A) $2 \left(\frac{6! 13!}{2 \cdot 20!} \right)$
- (B) $27132 \left(\frac{6! 13!}{2 \cdot 20!} \right)$
- (C) $\frac{6! 13!}{2 \cdot 20!}$
- (D) $20 \left(\frac{6! 13!}{2 \cdot 20!} \right)$
- (E) $\frac{1}{2} \left(\frac{6! 13!}{2 \cdot 20!} \right)$

48. Bisection starts with an interval of length 2^{23} . After 25 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{1}{8}$
- (B) $\frac{7}{8}$
- (C) $\frac{1}{16}$
- (D) $\frac{1}{24}$
- (E) $\frac{9}{16}$

Questions 49–51

49. Evaluate $\int_0^1 \frac{x^5}{1 + 13x^6} dx$.

- (A) $\frac{\log(14)}{13}$
- (B) $\log(14)$
- (C) $-\frac{\log(14)}{78}$
- (D) $\frac{\log(14)}{78}$
- (E) $\frac{\log(14)}{6}$

50. A permutation consists of disjoint cycles of lengths 4 and 5, together with a disjoint transposition when the sum of these two lengths is odd. It fixes all other points. What is its order?

- (A) 19
- (B) 21
- (C) 22
- (D) 18
- (E) 20

51. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-13)^j}{j^{\frac{5}{3}} (\log j)^m}$ converge absolutely at $x = 12$? Give the least such m , or 0 if none exists.

- (A) 0
- (B) 4
- (C) 1
- (D) 3
- (E) 2

Questions 52–54

52. What is the least positive multiplicative inverse of 2 modulo 47?

- (A) 32
- (B) 24
- (C) 8
- (D) 40
- (E) 16

53. Evaluate $\int_0^1 \int_0^x x^{13} y^6 dy dx$.

- (A) $\frac{1}{21}$
- (B) $\frac{1}{147}$
- (C) $\frac{1}{140}$
- (D) $\frac{1}{98}$
- (E) $\frac{1}{168}$

54. In $R = \mathbb{F}_{11}[x]/(x^{13})$, how many elements are units?

- (A) $11^{15} - 11^{14}$
- (B) $11^{14} - 11^{12}$
- (C) $11^{13} - 11^0$
- (D) $11^{13} - 11^{12}$
- (E) $11^{14} - 11^{13}$

Questions 55–57

55. Evaluate $\lim_{x \rightarrow 0} \frac{e^{13x} - \sum_{j=0}^5 (13x)^j / j!}{x^6}$.

- (A) $\frac{13^6}{6!}$
- (B) $\frac{1}{13} \left(\frac{13^6}{6!} \right)$
- (C) $720 \left(\frac{13^6}{6!} \right)$
- (D) $6 \left(\frac{13^6}{6!} \right)$
- (E) $-\frac{13^6}{6!}$

56. How many truth assignments to $p, q, r_1, \dots, r_{21}$ make $p \Rightarrow q$ false?

- (A) 2^{23}
- (B) 2^{21}
- (C) 2^{22}
- (D) 2^{24}
- (E) 2^{20}

57. The local inverse $y(x)$ of $x = 13y + y^3$ satisfies $y(0) = 0$. Find $y^{(7)}(0)$.

- (A) $13 \left(-\frac{60480}{13^{10}} \right)$
- (B) $-\frac{60480}{13^{10}}$
- (C) $\frac{1}{5040} \left(-\frac{60480}{13^{10}} \right)$
- (D) $\frac{1}{13} \left(-\frac{60480}{13^{10}} \right)$
- (E) $\frac{60480}{13^{10}}$

Questions 58–60

58. Integers x satisfy $x \equiv 2 \pmod{7}$ and $x \equiv 4 \pmod{9}$. Find the sum of all such x with $0 \leq x < 126$.

- (A) 181
- (B) 177
- (C) 178
- (D) 179
- (E) 180

59. Find the upward flux of $\mathbf{F} = (x, y, z)$ through $z = 13x^2 + 6y^2$, $0 \leq x, y \leq 1$.

- (A) $-\frac{19}{3}$
- (B) $-\frac{38}{3}$
- (C) $-\frac{16}{3}$
- (D) $-\frac{19}{6}$
- (E) $\frac{19}{3}$

60. Find the coefficient of x^3 in $(1+x)^{23}(1-x)^{23}(1+2x)^2$.

- (A) -32
- (B) -122
- (C) -62
- (D) -92
- (E) -152

Questions 61–63

61. Evaluate $\int_0^{\log 2} e^x(e^x + 13)^6 dx$.

- (A) $\frac{15^7}{7}$
- (B) $7 \left(\frac{15^7 - 14^7}{7} \right)$
- (C) $-\frac{15^7 - 14^7}{7}$
- (D) $15^6 - 14^6$
- (E) $\frac{15^7 - 14^7}{7}$

62. The symmetric group S_{63} acts on the two-element subsets of $\{1, \dots, 63\}$. What is the index of the setwise stabilizer of $\{1, 2\}$?

- (A) 1953
- (B) 1955
- (C) 1954
- (D) 1952
- (E) 1951

63. If $y' = y(1 - y/6)$ and $y(0) = 3$, find $y(-\log 13)$.

- (A) $\frac{6}{7}$
- (B) $\frac{3}{14}$
- (C) $\frac{3}{7}$
- (D) $\frac{10}{7}$
- (E) $-\frac{3}{7}$

Questions 64–66

64. How many distinct unoriented cycles of length 4 are in $K_{23,25}$?

- (A) 101200
- (B) 50600
- (C) 75900
- (D) 25300
- (E) 126500

65. Let $f(x, y) = x^2 + 26xy + (175)y^2$. What is the product of the eigenvalues of the Hessian of f ?

- (A) 12
- (B) 700
- (C) 175
- (D) 24
- (E) 6

66. In $\mathbb{Z}/1758\mathbb{Z}$, how many residues are not idempotent ($x^2 \neq x$)?

- (A) 1748
- (B) 1752
- (C) 1751
- (D) 1749
- (E) 1750

Answer Key

Practice Test 61 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	E	23	B	45	E
2	D	24	B	46	C
3	A	25	A	47	C
4	A	26	E	48	A
5	D	27	E	49	D
6	C	28	A	50	E
7	C	29	A	51	C
8	A	30	A	52	B
9	C	31	D	53	B
10	C	32	D	54	D
11	B	33	A	55	A
12	E	34	C	56	B
13	E	35	C	57	B
14	E	36	B	58	D
15	B	37	A	59	A
16	D	38	C	60	D
17	C	39	D	61	E
18	D	40	E	62	A
19	D	41	B	63	C
20	C	42	B	64	C
21	E	43	B	65	D
22	B	44	E	66	E

zarimion.com/gre

100 practice tests, complete answer keys, and future updates.
Please retain the source attribution when sharing this material.

