

GRE Mathematics Practice Test 62

Advanced Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A demanding practice form with frequent multi-step arguments and a mix of computational and conceptual questions. This is cycle 12 of 19, position 2 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. If $y' = 26xy$ and $y(0) = 1$, find the coefficient of x^{10} in the Maclaurin series of y .
- (A) $-\frac{371293}{120}$
(B) $\frac{371293}{120}$
(C) $\frac{371293}{240}$
(D) $\frac{371293}{60}$
(E) $\frac{371413}{120}$
2. Fix a nonzero vector $u \in \mathbb{R}^{64}$. What is the dimension of the space of skew-symmetric 64×64 matrices A satisfying $Au = 0$?
- (A) 1952
(B) 1954
(C) 1955
(D) 1953
(E) 1951
3. Evaluate $\int_C (5x^4 + 13y) dx + (13x + 5y^4) dy$, where C is $r(t) = (t, 3t^2)$, $0 \leq t \leq 1$.
- (A) 284
(B) $\frac{283}{2}$
(C) 283
(D) -283
(E) 566

Questions 4–6

4. Independent exponential waiting times X, Y have rates 24 and 26. Find $P(X < Y)$.

- (A) $\frac{37}{50}$
- (B) $\frac{12}{25}$
- (C) $\frac{6}{25}$
- (D) $\frac{4}{25}$
- (E) $\frac{13}{25}$

5. Let $F(x) = \int_0^{x^5} e^{13t} dt$. Find $F'(1)$.

- (A) $65e^{13}$
- (B) e^{13}
- (C) $5e^{13}$
- (D) $\frac{e^{13}}{5}$
- (E) e^{65}

6. Let $A = \begin{bmatrix} 1 & 58 & 0 \\ 0 & 1 & 8 \\ 0 & 0 & 1 \end{bmatrix}$. What is the $(1, 3)$ entry of A^5 ?

- (A) 4638
- (B) 4641
- (C) 4639
- (D) 4642
- (E) 4640

Questions 7–9

7. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(13+5j)(18+5j)}$.

- (A) $\frac{5}{13}$
- (B) $\frac{1}{5}$
- (C) $\frac{1}{65}$
- (D) $\frac{1}{13}$
- (E) $\frac{1}{18}$

8. $X_1, \dots, X_{24}$ are independent with common mean and variance 26. With $\bar{X}$ their sample mean, find $E[\sum_i (X_i - \bar{X})^2]$.

- (A) 399
- (B) 797
- (C) 200
- (D) 996
- (E) 598

9. Find the curvature of the helix $r(t) = (13 \cos t, 13 \sin t, 5t)$.

- (A) $\frac{13}{388}$
- (B) $\frac{13}{97}$
- (C) $\frac{207}{194}$
- (D) $-\frac{13}{194}$
- (E) $\frac{13}{194}$

Questions 10–12

10. An invertible 64×64 matrix A has determinant 4. Find $\det(\text{adj } A)$.

- (A) $2^{126} \cdot 5$
- (B) 2^{127}
- (C) $2^{126} \cdot 3$
- (D) 2^{126}
- (E) 2^{125}

11. The solution of $y' = 13x^5y^3$ has $y(0) = 1/14$. Find $y(1)$.

- (A) $-\frac{\sqrt{69}}{115}$
- (B) $\frac{\sqrt{69}}{115}$
- (C) $\frac{\sqrt{69}}{230}$
- (D) $\frac{2\sqrt{69}}{115}$
- (E) $\frac{\sqrt{69}}{115} + 1$

12. A permutation of 26 objects is chosen uniformly. Let Z be the number of fixed points. Find $E[Z^2]$.

- (A) 4
- (B) 1
- (C) 3
- (D) 2
- (E) 0

Questions 13–15

13. For $f(x, y) = x^2 + 13xy + 5y^2$, what is its maximum directional derivative at $(1, -1)$?
- (A) 3
(B) 14
(C) 130
(D) $\sqrt{130}$
(E) -11
14. On P_{65} , the real polynomials of degree at most 65, define $T(p) = x^2p'' - 20xp' + 110p$. Find its rank.
- (A) 62
(B) 66
(C) 64
(D) 63
(E) 65
15. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(13x) - \sum_{j=0}^3 (-1)^j (13x)^{2j+1} / (2j+1)!}{x^9}$.
- (A) $\frac{1}{13} \left(\frac{(-1)^4 13^9}{9!} \right)$
(B) $9 \left(\frac{(-1)^4 13^9}{9!} \right)$
(C) $\frac{(-1)^4 13^9}{9!}$
(D) $-\frac{(-1)^4 13^9}{9!}$
(E) $13 \left(\frac{(-1)^4 13^9}{9!} \right)$

Questions 16–18

16. A coin success probability p has uniform prior on $[0, 1]$. After observing 24 successes and 2 failures in independent tosses conditional on p , what is the predictive probability of success on the next toss?

- (A) $\frac{25}{28}$
- (B) $\frac{53}{56}$
- (C) $\frac{25}{56}$
- (D) $\frac{3}{28}$
- (E) $\frac{25}{84}$

17. Let $f(x) = \log(1 + 13x)$. Find $f^{(5)}(0)$.

- (A) $(-1)^4 4! 13^5$
- (B) $\frac{1}{13} \left((-1)^4 4! 13^5 \right)$
- (C) $\frac{1}{5} \left((-1)^4 4! 13^5 \right)$
- (D) $-(-1)^4 4! 13^5$
- (E) $5 \left((-1)^4 4! 13^5 \right)$

18. Subspaces U, V of $\mathbb{R}^{66}$ have dimensions 6 and 7. What is the smallest possible value of $\dim(U \cap V)$?

- (A) 3
- (B) 4
- (C) 1
- (D) 0
- (E) 2

Questions 19–21

19. If $y' + 13y = 5x^5e^{-13x}$ and $y(0) = 3$, find $y(1)$.

- (A) $\frac{23}{12e^{13}}$
- (B) $\frac{23}{3e^{13}}$
- (C) $\frac{23}{6e^{13}}$
- (D) $-\frac{23}{6e^{13}}$
- (E) $\frac{23}{6e^{13}} + 1$

20. A random variable has mean 0 and variance 24. What upper bound does the Chebyshev inequality give for $P(|X| \geq 48)$?

- (A) $\frac{1}{192}$
- (B) $\frac{1}{288}$
- (C) $\frac{95}{96}$
- (D) $\frac{1}{96}$
- (E) $\frac{97}{192}$

21. The solution of $y' = 13x^4y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\frac{13^{\frac{4}{5}}\sqrt[5]{5}}{13}$
- (B) $\frac{\sqrt[5]{13} \cdot 5^{\frac{4}{5}}}{5}$
- (C) $\frac{26^{\frac{4}{5}}\sqrt[5]{5}}{26}$
- (D) $\frac{13^{\frac{4}{5}}}{13}$
- (E) $\frac{13}{5}$

Questions 22–24

22. Let V consist of real polynomials of degree at most 13 that have both value and first derivative zero at 58, 59, and 61. Find $\dim V$.

- (A) 7
- (B) 10
- (C) 8
- (D) 6
- (E) 9

23. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 13x$ and has density $\rho(x, y) = x^3y$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{12}{7}$
- (B) $\frac{6}{7}$
- (C) $-\frac{6}{7}$
- (D) $\frac{3}{7}$
- (E) $\frac{13}{7}$

24. A continuous $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(q) = q^{19}$ for every rational q . Find $f(2)$.

- (A) 2^{20}
- (B) 2^{19}
- (C) 2^{22}
- (D) 2^{21}
- (E) 2^{18}

Questions 25–27

25. For $f(x) = x^5 e^{13x}$, find $f^{(7)}(0)$.

- (A) $\frac{1}{2520} \left(\frac{7! 13^2}{2} \right)$
- (B) $\frac{1}{42} \left(\frac{7! 13^2}{2} \right)$
- (C) $\frac{2}{13} \left(\frac{7! 13^2}{2} \right)$
- (D) $\frac{7! 13^2}{2}$
- (E) $2 \left(\frac{7! 13^2}{2} \right)$

26. The roots r, s of $x^2 - 66x + 464 = 0$ are counted with multiplicity. Find $r^3 + s^3$.

- (A) 195625
- (B) 195624
- (C) 195623
- (D) 195626
- (E) 195622

27. What is the maximum of $x^5 y^{13}$ over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\left(\frac{5}{18} \right)^{13} \left(\frac{13}{18} \right)^5$
- (B) $18 \left(\left(\frac{5}{18} \right)^5 \left(\frac{13}{18} \right)^{13} \right)$
- (C) $\frac{1}{2^{18}}$
- (D) $\frac{65}{324}$
- (E) $\left(\frac{5}{18} \right)^5 \left(\frac{13}{18} \right)^{13}$

Questions 28–30

28. Find the radius of convergence of $\sum_{k \geq 1} k^2 z^{2k} / 19^{2k} + \sum_{k \geq 0} z^{2k+1} / (20)^{2k+1}$.

- (A) 19
- (B) 13
- (C) 7
- (D) 25
- (E) 31

29. If $y' = y(1 - y/5)$ and $y(0) = 5/2$, find $y(-\log 13)$.

- (A) $-\frac{5}{14}$
- (B) $\frac{19}{14}$
- (C) $\frac{5}{7}$
- (D) $\frac{5}{28}$
- (E) $\frac{5}{14}$

30. For real t , let $M(t) = \begin{pmatrix} t-58 & 8 & 0 \\ 0 & t-58 & 1 \\ 0 & 0 & t-58 \end{pmatrix}$. At $t = 58$, what is $\dim \ker(M(t)^2) - \dim \ker M(t)$?

- (A) 4
- (B) 0
- (C) 1
- (D) 2
- (E) 3

Questions 31–33

31. If $y'' - 26y' + 169y = 5e^{13x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) $1 + \frac{5e^{13}}{2}$
- (B) $\frac{5e^{13}}{2}$
- (C) $\frac{5e^{13}}{4}$
- (D) $-\frac{5e^{13}}{2}$
- (E) $5e^{13}$

32. On $[0, 1]$, f is 19 on rationals and $1/19$ on irrationals. Find its upper Darboux integral minus its lower Darboux integral.

- (A) $\frac{180}{19}$
- (B) $\frac{360}{19}$
- (C) $\frac{398}{19}$
- (D) $\frac{720}{19}$
- (E) $\frac{379}{19}$

33. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-13)^j}{j^{\frac{4}{3}} (\log j)^m}$ converge absolutely at $x = 12$? Give the least such m , or 0 if none exists.

- (A) 1
- (B) 2
- (C) 3
- (D) 4
- (E) 0

Questions 34–36

34. Let $u = (12, 5)^T$ and $v = (1, 0)^T$. Find the squared distance from $(12, 0)^T$ to $\text{span}(u)$.
- (A) $\frac{3431}{169}$
(B) $\frac{7031}{338}$
(C) $\frac{3769}{169}$
(D) $\frac{7369}{338}$
(E) $\frac{3600}{169}$
35. Find the arc length of $r(t) = (13(t - t^3/3), 13t^2)$ for $0 \leq t \leq 1/5$.
- (A) $\frac{1976}{375}$
(B) $\frac{988}{375}$
(C) $-\frac{988}{375}$
(D) $\frac{494}{375}$
(E) $\frac{1363}{375}$
36. A set with 19 elements has the discrete topology. How many of its subsets are simultaneously open and closed?
- (A) 2^{18}
(B) 2^{21}
(C) 2^{19}
(D) 2^{22}
(E) 2^{20}

Questions 37–39

37. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{13t} \cos(5t)$, and $y = e^{13t} \sin(5t)$, find $d^2 f/dt^2$ at $t = 1$.

- (A) 26
- (B) 0
- (C) 388
- (D) 676
- (E) 10

38. An idempotent 64×64 real matrix P has rank 4. What is $\det(I + P)$?

- (A) 14
- (B) 15
- (C) 17
- (D) 16
- (E) 18

39. Evaluate $\iiint_{x^2+y^2+z^2 \leq 13} (x^2 + y^2 + z^2)^4 dV$.

- (A) $11 \left(\frac{4\pi 13^{\frac{11}{2}}}{11} \right)$
- (B) $\frac{11}{117} \left(\frac{4\pi 13^{\frac{11}{2}}}{11} \right)$
- (C) $\frac{1}{3} \left(\frac{4\pi 13^{\frac{11}{2}}}{11} \right)$
- (D) $\frac{1}{2} \left(\frac{4\pi 13^{\frac{11}{2}}}{11} \right)$
- (E) $\frac{4\pi 13^{\frac{11}{2}}}{11}$

Questions 40–42

40. What is the coefficient of z^{-1} in the Laurent expansion of e^{14z}/z^3 about zero?
- (A) 66
(B) 130
(C) 34
(D) 162
(E) 98
41. Find the directional derivative of $f(x, y, z) = x^2y + 5xz^2 + 13y^2z$ at $(1, 1, 1)$ in the direction $(1, 3, 0)$.
- (A) $\frac{44\sqrt{10}}{5}$
(B) $\frac{88\sqrt{10}}{5}$
(C) $\frac{22\sqrt{10}}{5}$
(D) $1 + \frac{44\sqrt{10}}{5}$
(E) $-\frac{44\sqrt{10}}{5}$
42. In the additive group $\mathbb{Z}/90\mathbb{Z}$, find the index of the subgroup generated by $[3]$.
- (A) 5
(B) 1
(C) 2
(D) 4
(E) 3

Questions 43–45

43. For $D = \{(x, y) : x^2 + y^2 \leq 13\}$, evaluate $\iint_D (x^2 + y^2)^4 dA$.

- (A) $\frac{742586\pi}{5}$
- (B) 371293π
- (C) $\frac{28561\pi}{5}$
- (D) $\frac{371293\pi}{5}$
- (E) $\frac{371293\pi}{9}$

44. An entire function satisfies $|f(z)| \leq C(1 + |z|^{14})$ for some $C > 0$. What is the largest possible degree of f ?

- (A) 6
- (B) 14
- (C) 10
- (D) 18
- (E) 22

45. If $y'' + 25y = 13$ and $y(0) = y'(0) = 0$, find $y(\pi/5)$.

- (A) $\frac{26}{25}$
- (B) $-\frac{26}{25}$
- (C) $\frac{51}{25}$
- (D) $\frac{13}{25}$
- (E) $\frac{52}{25}$

Questions 46–48

46. How many invertible 3×3 matrices over $\mathbb{F}_{37}$ are there with determinant 1?

- (A) $\frac{(37^3 - 37^0)(37^3 - 37^1)}{36}$
(B) $\frac{(37^3 - 37^0)(37^3 - 37^0)(37^3 - 37^0)}{36}$
(C) $\frac{(37^3 - 37^0)(37^3 - 37^1)(37^3 - 37^2)}{36}$
(D) $\frac{(37^3 - 37^1)(37^3 - 37^2)(37^3 - 37^3)}{36}$
(E) $\frac{37^3 37^3 37^3}{36}$

47. Evaluate $\lim_{x \rightarrow 0} \frac{e^{13x} - \sum_{j=0}^4 (13x)^j / j!}{x^5}$.

- (A) $\frac{13^5}{5!}$
(B) $5 \left(\frac{13^5}{5!} \right)$
(C) $-\frac{13^5}{5!}$
(D) $\frac{1}{13} \left(\frac{13^5}{5!} \right)$
(E) $120 \left(\frac{13^5}{5!} \right)$

48. Bisection starts with an interval of length 2^{24} . After 26 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{9}{16}$
(B) $\frac{7}{8}$
(C) $\frac{1}{16}$
(D) $\frac{1}{8}$
(E) $\frac{1}{24}$

Questions 49–51

49. Find the coefficient of x^4 in $e^{4x} \log(1 + 13x)$.

- (A) -113958
- (B) $-\frac{18993}{4}$
- (C) $-\frac{18941}{4}$
- (D) $-\frac{28561}{4}$
- (E) $\frac{18993}{4}$

50. How many permutations in S_{64} consist of one 6-cycle and 58 fixed points?

- (A) $2^7 \cdot 3^2 \cdot 5 \cdot 7 \cdot 31 \cdot 59 \cdot 61$
- (B) $2^8 \cdot 3^3 \cdot 5 \cdot 7 \cdot 31 \cdot 59 \cdot 61$
- (C) $2^9 \cdot 3^2 \cdot 5 \cdot 7 \cdot 31 \cdot 59 \cdot 61$
- (D) $2^8 \cdot 3^2 \cdot 5^2 \cdot 7 \cdot 31 \cdot 59 \cdot 61$
- (E) $2^8 \cdot 3^2 \cdot 5 \cdot 7 \cdot 31 \cdot 59 \cdot 61$

51. Solve $y' + 13y = x^5 e^{-13x}$, $y(0) = 0$, and find $y(1)$.

- (A) $\frac{1}{6e^{13}}$
- (B) e^{-13}
- (C) $\frac{e^{13}}{6}$
- (D) $\frac{1}{5e^{13}}$
- (E) $\frac{13}{6e^{13}}$

Questions 52–54

52. What is the least positive multiplicative inverse of 2 modulo 49?

- (A) 17
- (B) 9
- (C) 33
- (D) 41
- (E) 25

53. Let C be $x^2/25 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C \left(-\frac{13y^3}{3}\right) dx + \left(\frac{x^3}{3}\right) dy$.

- (A) $-\frac{95\pi}{2}$
- (B) 95π
- (C) $\frac{95\pi}{4}$
- (D) $1 + \frac{95\pi}{2}$
- (E) $\frac{95\pi}{2}$

54. How many maximal proper subgroups does the cyclic group of order 744 have?

- (A) 2
- (B) 3
- (C) 4
- (D) 1
- (E) 5

Questions 55–57

55. Evaluate $\int_0^1 \int_0^x x^{13} y^5 dy dx$.

- (A) $\frac{1}{140}$
- (B) $\frac{1}{120}$
- (C) $\frac{1}{84}$
- (D) $\frac{1}{20}$
- (E) $\frac{1}{114}$

56. How many truth assignments to $p, q, r_1, \dots, r_{22}$ make $p \Rightarrow q$ false?

- (A) 2^{21}
- (B) 2^{25}
- (C) 2^{23}
- (D) 2^{22}
- (E) 2^{24}

57. Let $F(x) = \int_0^x (x-t)^4 e^{13t} dt$. Find $F^{(5)}(0)$.

- (A) 312
- (B) 0
- (C) 24
- (D) $\frac{1}{24}$
- (E) 120

Questions 58–60

58. How many residues x modulo 90 satisfy $3x \equiv 12 \pmod{90}$ and $2x \equiv 0 \pmod{90}$?

- (A) 2
- (B) 1
- (C) 0
- (D) 3
- (E) 4

59. The region $0 \leq x \leq 1$, $0 \leq y \leq 13x^5$ is revolved about the line $x = -3$. Find its volume.

- (A) $\frac{117\pi}{14}$
- (B) $-\frac{117\pi}{7}$
- (C) $\frac{234\pi}{7}$
- (D) $\frac{117\pi}{7}$
- (E) $1 + \frac{117\pi}{7}$

60. Find the coefficient of x^3 in $(1+x)^{24}(1-x)^{24}(1+2x)^2$.

- (A) -96
- (B) -64
- (C) -160
- (D) -32
- (E) -128

Questions 61–63

61. Evaluate $\int_0^{\log 2} e^x (e^x + 13)^5 dx$.

- (A) $-\frac{15^6 - 14^6}{6}$
- (B) $\frac{15^6}{6}$
- (C) $6 \left(\frac{15^6 - 14^6}{6} \right)$
- (D) $15^5 - 14^5$
- (E) $\frac{15^6 - 14^6}{6}$

62. The field K has 7^{13} elements. Find the largest possible multiplicative order of a nonzero element of K .

- (A) $7^{13} - 1$
- (B) $7^{13} + 1$
- (C) $7^{14} - 1$
- (D) $7^{12} - 1$
- (E) 7^{13}

63. For $x > 0$, $x^2 y'' + (1 - 18)xy' + 65y = 0$, $y(1) = 0$, $y'(1) = 13$. Find $y(e)$.

- (A) $-\frac{13(1 - e^8)e^5}{8}$
- (B) $e^5 + e^{13}$
- (C) $-e^5 + e^{13}$
- (D) $\frac{13(1 - e^8)e^5}{8}$
- (E) $\frac{e^{13}}{18}$

Questions 64–66

64. How many edges does the complete bipartite graph $K_{24,26}$ have?

- (A) 1040
- (B) 208
- (C) 624
- (D) 832
- (E) 416

65. Evaluate $\int_0^1 x^{12}(\log x)^4 dx$.

- (A) $\frac{1}{5} \left(\frac{(-1)^4 4!}{13^5} \right)$
- (B) $\frac{28561}{24} \left(\frac{(-1)^4 4!}{13^5} \right)$
- (C) $13 \left(\frac{(-1)^4 4!}{13^5} \right)$
- (D) $\frac{(-1)^4 4!}{13^5}$
- (E) $-\frac{(-1)^4 4!}{13^5}$

66. Let $G = \mathbb{Z}/378\mathbb{Z} \times \mathbb{Z}/256\mathbb{Z}$. Find the order of G/H , where $H = \{g \in G : 2g = 0\}$.

- (A) 24192
- (B) 24193
- (C) 24191
- (D) 24194
- (E) 24190

Answer Key

Practice Test 62 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	B	23	B	45	A
2	D	24	B	46	C
3	C	25	D	47	A
4	B	26	B	48	D
5	C	27	E	49	B
6	E	28	A	50	E
7	C	29	E	51	A
8	E	30	C	52	E
9	E	31	B	53	E
10	D	32	B	54	B
11	B	33	A	55	B
12	D	34	E	56	D
13	D	35	B	57	C
14	C	36	C	58	C
15	C	37	B	59	D
16	A	38	D	60	A
17	A	39	E	61	E
18	D	40	E	62	A
19	C	41	A	63	A
20	D	42	E	64	C
21	A	43	D	65	D
22	C	44	B	66	A

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