

GRE Mathematics Practice Test 63

Upper-Intermediate Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

Intermediate reinforcement, combining standard applications with selected more demanding arguments. This is cycle 12 of 19, position 3 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

zarimion.com/gre

100 practice tests, complete answer keys, and future updates.

Please retain the source attribution when sharing this material.



This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 13x$ and has density $\rho(x, y) = x^2y$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{5}{6}$
- (B) $-\frac{5}{6}$
- (C) $\frac{11}{6}$
- (D) $\frac{5}{12}$
- (E) $\frac{5}{3}$

2. Let V consist of real polynomials of degree at most 12 that vanish at 59, 60, and 62. Find $\dim V$.

- (A) 12
- (B) 11
- (C) 9
- (D) 10
- (E) 8

3. The solution of $y' = 13x^4y^3$ has $y(0) = 1/14$. Find $y(1)$.

- (A) $\frac{\sqrt{530}}{318} + 1$
- (B) $-\frac{\sqrt{530}}{318}$
- (C) $\frac{\sqrt{530}}{636}$
- (D) $\frac{\sqrt{530}}{318}$
- (E) $\frac{\sqrt{530}}{159}$

Questions 4–6

4. Independent exponential waiting times X, Y have rates 25 and 27. Find $P(X < Y)$.

- (A) $\frac{27}{52}$
- (B) $\frac{25}{104}$
- (C) $\frac{25}{52}$
- (D) $\frac{77}{104}$
- (E) $\frac{25}{156}$

5. Under $x = u + 13v$, $y = u - 4v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) 289
- (B) $\frac{1}{17}$
- (C) 17
- (D) 9
- (E) 52

6. The roots r, s of $x^2 - 70x + 649 = 0$ are counted with multiplicity. Find $r^3 + s^3$.

- (A) 206710
- (B) 206711
- (C) 206709
- (D) 206708
- (E) 206712

Questions 7–9

7. Evaluate $\int_0^1 x^{12}(\log x)^3 dx$.

- (A) $\frac{(-1)^3 3!}{13^4}$
- (B) $\frac{2197}{6} \left(\frac{(-1)^3 3!}{13^4} \right)$
- (C) $\frac{1}{4} \left(\frac{(-1)^3 3!}{13^4} \right)$
- (D) $13 \left(\frac{(-1)^3 3!}{13^4} \right)$
- (E) $-\frac{(-1)^3 3!}{13^4}$

8. $X_1, \dots, X_{25}$ are independent with common mean and variance 27. With $\bar{X}$ their sample mean, find $E[\sum_i (X_i - \bar{X})^2]$.

- (A) 864
- (B) 216
- (C) 1080
- (D) 648
- (E) 432

9. Evaluate $\lim_{x \rightarrow 0} \frac{(1 + 13x)^{1/(3x)} - e^{13/3}}{x}$.

- (A) $-\frac{169e^{\frac{13}{3}}}{3}$
- (B) $-\frac{169e^{\frac{13}{3}}}{6}$
- (C) $e^{\frac{13}{3}}$
- (D) $\frac{169e^{\frac{13}{3}}}{6}$
- (E) $-\frac{13e^{\frac{13}{3}}}{6}$

Questions 10–12

10. For real t , let $M(t) = \begin{pmatrix} t-59 & 11 & 0 \\ 0 & t-59 & 1 \\ 0 & 0 & t-59 \end{pmatrix}$. At $t = 59$, what is $\dim \ker(M(t)^2) - \dim \ker M(t)$?

- (A) 4
- (B) 0
- (C) 3
- (D) 2
- (E) 1

11. Solve $y' + 13y = x^4 e^{-13x}$, $y(0) = 0$, and find $y(1)$.

- (A) $\frac{13}{5e^{13}}$
- (B) $\frac{1}{5e^{13}}$
- (C) e^{-13}
- (D) $\frac{e^{13}}{5}$
- (E) $\frac{1}{4e^{13}}$

12. A permutation of 27 objects is chosen uniformly. Let Z be the number of fixed points. Find $E[Z^2]$.

- (A) 3
- (B) 4
- (C) 1
- (D) 0
- (E) 2

Questions 13–15

13. Let $f(x, y) = x^2 + 26xy + (173)y^2$. What is the product of the eigenvalues of the Hessian of f ?

- (A) 16
- (B) 8
- (C) 173
- (D) 692
- (E) 4

14. Let $u = (12, 4)^T$ and $v = (1, 0)^T$. Find the squared distance from $(12, 0)^T$ to $\text{span}(u)$.

- (A) $\frac{67}{5}$
- (B) $\frac{139}{10}$
- (C) $\frac{72}{5}$
- (D) $\frac{77}{5}$
- (E) $\frac{149}{10}$

15. Evaluate $\iiint_B 13z^2(x^2 + y^2 + z^2)^4 dV$, where B is the unit ball.

- (A) $1 + \frac{4\pi}{3}$
- (B) $\frac{8\pi}{3}$
- (C) $\frac{4\pi}{3}$
- (D) $\frac{2\pi}{3}$
- (E) $-\frac{4\pi}{3}$

Questions 16–18

16. A coin success probability p has uniform prior on $[0, 1]$. After observing 25 successes and 2 failures in independent tosses conditional on p , what is the predictive probability of success on the next toss?

- (A) $\frac{13}{29}$
- (B) $\frac{26}{29}$
- (C) $\frac{26}{87}$
- (D) $\frac{55}{58}$
- (E) $\frac{3}{29}$

17. If $y' = y(1 - y/4)$ and $y(0) = 2$, find $y(-\log 13)$.

- (A) $\frac{4}{7}$
- (B) $-\frac{2}{7}$
- (C) $\frac{9}{7}$
- (D) $\frac{1}{7}$
- (E) $\frac{2}{7}$

18. An idempotent 65×65 real matrix P has rank 5. What is $\det(I + P)$?

- (A) 32
- (B) 34
- (C) 30
- (D) 31
- (E) 33

Questions 19–21

19. Find the coefficient of x^6 in $\cos(13x)/(1 - 3x^2)$.

- (A) -2784439
(B) $\frac{2784439}{720}$
(C) $-\frac{4826809}{720}$
(D) $-\frac{2784439}{720}$
(E) $-\frac{2785159}{720}$

20. A random variable has mean 0 and variance 25. What upper bound does the Chebyshev inequality give for $P(|X| \geq 50)$?

- (A) $\frac{1}{200}$
(B) $\frac{99}{100}$
(C) $\frac{1}{300}$
(D) $\frac{101}{200}$
(E) $\frac{1}{100}$

21. Evaluate $\iiint_{x^2+y^2+z^2 \leq 13} (x^2 + y^2 + z^2)^3 dV$.

- (A) $\frac{1}{3} \left(\frac{4\pi 13^{\frac{9}{2}}}{9} \right)$
(B) $9 \left(\frac{4\pi 13^{\frac{9}{2}}}{9} \right)$
(C) $\frac{9}{91} \left(\frac{4\pi 13^{\frac{9}{2}}}{9} \right)$
(D) $\frac{1}{2} \left(\frac{4\pi 13^{\frac{9}{2}}}{9} \right)$
(E) $\frac{4\pi 13^{\frac{9}{2}}}{9}$

Questions 22–24

22. For $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} 59 \\ 11 \\ 70 \end{bmatrix}$, let x minimize $\|Ax - b\|^2$. Find x_2 .

- (A) 6
- (B) $\frac{11}{2}$
- (C) 5
- (D) $\frac{9}{2}$
- (E) $\frac{13}{2}$

23. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{13t} \cos(4t)$, and $y = e^{13t} \sin(4t)$, find d^2f/dt^2 at $t = 1$.

- (A) 8
- (B) 26
- (C) 370
- (D) 0
- (E) 676

24. A continuous $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(q) = q^{20}$ for every rational q . Find $f(2)$.

- (A) 2^{21}
- (B) 2^{19}
- (C) 2^{20}
- (D) 2^{22}
- (E) 2^{23}

Questions 25–27

- 25.** Find the coefficient of x^4 in $e^{3x} \log(1 + 13x)$.
- (A) -126360
(B) $-\frac{28561}{4}$
(C) -5265
(D) 5265
(E) -5252
- 26.** What is the coefficient of x^7 in $(x + 12)^6(1 - x)^3$?
- (A) -1949
(B) -1947
(C) -1946
(D) -1945
(E) -1948
- 27.** Let $F(x) = \int_0^x (x - t)^3 e^{13t} dt$. Find $F^{(4)}(0)$.
- (A) 0
(B) 78
(C) 6
(D) $\frac{1}{6}$
(E) 24

Questions 28–30

28. Find the radius of convergence of $\sum_{k \geq 1} k^2 z^{2k} / 20^{2k} + \sum_{k \geq 0} z^{2k+1} / (21)^{2k+1}$.

- (A) 32
- (B) 14
- (C) 20
- (D) 8
- (E) 26

29. Let $F(x) = \int_0^{x^4} e^{13t} dt$. Find $F'(1)$.

- (A) e^{52}
- (B) $4e^{13}$
- (C) $\frac{e^{13}}{4}$
- (D) $52e^{13}$
- (E) e^{13}

30. A matrix A satisfies $(A - 60I)(A - 62I) = 0$, and its minimal polynomial has degree two. If $A^4 = uA + vI$, find u .

- (A) 908168
- (B) 908169
- (C) 908166
- (D) 908167
- (E) 908170

Questions 31–33

31. Evaluate $\int_0^{\pi/2} \sin^9 t \cos^{27} t \, dt$.

- (A) $2380 \left(\frac{4! 13!}{2 \cdot 18!} \right)$
- (B) $\frac{1}{2} \left(\frac{4! 13!}{2 \cdot 18!} \right)$
- (C) $2 \left(\frac{4! 13!}{2 \cdot 18!} \right)$
- (D) $18 \left(\frac{4! 13!}{2 \cdot 18!} \right)$
- (E) $\frac{4! 13!}{2 \cdot 18!}$

32. On $[0, 1]$, f is 20 on rationals and $1/20$ on irrationals. Find its upper Darboux integral minus its lower Darboux integral.

- (A) $\frac{419}{20}$
- (B) $\frac{399}{20}$
- (C) $\frac{439}{20}$
- (D) $\frac{399}{10}$
- (E) $\frac{399}{40}$

33. For $f(x) = x^4 e^{13x}$, find $f^{(6)}(0)$.

- (A) $\frac{1}{360} \left(\frac{6! 13^2}{2} \right)$
- (B) $\frac{2}{13} \left(\frac{6! 13^2}{2} \right)$
- (C) $2 \left(\frac{6! 13^2}{2} \right)$
- (D) $\frac{6! 13^2}{2}$
- (E) $\frac{1}{30} \left(\frac{6! 13^2}{2} \right)$

Questions 34–36

34. Let $V = \begin{bmatrix} 1 & 59 & 3481 \\ 1 & 61 & 3721 \\ 1 & 64 & 4096 \end{bmatrix}$. Find $\det(V^T V)$.

- (A) 898
- (B) 899
- (C) 900
- (D) 901
- (E) 902

35. If $y'' - 26y' + 169y = 4e^{13x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) $4e^{13}$
- (B) e^{13}
- (C) $1 + 2e^{13}$
- (D) $-2e^{13}$
- (E) $2e^{13}$

36. A set with 20 elements has the discrete topology. How many of its subsets are simultaneously open and closed?

- (A) 2^{23}
- (B) 2^{19}
- (C) 2^{22}
- (D) 2^{21}
- (E) 2^{20}

Questions 37–39

37. Two solutions of $y'' + 13x^4y' + 4y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $\frac{2}{e^{\frac{13}{5}}}$
- (B) $e^{-\frac{13}{5}}$
- (C) $-\frac{1}{e^{\frac{13}{5}}}$
- (D) $\frac{1}{2e^{\frac{13}{5}}}$
- (E) $e^{-\frac{13}{5}} + 1$

38. Let $u = (1, 2, \dots, 65)^T$, $v = (1, 1, \dots, 1)^T$, and $B = I + uv^T$. Find $\det B$.

- (A) 2148
- (B) 2147
- (C) 2145
- (D) 2146
- (E) 2144

39. Let $\mathbf{F} = (13y^4, z^4, x^4)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (1, 1, 4)/\sqrt{18}$.

- (A) $1 - 36\sqrt{2}$
- (B) $36\sqrt{2}$
- (C) $-18\sqrt{2}$
- (D) $-72\sqrt{2}$
- (E) $-36\sqrt{2}$

Questions 40–42

40. What is the coefficient of z^{-1} in the Laurent expansion of e^{15z}/z^3 about zero?

- (A) $\frac{227}{2}$
- (B) $\frac{229}{2}$
- (C) $\frac{225}{4}$
- (D) $\frac{225}{2}$
- (E) 225

41. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(13j)!}{(j!)^{13}} x^{4j}$?

- (A) $13^{-\frac{13}{4}}$
- (B) $\frac{1}{13}$
- (C) $13^{\frac{13}{4}}$
- (D) 13^{-13}
- (E) $13^{-\frac{4}{13}}$

42. How many group homomorphisms $\mathbb{Z}/384\mathbb{Z} \rightarrow \mathbb{Z}/260\mathbb{Z}$ are there?

- (A) 4
- (B) 5
- (C) 6
- (D) 2
- (E) 3

Questions 43–45

43. What is the maximum of x^4y^{13} over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\left(\frac{4}{17}\right)^4 \left(\frac{13}{17}\right)^{13}$
- (B) $\frac{1}{2^{17}}$
- (C) $17 \left(\left(\frac{4}{17}\right)^4 \left(\frac{13}{17}\right)^{13} \right)$
- (D) $\left(\frac{4}{17}\right)^{13} \left(\frac{13}{17}\right)^4$
- (E) $\frac{52}{289}$

44. An entire function satisfies $|f(z)| \leq C(1 + |z|^{15})$ for some $C > 0$. What is the largest possible degree of f ?

- (A) 20
- (B) 5
- (C) 25
- (D) 15
- (E) 10

45. For $\mathbf{F} = (13x^4y, y^4z, xz^4)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) -140
- (B) 70
- (C) 280
- (D) 140
- (E) 141

Questions 46–48

46. How many 1-dimensional subspaces does $\mathbb{F}_{37}^5$ have?

- (A) $\frac{37^4 - 1}{37 - 1}$
- (B) $\frac{37^8 - 1}{37 - 1}$
- (C) $\frac{37^7 - 1}{37 - 1}$
- (D) $\frac{37^5 - 1}{37 - 1}$
- (E) $\frac{37^6 - 1}{37 - 1}$

47. For the curve $x = t^4$, $y = t^5 + 13t$, find d^2y/dx^2 at $t = 1$.

- (A) $-\frac{17}{2}$
- (B) $\frac{9}{2}$
- (C) $-\frac{17}{8}$
- (D) $-\frac{13}{4}$
- (E) $\frac{11}{4}$

48. Bisection starts with an interval of length 2^{25} . After 27 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{7}{8}$
- (B) $\frac{9}{16}$
- (C) $\frac{1}{8}$
- (D) $\frac{1}{24}$
- (E) $\frac{1}{16}$

Questions 49–51

49. If $y' = 26xy$ and $y(0) = 1$, find the coefficient of x^8 in the Maclaurin series of y .
- (A) $\frac{28561}{24}$
(B) $-\frac{28561}{24}$
(C) $\frac{28585}{24}$
(D) $\frac{28561}{48}$
(E) $\frac{28561}{12}$
50. How many monic irreducible polynomials of degree 3 are there over $\mathbb{F}_{37}$?
- (A) 16873
(B) 16872
(C) 16870
(D) 16871
(E) 16874
51. Find the total outward flux of $\mathbf{F} = (13x^4y, y^4z, xz^4)$ through the boundary of $[0, 1]^3$.
- (A) $-\frac{15}{2}$
(B) $\frac{17}{2}$
(C) 15
(D) $\frac{15}{2}$
(E) $\frac{15}{4}$

Questions 52–54

52. What is the least positive multiplicative inverse of 2 modulo 51?

- (A) 42
- (B) 26
- (C) 18
- (D) 34
- (E) 10

53. For $x > 0$, $x^2 y'' + (-29)xy' + 221y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

- (A) $\frac{(1 - e^4) e^{13}}{4}$
- (B) $-\frac{e^{13}}{4} + 1 + \frac{e^{17}}{4}$
- (C) $-\frac{(1 - e^4) e^{13}}{4}$
- (D) $-\frac{(1 - e^4) e^{13}}{2}$
- (E) $-\frac{(1 - e^4) e^{13}}{8}$

54. How many elements z of $\mathbb{F}_{5^{13}}$ satisfy $z^{25} = z$?

- (A) 3
- (B) 5
- (C) 6
- (D) 4
- (E) 7

Questions 55–57

55. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^{13}y^3 dV$.

- (A) $6 \left(\frac{13! 3!}{19!} \right)$
- (B) $\frac{13! 3!}{19!}$
- (C) $19 \left(\frac{13! 3!}{19!} \right)$
- (D) $\frac{1}{6} \left(\frac{13! 3!}{19!} \right)$
- (E) $\frac{1}{56}$

56. How many truth assignments to $p, q, r_1, \dots, r_{23}$ make $p \Rightarrow q$ false?

- (A) 2^{23}
- (B) 2^{26}
- (C) 2^{25}
- (D) 2^{24}
- (E) 2^{22}

57. Evaluate $\int_0^1 x^{13}(1-x)^3 dx$.

- (A) $17 \left(\frac{13! 3!}{17!} \right)$
- (B) $14 \left(\frac{13! 3!}{17!} \right)$
- (C) $\frac{13! 3!}{17!}$
- (D) $\frac{1}{4} \left(\frac{13! 3!}{17!} \right)$
- (E) $170 \left(\frac{13! 3!}{17!} \right)$

Questions 58–60

58. How many generators does the cyclic group of order 1874161 have?

- (A) $37^3(37 - 1)$
- (B) $37^6(37 - 1)$
- (C) $37^5(37 - 1)$
- (D) $37^2(37 - 1)$
- (E) $37^4(37 - 1)$

59. Find the upward flux of $\mathbf{F} = (x, y, z)$ through $z = 13x^2 + 4y^2$, $0 \leq x, y \leq 1$.

- (A) $-\frac{17}{3}$
- (B) $-\frac{17}{6}$
- (C) $-\frac{34}{3}$
- (D) $\frac{17}{3}$
- (E) $-\frac{14}{3}$

60. What is the coefficient of x^2 in $(1 + 2x)^{25}$?

- (A) 400
- (B) 800
- (C) 1600
- (D) 1200
- (E) 2000

Questions 61–63

61. The region $0 \leq x \leq 1$, $0 \leq y \leq 13x^4$ is revolved about the line $x = -4$. Find its volume.

- (A) $\frac{377\pi}{30}$
- (B) $-\frac{377\pi}{15}$
- (C) $\frac{754\pi}{15}$
- (D) $\frac{377\pi}{15}$
- (E) $1 + \frac{377\pi}{15}$

62. Let $R = (\mathbb{Z}/96\mathbb{Z})/([16])$, where $([16])$ is the ideal generated by $[16]$. How many units does R have?

- (A) 10
- (B) 8
- (C) 7
- (D) 9
- (E) 6

63. Evaluate $\int_0^\infty x^4 e^{-13x} dx$.

- (A) 24
- (B) $\frac{1}{13} \left(\frac{4!}{13^5} \right)$
- (C) $13 \left(\frac{4!}{13^5} \right)$
- (D) $\frac{1}{4} \left(\frac{4!}{13^5} \right)$
- (E) $\frac{4!}{13^5}$

Questions 64–66

64. How many edges does the complete bipartite graph $K_{25,27}$ have?

- (A) 900
- (B) 225
- (C) 1125
- (D) 450
- (E) 675

65. Evaluate $\int_0^1 \int_0^x x^{13} y^4 dy dx$.

- (A) $\frac{1}{90}$
- (B) $\frac{1}{114}$
- (C) $\frac{1}{70}$
- (D) $\frac{1}{19}$
- (E) $\frac{1}{95}$

66. What is the largest integer k such that 3^k divides $68!$?

- (A) 30
- (B) 29
- (C) 32
- (D) 33
- (E) 31

Answer Key

Practice Test 63 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	A	23	D	45	D
2	D	24	C	46	D
3	D	25	C	47	C
4	C	26	B	48	C
5	C	27	C	49	A
6	A	28	C	50	B
7	A	29	B	51	D
8	D	30	A	52	B
9	B	31	E	53	C
10	E	32	B	54	B
11	B	33	D	55	B
12	E	34	C	56	A
13	A	35	E	57	C
14	C	36	E	58	A
15	C	37	B	59	A
16	B	38	D	60	D
17	E	39	E	61	D
18	A	40	D	62	B
19	D	41	A	63	E
20	E	42	A	64	E
21	E	43	A	65	E
22	B	44	D	66	E

zarimion.com/gre

100 practice tests, complete answer keys, and future updates.
Please retain the source attribution when sharing this material.

