

GRE Mathematics Practice Test 64

Bridge to Standard

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A bridge toward standard practice, with routine multi-step questions and a smaller number of challenging concepts. This is cycle 12 of 19, position 4 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Evaluate $\iiint_B 13(x^2 + y^2 + z^2)^{3/2} dV$, where B is the unit ball.

- (A) $-\frac{26\pi}{3}$
- (B) $\frac{13\pi}{3}$
- (C) $\frac{26\pi}{3}$
- (D) $1 + \frac{26\pi}{3}$
- (E) $\frac{52\pi}{3}$

2. For $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $b = \begin{bmatrix} 60 \\ 9 \\ 69 \end{bmatrix}$, let x minimize $\|Ax - b\|^2$. Find x_2 .

- (A) 10
- (B) 7
- (C) 8
- (D) 11
- (E) 9

3. For $y' = 13y(1 - y/3)$, what is the derivative of the right-hand side with respect to y at the positive equilibrium?

- (A) $-\frac{13}{2}$
- (B) -26
- (C) -12
- (D) -13
- (E) 13

Questions 4–6

4. Independent exponential waiting times X, Y have rates 26 and 28. Find $P(X < Y)$.

- (A) $\frac{13}{81}$
- (B) $\frac{20}{27}$
- (C) $\frac{13}{27}$
- (D) $\frac{13}{54}$
- (E) $\frac{14}{27}$

5. The solution of $y' = 13x^3y^2$ has $y(0) = 1/14$. Find $y(1)$.

- (A) $\frac{47}{43}$
- (B) $\frac{4}{43}$
- (C) $\frac{8}{43}$
- (D) $\frac{2}{43}$
- (E) $-\frac{4}{43}$

6. What is the coefficient of x^3 in $(x + 12)^5$?

- (A) 1438
- (B) 1439
- (C) 1441
- (D) 1442
- (E) 1440

Questions 7–9

7. Evaluate $\int_0^1 \int_x^1 13y^3 \, dy \, dx$.

- (A) $\frac{13}{5}$
- (B) $\frac{13}{2}$
- (C) $\frac{13}{20}$
- (D) $\frac{13}{3}$
- (E) $\frac{13}{4}$

8. $X_1, \dots, X_{26}$ are independent with common mean and variance 28. With $\bar{X}$ their sample mean, find $E[\sum_i (X_i - \bar{X})^2]$.

- (A) 1166
- (B) 467
- (C) 234
- (D) 700
- (E) 933

9. For $x > 0$, $xy' = 13y$ and $y(1) = 1$. Find $y(e^3)$.

- (A) e^{39}
- (B) $\frac{e^{39}}{2}$
- (C) $2e^{39}$
- (D) $-e^{39}$
- (E) $1 + e^{39}$

Questions 10–12

10. A matrix A satisfies $(A - 61I)(A - 63I) = 0$, and its minimal polynomial has degree two. If $A^2 = uA + vI$, find u .

- (A) 125
- (B) 124
- (C) 126
- (D) 122
- (E) 123

11. Let $f(x, y) = e^{13x+3y}$. Find $\partial^3 f / \partial x^2 \partial y$ at $(0, 0)$.

- (A) $\frac{13}{3} (3 \cdot 13^2)$
- (B) $3 \cdot 13^2$
- (C) $\frac{3}{13} (3 \cdot 13^2)$
- (D) $\frac{9}{169} (3 \cdot 13^2)$
- (E) $\frac{1}{3} (3 \cdot 13^2)$

12. A permutation of 28 objects is chosen uniformly. Let Z be the number of fixed points. Find $E[Z]$.

- (A) 1
- (B) 3
- (C) 2
- (D) 0
- (E) 4

Questions 13–15

13. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-13)^j}{j^{\frac{2}{3}} (\log j)^m}$ converge absolutely at $x = 12$? Give the least such m , or 0 if none exists.

- (A) 1
- (B) 2
- (C) 3
- (D) 4
- (E) 0

14. Let $V = \begin{bmatrix} 1 & 60 & 3600 \\ 1 & 62 & 3844 \\ 1 & 65 & 4225 \end{bmatrix}$. Find $\det V$.

- (A) 29
- (B) 32
- (C) 31
- (D) 28
- (E) 30

15. Solve $y' + 13y = x^3 e^{-13x}$, $y(0) = 0$, and find $y(1)$.

- (A) e^{-13}
- (B) $\frac{e^{13}}{4}$
- (C) $\frac{1}{3e^{13}}$
- (D) $\frac{13}{4e^{13}}$
- (E) $\frac{1}{4e^{13}}$

Questions 16–18

16. A coin success probability p has uniform prior on $[0, 1]$. After observing 26 successes and 2 failures in independent tosses conditional on p , what is the predictive probability of success on the next toss?

- (A) $\frac{3}{10}$
- (B) $\frac{19}{20}$
- (C) $\frac{9}{10}$
- (D) $\frac{9}{20}$
- (E) $\frac{1}{10}$

17. Two solutions of $y'' + 13x^1y' + 3y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $e^{-\frac{13}{2}}$
- (B) $\frac{1}{2e^{\frac{13}{2}}}$
- (C) $e^{-\frac{13}{2}} + 1$
- (D) $\frac{2}{e^{\frac{13}{2}}}$
- (E) $-\frac{1}{e^{\frac{13}{2}}}$

18. Let $u = (1, 2, \dots, 66)^T$, $v = (1, 1, \dots, 1)^T$, and $B = I + uv^T$. Find $\det B$.

- (A) 2211
- (B) 2210
- (C) 2212
- (D) 2213
- (E) 2214

Questions 19–21

19. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 13z^2 = x + 3y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.
- (A) -26
(B) -39
(C) -3
(D) -78
(E) 78
20. A random variable has mean 0 and variance 26. What upper bound does the Chebyshev inequality give for $P(|X| \geq 52)$?
- (A) $\frac{1}{208}$
(B) $\frac{1}{312}$
(C) $\frac{1}{104}$
(D) $\frac{105}{208}$
(E) $\frac{103}{104}$
21. If $y' = 13y$, $y(0) = 1$, what is the coefficient of x^3 in the Maclaurin series of y ?
- (A) $\frac{2197}{3}$
(B) $\frac{2197}{6}$
(C) $\frac{2203}{6}$
(D) $\frac{2197}{12}$
(E) $-\frac{2197}{6}$

Questions 22–24

22. Let D be a diagonal 66×66 real matrix with distinct diagonal entries. What is the dimension of the space of matrices X satisfying $XD = DX$?

- (A) 68
- (B) 67
- (C) 64
- (D) 65
- (E) 66

23. Evaluate $\int_0^{\pi/2} \sin^7 t \cos^{27} t \, dt$.

- (A) $2 \left(\frac{3! 13!}{2 \cdot 17!} \right)$
- (B) $17 \left(\frac{3! 13!}{2 \cdot 17!} \right)$
- (C) $\frac{1}{2} \left(\frac{3! 13!}{2 \cdot 17!} \right)$
- (D) $560 \left(\frac{3! 13!}{2 \cdot 17!} \right)$
- (E) $\frac{3! 13!}{2 \cdot 17!}$

24. A continuous $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(q) = q^{21}$ for every rational q . Find $f(2)$.

- (A) 2^{21}
- (B) 2^{22}
- (C) 2^{24}
- (D) 2^{23}
- (E) 2^{20}

Questions 25–27

25. If $y'' + 9y = 0$, $y(0) = 13$, and $y'(0) = 5$, find $y(\pi/6)$.

- (A) $\frac{8}{3}$
- (B) $\frac{5}{6}$
- (C) $\frac{5}{3}$
- (D) $\frac{10}{3}$
- (E) $-\frac{5}{3}$

26. What is the remainder when $x^2 + 1$ is divided by $x - 61$?

- (A) 3723
- (B) 3722
- (C) 3721
- (D) 3720
- (E) 3724

27. Let C be $x^2/9 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C (-y) dx + (13x) dy$.

- (A) -42π
- (B) 42π
- (C) 84π
- (D) $1 + 42\pi$
- (E) 21π

Questions 28–30

28. Find the radius of convergence of $\sum_{k \geq 1} k^2 z^{2k} / 21^{2k} + \sum_{k \geq 0} z^{2k+1} / (22)^{2k+1}$.

- (A) 7
- (B) 14
- (C) 21
- (D) 35
- (E) 28

29. For $\mathbf{F} = (13x^3, y^3, z^3)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) 108
- (B) 27
- (C) -54
- (D) 54
- (E) 55

30. Let $P = \text{diag}(\underbrace{1, \dots, 1}_4, \underbrace{0, \dots, 0}_{63})$. What is the dimension of the space of 67×67 real matrices X satisfying $PX = X$?

- (A) 268
- (B) 270
- (C) 269
- (D) 266
- (E) 267

Questions 31–33

31. Let $F(x) = \int_0^{x^3} e^{13t} dt$. Find $F'(1)$.

- (A) e^{39}
- (B) $3e^{13}$
- (C) $\frac{e^{13}}{3}$
- (D) e^{13}
- (E) $39e^{13}$

32. On $[0, 1]$, f is 21 on rationals and $1/21$ on irrationals. Find its upper Darboux integral minus its lower Darboux integral.

- (A) $\frac{220}{21}$
- (B) $\frac{482}{21}$
- (C) $\frac{440}{21}$
- (D) $\frac{880}{21}$
- (E) $\frac{461}{21}$

33. Let C be the unit circle $x^2 + y^2 = 1$, $z = 3$, counterclockwise viewed from above. Evaluate $\int_C -13y(x^2 + y^2)^2 dx + (13x(x^2 + y^2)^2) dy$.

- (A) 26π
- (B) $1 + 26\pi$
- (C) 52π
- (D) 13π
- (E) -26π

Questions 34–36

34. A 3×3 matrix A has characteristic polynomial $(t - 60)(t + 1)(t - 2)$. What is $\text{tr}(A^2)$?

- (A) 3604
- (B) 3606
- (C) 3605
- (D) 3603
- (E) 3607

35. Let $f(x) = \log(1 + 13x)$. Find $f^{(3)}(0)$.

- (A) $\frac{1}{3} \left((-1)^2 2! 13^3 \right)$
- (B) $3 \left((-1)^2 2! 13^3 \right)$
- (C) $-(-1)^2 2! 13^3$
- (D) $\frac{1}{13} \left((-1)^2 2! 13^3 \right)$
- (E) $(-1)^2 2! 13^3$

36. A set with 21 elements has the discrete topology. How many of its subsets are simultaneously open and closed?

- (A) 2^{23}
- (B) 2^{24}
- (C) 2^{22}
- (D) 2^{21}
- (E) 2^{20}

Questions 37–39

37. For $f(x, y) = x^2 + 13xy + 3y^2$, what is its maximum directional derivative at $(1, -1)$?

- (A) -11
- (B) 7
- (C) 170
- (D) $\sqrt{170}$
- (E) 18

38. Let N be the 66×66 matrix with ones immediately above the diagonal and zeros elsewhere. Find $\text{rank } N^{10}$.

- (A) 55
- (B) 54
- (C) 58
- (D) 56
- (E) 57

39. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(13x) - \sum_{j=0}^1 (-1)^j (13x)^{2j+1} / (2j+1)!}{x^5}$.

- (A) $\frac{1}{13} \left(\frac{(-1)^2 13^5}{5!} \right)$
- (B) $\frac{(-1)^2 13^5}{5!}$
- (C) $-\frac{(-1)^2 13^5}{5!}$
- (D) $5 \left(\frac{(-1)^2 13^5}{5!} \right)$
- (E) $13 \left(\frac{(-1)^2 13^5}{5!} \right)$

Questions 40–42

40. What is the coefficient of z^{-1} in the Laurent expansion of e^{16z}/z^3 about zero?

- (A) 44
- (B) 128
- (C) 212
- (D) 86
- (E) 170

41. Evaluate $\int_0^\infty x^3 e^{-13x} dx$.

- (A) $28561 \left(\frac{3!}{13^4} \right)$
- (B) $\frac{1}{13} \left(\frac{3!}{13^4} \right)$
- (C) $\frac{1}{3} \left(\frac{3!}{13^4} \right)$
- (D) $13 \left(\frac{3!}{13^4} \right)$
- (E) $\frac{3!}{13^4}$

42. Let A be the 66×66 matrix whose diagonal entries are 63 and whose off-diagonal entries are 1. What is its smallest eigenvalue?

- (A) 60
- (B) 61
- (C) 62
- (D) 64
- (E) 63

Questions 43–45

43. The solution of $y' = 13x^2y^2$, $y(0) = 1$, has its first positive singularity at which x ?
- (A) $\frac{3}{13}$
(B) $\frac{13^{\frac{2}{3}}}{13}$
(C) $\frac{26^{\frac{2}{3}}\sqrt[3]{3}}{26}$
(D) $\frac{13^{\frac{2}{3}}\sqrt[3]{3}}{13}$
(E) $\frac{\sqrt[3]{13} \cdot 3^{\frac{2}{3}}}{3}$
44. An entire function satisfies $|f(z)| \leq C(1 + |z|^{16})$ for some $C > 0$. What is the largest possible degree of f ?
- (A) 6
(B) 26
(C) 11
(D) 21
(E) 16
45. Evaluate $\iiint_{x^2+y^2+z^2 \leq 13} (x^2 + y^2 + z^2)^2 dV$.
- (A) $\frac{1}{2} \left(\frac{4\pi 13^{\frac{7}{2}}}{7} \right)$
(B) $\frac{1}{3} \left(\frac{4\pi 13^{\frac{7}{2}}}{7} \right)$
(C) $\frac{7}{65} \left(\frac{4\pi 13^{\frac{7}{2}}}{7} \right)$
(D) $\frac{4\pi 13^{\frac{7}{2}}}{7}$
(E) $7 \left(\frac{4\pi 13^{\frac{7}{2}}}{7} \right)$

Questions 46–48

46. In the ring $\mathbb{Z}/384\mathbb{Z}$, how many elements are units?

- (A) 127
- (B) 126
- (C) 128
- (D) 129
- (E) 130

47. Find the curvature at $t = 0$ of $r(t) = (3t, 13t^2)$.

- (A) $\frac{26}{9}$
- (B) $\frac{35}{9}$
- (C) $\frac{52}{9}$
- (D) $-\frac{26}{9}$
- (E) $\frac{13}{9}$

48. Bisection starts with an interval of length 2^{26} . After 28 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{1}{24}$
- (B) $\frac{1}{16}$
- (C) $\frac{7}{8}$
- (D) $\frac{9}{16}$
- (E) $\frac{1}{8}$

Questions 49–51

49. Under $x = u + 13v$, $y = u - 3v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) 10
- (B) 256
- (C) $\frac{1}{16}$
- (D) 39
- (E) 16

50. A permutation consists of disjoint cycles of lengths 7 and 8 and fixes all other points. What is its order?

- (A) 56
- (B) 58
- (C) 54
- (D) 57
- (E) 55

51. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(13 + 3j)(16 + 3j)}$.

- (A) $\frac{3}{13}$
- (B) $\frac{1}{39}$
- (C) $\frac{1}{16}$
- (D) $\frac{1}{3}$
- (E) $\frac{1}{13}$

Questions 52–54

52. What is the least positive multiplicative inverse of 2 modulo 53?

- (A) 27
- (B) 9
- (C) 45
- (D) 18
- (E) 36

53. What is the maximum of x^3y^{13} over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\left(\frac{3}{16}\right)^{13} \left(\frac{13}{16}\right)^3$
- (B) $\left(\frac{3}{16}\right)^3 \left(\frac{13}{16}\right)^{13}$
- (C) $\frac{1}{2^{16}}$
- (D) $16 \left(\left(\frac{3}{16}\right)^3 \left(\frac{13}{16}\right)^{13} \right)$
- (E) $\frac{39}{256}$

54. In $R = \mathbb{F}_3[x]/(x^{13})$, how many elements are there?

- (A) 3^{13}
- (B) 3^{14}
- (C) 3^{12}
- (D) 3^{15}
- (E) 3^{16}

Questions 55–57

55. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{13t} \cos(3t)$, and $y = e^{13t} \sin(3t)$, find $d^2 f/dt^2$ at $t = 1$.

- (A) 0
- (B) 356
- (C) 676
- (D) 26
- (E) 6

56. How many truth assignments to $p, q, r_1, \dots, r_{24}$ make $p \Rightarrow q$ false?

- (A) 2^{26}
- (B) 2^{25}
- (C) 2^{27}
- (D) 2^{24}
- (E) 2^{23}

57. Find the coefficient of x^3 in the Taylor series of $(1 - 13x)^{-2}$ at zero.

- (A) $-4 \cdot 13^3$
- (B) $\frac{1}{13} (4 \cdot 13^3)$
- (C) $4 \cdot 13^3$
- (D) $\frac{1}{4} (4 \cdot 13^3)$
- (E) $\frac{3}{4} (4 \cdot 13^3)$

Questions 58–60

58. Integers x satisfy $x \equiv 5 \pmod{7}$ and $x \equiv 7 \pmod{9}$. Find the least nonnegative solution.

- (A) 60
- (B) 63
- (C) 62
- (D) 61
- (E) 59

59. Evaluate $\int_0^1 x^{12}(\log x)^2 dx$.

- (A) $\frac{1}{3} \left(\frac{(-1)^2 2!}{13^3} \right)$
- (B) $-\frac{(-1)^2 2!}{13^3}$
- (C) $13 \left(\frac{(-1)^2 2!}{13^3} \right)$
- (D) $\frac{(-1)^2 2!}{13^3}$
- (E) $\frac{169}{2} \left(\frac{(-1)^2 2!}{13^3} \right)$

60. What is the coefficient of x^2 in $(1 + 2x)^{26}$?

- (A) 1733
- (B) 1300
- (C) 434
- (D) 867
- (E) 2166

Questions 61–63

61. Evaluate $\lim_{x \rightarrow 0} (1 + 13x)^{1/(2x)}$.

- (A) $e^{\frac{13}{2}}$
- (B) e^{169}
- (C) e^{14}
- (D) 13
- (E) $e^{\frac{2}{13}}$

62. The symmetric group S_{66} acts on $\{1, \dots, 66\}$. What is the index of the stabilizer of 1?

- (A) 67
- (B) 65
- (C) 64
- (D) 66
- (E) 68

63. Let $F(x) = \int_0^x (x - t)^2 e^{13t} dt$. Find $F^{(3)}(0)$.

- (A) 0
- (B) 2
- (C) 6
- (D) 26
- (E) $\frac{1}{2}$

Questions 64–66

64. How many edges does the complete bipartite graph $K_{26,28}$ have?

- (A) 244
- (B) 728
- (C) 486
- (D) 970
- (E) 1212

65. The region $0 \leq x \leq 1$, $0 \leq y \leq 13x^3$ is revolved about the x -axis. Find its volume.

- (A) $1 + \frac{169\pi}{7}$
- (B) $\frac{169\pi}{14}$
- (C) $\frac{338\pi}{7}$
- (D) $-\frac{169\pi}{7}$
- (E) $\frac{169\pi}{7}$

66. In $\mathbb{Z}/1878\mathbb{Z}$, how many residues are idempotent ($x^2 = x$)?

- (A) 10
- (B) 9
- (C) 8
- (D) 6
- (E) 7

Answer Key

Practice Test 64 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	C	23	E	45	D
2	E	24	A	46	C
3	D	25	C	47	A
4	C	26	B	48	E
5	B	27	B	49	E
6	E	28	C	50	A
7	A	29	D	51	B
8	D	30	A	52	A
9	A	31	B	53	B
10	B	32	C	54	A
11	B	33	A	55	A
12	A	34	C	56	D
13	E	35	E	57	C
14	E	36	D	58	D
15	E	37	D	59	D
16	C	38	D	60	B
17	A	39	B	61	A
18	C	40	B	62	D
19	D	41	E	63	B
20	C	42	C	64	B
21	B	43	D	65	E
22	E	44	E	66	C

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