

GRE Mathematics Practice Test 65

Standard-Level Target

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A mixed standard-level target, intended to approximate public GRE Mathematics practice in scope and overall demand. This is cycle 12 of 19, position 5 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Let $\mathbf{F} = (13y^2, z^2, x^2)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (0, 0, 1)$.

- (A) -13
- (B) 26
- (C) -25
- (D) -52
- (E) -26

2. Let D be a diagonal 67×67 real matrix with distinct diagonal entries. What is the dimension of the space of matrices X satisfying $XD = DX$?

- (A) 68
- (B) 65
- (C) 67
- (D) 66
- (E) 69

3. Let $F(x) = \int_0^{x^2} e^{13t} dt$. Find $F'(1)$.

- (A) $26e^{13}$
- (B) e^{13}
- (C) $2e^{13}$
- (D) e^{26}
- (E) $\frac{e^{13}}{2}$

Questions 4–6

4. Independent exponential waiting times X, Y have rates 27 and 29. Find $P(X < Y)$.

- (A) $\frac{27}{56}$
- (B) $\frac{9}{56}$
- (C) $\frac{83}{112}$
- (D) $\frac{29}{56}$
- (E) $\frac{27}{112}$

5. Evaluate $\int_0^\infty x^2 e^{-13x} dx$.

- (A) $13 \left(\frac{2!}{13^3} \right)$
- (B) $\frac{1}{13} \left(\frac{2!}{13^3} \right)$
- (C) $2197 \left(\frac{2!}{13^3} \right)$
- (D) $\frac{1}{2} \left(\frac{2!}{13^3} \right)$
- (E) $\frac{2!}{13^3}$

6. What is the remainder when $x^2 + 1$ is divided by $x - 62$?

- (A) 3846
- (B) 3843
- (C) 3845
- (D) 3844
- (E) 3847

Questions 7–9

7. Find the curvature at $t = 0$ of $r(t) = (2t, 13t^2)$.

- (A) 13
- (B) $-\frac{13}{2}$
- (C) $\frac{13}{4}$
- (D) $\frac{15}{2}$
- (E) $\frac{13}{2}$

8. $X_1, \dots, X_{27}$ are independent with common mean and variance 29. With $\bar{X}$ their sample mean, find $E[\sum_i (X_i - \bar{X})^2]$.

- (A) 1256
- (B) 754
- (C) 252
- (D) 503
- (E) 1005

9. Find the upward flux of $\mathbf{F} = (0, 0, z)$ through $z = 13x^2 + 2y^2$, $0 \leq x, y \leq 1$.

- (A) 10
- (B) 6
- (C) $\frac{5}{2}$
- (D) -5
- (E) 5

Questions 10–12

10. Let $P = \text{diag}(\underbrace{1, \dots, 1}_1, \underbrace{0, \dots, 0}_{67})$. What is the dimension of the space of 68×68 real matrices X satisfying $PX = X$?

- (A) 68
- (B) 70
- (C) 66
- (D) 69
- (E) 67

11. Let $f(x, y) = e^{13x+2y}$. Find $\partial^2 f / \partial x^1 \partial y$ at $(0, 0)$.

- (A) $\frac{1}{2} (2 \cdot 13^1)$
- (B) $\frac{13}{2} (2 \cdot 13^1)$
- (C) $2 (2 \cdot 13^1)$
- (D) $2 \cdot 13^1$
- (E) $\frac{2}{13} (2 \cdot 13^1)$

12. A permutation of 29 objects is chosen uniformly. Let Z be the number of fixed points. Find $E[Z]$.

- (A) 4
- (B) 1
- (C) 3
- (D) 2
- (E) 0

Questions 13–15

13. If $y' + 13y = 2e^{-13x}$ and $y(0) = 6$, find $y(1)$.

- (A) $\frac{8}{e^{13}}$
- (B) $\frac{16}{e^{13}}$
- (C) $\frac{4}{e^{13}}$
- (D) $\frac{8}{e^{13}} + 1$
- (E) $-\frac{8}{e^{13}}$

14. A 3×3 matrix A has characteristic polynomial $(t - 61)(t + 1)(t - 2)$. What is $\text{tr}(A^2)$?

- (A) 3725
- (B) 3727
- (C) 3726
- (D) 3728
- (E) 3724

15. The region $0 \leq x \leq 1$, $0 \leq y \leq 13x^2$ is revolved about the x -axis. Find its volume.

- (A) $\frac{338\pi}{5}$
- (B) $\frac{169\pi}{5}$
- (C) $-\frac{169\pi}{5}$
- (D) $1 + \frac{169\pi}{5}$
- (E) $\frac{169\pi}{10}$

Questions 16–18

16. A coin success probability p has uniform prior on $[0, 1]$. After observing 27 successes and 2 failures in independent tosses conditional on p , what is the predictive probability of success on the next toss?

- (A) $\frac{28}{93}$
- (B) $\frac{3}{31}$
- (C) $\frac{59}{62}$
- (D) $\frac{28}{31}$
- (E) $\frac{14}{31}$

17. Let C be $x^2/4 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C (-y) dx + (13x) dy$.

- (A) 28π
- (B) 14π
- (C) $1 + 28\pi$
- (D) -28π
- (E) 56π

18. Let N be the 67×67 matrix with ones immediately above the diagonal and zeros elsewhere. Find $\text{rank } N^{10}$.

- (A) 56
- (B) 55
- (C) 57
- (D) 58
- (E) 59

Questions 19–21

19. Solve $y' + 13y = x^2e^{-13x}$, $y(0) = 0$, and find $y(1)$.

- (A) $\frac{1}{2e^{13}}$
- (B) $\frac{13}{3e^{13}}$
- (C) $\frac{e^{13}}{3}$
- (D) $\frac{1}{3e^{13}}$
- (E) e^{-13}

20. A random variable has mean 0 and variance 27. What upper bound does the Chebyshev inequality give for $P(|X| \geq 54)$?

- (A) $\frac{1}{108}$
- (B) $\frac{1}{324}$
- (C) $\frac{1}{216}$
- (D) $\frac{109}{216}$
- (E) $\frac{107}{108}$

21. Evaluate $\iiint_{x^2+y^2+z^2 \leq 13} (x^2 + y^2 + z^2)^1 dV$.

- (A) $\frac{1}{2} \left(\frac{4\pi 13^{\frac{5}{2}}}{5} \right)$
- (B) $5 \left(\frac{4\pi 13^{\frac{5}{2}}}{5} \right)$
- (C) $\frac{1}{3} \left(\frac{4\pi 13^{\frac{5}{2}}}{5} \right)$
- (D) $\frac{4\pi 13^{\frac{5}{2}}}{5}$
- (E) $\frac{5}{39} \left(\frac{4\pi 13^{\frac{5}{2}}}{5} \right)$

Questions 22–24

22. Let A be the 67×67 matrix whose diagonal entries are 64 and whose off-diagonal entries are 1. What is its smallest eigenvalue?

- (A) 63
- (B) 62
- (C) 64
- (D) 61
- (E) 65

23. Find the coefficient of x^2 in the Taylor series of $(1 - 13x)^{-2}$ at zero.

- (A) $\frac{2}{3} (3 \cdot 13^2)$
- (B) $2 (3 \cdot 13^2)$
- (C) $3 \cdot 13^2$
- (D) $\frac{1}{13} (3 \cdot 13^2)$
- (E) $\frac{1}{3} (3 \cdot 13^2)$

24. A continuous $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(q) = q^{22}$ for every rational q . Find $f(2)$.

- (A) 2^{23}
- (B) 2^{22}
- (C) 2^{21}
- (D) 2^{25}
- (E) 2^{24}

Questions 25–27

25. Evaluate $\int_0^1 \int_0^x x^{13} y^2 dy dx$.

- (A) $\frac{1}{68}$
- (B) $\frac{1}{51}$
- (C) $\frac{1}{48}$
- (D) $\frac{1}{17}$
- (E) $\frac{1}{42}$

26. Positive real numbers x, y satisfy $\log_2 x + \log_2 y = \log_2 427$ and $x + y = 68$. Find xy .

- (A) 428
- (B) 426
- (C) 429
- (D) 427
- (E) 425

27. The function $y(x)$ satisfies $y^3 + 13y = 1x$ and $y(0) = 0$. Find $y'(0)$.

- (A) $\frac{1}{13}$
- (B) 0
- (C) $\frac{1}{16}$
- (D) 13
- (E) $-\frac{1}{13}$

Questions 28–30

28. Find the radius of convergence of $\sum_{k=1}^{\infty} k^{22}x^k/22^k$.

- (A) 29
- (B) 15
- (C) 36
- (D) 8
- (E) 22

29. Let $F(x) = \int_0^x (x-t)^1 e^{13t} dt$. Find $F^{(2)}(0)$.

- (A) 13
- (B) 2
- (C) 0
- (D) $\frac{1}{2}$
- (E) 1

30. A sequence satisfies $u_0 = 61$ and $u_{k+1} = 1u_k + 7$ for all $k \geq 0$. Find u_5 .

- (A) 94
- (B) 97
- (C) 95
- (D) 96
- (E) 98

Questions 31–33

31. For $f(x) = x^2 e^{13x}$, find $f^{(4)}(0)$.

- (A) $\frac{1}{12} \left(\frac{4! 13^2}{2} \right)$
- (B) $2 \left(\frac{4! 13^2}{2} \right)$
- (C) $\frac{1}{2} \left(\frac{4! 13^2}{2} \right)$
- (D) $\frac{4! 13^2}{2}$
- (E) $\frac{2}{13} \left(\frac{4! 13^2}{2} \right)$

32. On $[0, 1]$, f is 22 on rationals and $1/22$ on irrationals. Find its upper Darboux integral minus its lower Darboux integral.

- (A) $\frac{505}{22}$
- (B) $\frac{527}{22}$
- (C) $\frac{483}{22}$
- (D) $\frac{11}{483}$
- (E) $\frac{22}{483}$

33. Evaluate $\lim_{x \rightarrow 0} \frac{e^{13x} - \sum_{j=0}^1 (13x)^j / j!}{x^2}$.

- (A) $-\frac{13^2}{2!}$
- (B) $\frac{1}{2} \left(\frac{13^2}{2!} \right)$
- (C) $2 \left(\frac{13^2}{2!} \right)$
- (D) $\frac{1}{13} \left(\frac{13^2}{2!} \right)$
- (E) $\frac{13^2}{2!}$

Questions 34–36

34. A linear map $T : \mathbb{R}^{68} \rightarrow \mathbb{R}^{70}$ has rank 5. What is its nullity?

- (A) 64
- (B) 61
- (C) 63
- (D) 62
- (E) 65

35. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(13+2j)(15+2j)}$.

- (A) $\frac{1}{13}$
- (B) $\frac{1}{26}$
- (C) $\frac{1}{15}$
- (D) $\frac{1}{2}$
- (E) $\frac{2}{13}$

36. A set with 22 elements has the discrete topology. How many of its subsets are simultaneously open and closed?

- (A) 2^{24}
- (B) 2^{22}
- (C) 2^{23}
- (D) 2^{21}
- (E) 2^{25}

Questions 37–39

37. The solution of $y' = 13x^1y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\frac{\sqrt{13}}{13}$
- (B) $\frac{\sqrt{26}}{13}$
- (C) $\frac{2\sqrt{26}}{13}$
- (D) $\frac{2}{13}$
- (E) $\frac{\sqrt{26}}{2}$

38. What is the dimension of the real vector space of 67×67 symmetric matrices?

- (A) 2276
- (B) 2280
- (C) 2277
- (D) 2278
- (E) 2279

39. For $\mathbf{F} = (13x^2, y^2, z^2)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) 33
- (B) -32
- (C) 32
- (D) 64
- (E) 16

Questions 40–42

40. What is the coefficient of z^{-1} in the Laurent expansion of e^{17z}/z^3 about zero?

- (A) $\frac{291}{2}$
- (B) $\frac{293}{2}$
- (C) $\frac{289}{4}$
- (D) $\frac{289}{2}$
- (E) 289

41. Let $f(x, y) = x^2 + 26xy + (171)y^2$. What is the product of the eigenvalues of the Hessian of f ?

- (A) 684
- (B) 171
- (C) 8
- (D) 4
- (E) 2

42. Let $A = \begin{bmatrix} 1 & 61 & 0 \\ 0 & 1 & 7 \\ 0 & 0 & 1 \end{bmatrix}$. What is the $(1, 2)$ entry of A^3 ?

- (A) 184
- (B) 181
- (C) 182
- (D) 183
- (E) 185

Questions 43–45

43. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^{13}y^1 dV$.

- (A) $6 \left(\frac{13! 1!}{17!} \right)$
- (B) $\frac{1}{28}$
- (C) $17 \left(\frac{13! 1!}{17!} \right)$
- (D) $\frac{13! 1!}{17!}$
- (E) $\frac{1}{6} \left(\frac{13! 1!}{17!} \right)$

44. An entire function satisfies $|f(z)| \leq 17$ for all complex z . What is $f'(1)$?

- (A) 2
- (B) 4
- (C) 3
- (D) 1
- (E) 0

45. Find the coefficient of x^2 in $e^{1x} \log(1 + 13x)$.

- (A) $\frac{143}{2}$
- (B) $-\frac{117}{2}$
- (C) $-\frac{169}{2}$
- (D) -143
- (E) $-\frac{143}{2}$

Questions 46–48

46. An invertible 67×67 matrix A has determinant 4. Find $\det(A^T A)$.

- (A) 15
- (B) 18
- (C) 17
- (D) 14
- (E) 16

47. Evaluate $\int_0^1 \frac{x^1}{1 + 13x^2} dx$.

- (A) $\frac{\log(14)}{2}$
- (B) $-\frac{\log(14)}{26}$
- (C) $\frac{\log(14)}{26}$
- (D) $\frac{\log(14)}{13}$
- (E) $\log(14)$

48. Bisection starts with an interval of length 2^{27} . After 29 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{1}{16}$
- (B) $\frac{1}{24}$
- (C) $\frac{9}{16}$
- (D) $\frac{7}{8}$
- (E) $\frac{1}{8}$

Questions 49–51

49. For $x > 0$, $xy' = 13y$ and $y(1) = 1$. Find $y(e^2)$.

- (A) e^{26}
- (B) $2e^{26}$
- (C) $-e^{26}$
- (D) $1 + e^{26}$
- (E) $\frac{e^{26}}{2}$

50. How many invertible 2×2 matrices over $\mathbb{F}_{37}$ are there?

- (A) $(37^2 - 37^0)(37^2 - 37^1)$
- (B) $(37^2 - 37^0)(37^2 - 37^0)$
- (C) $37^2 37^2$
- (D) $(37^2 - 37^0)$
- (E) $(37^2 - 37^1)(37^2 - 37^2)$

51. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(13j)!}{(j!)^{13}} x^{2j}$?

- (A) $13^{-\frac{2}{13}}$
- (B) 13^{-13}
- (C) $13^{-\frac{13}{2}}$
- (D) $\frac{1}{13}$
- (E) $13^{\frac{13}{2}}$

Questions 52–54

52. What is the least positive multiplicative inverse of 2 modulo 55?

- (A) 46
- (B) 19
- (C) 28
- (D) 10
- (E) 37

53. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 13x$ and has density $\rho(x, y) = 1$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{5}{3}$
- (B) $\frac{2}{3}$
- (C) $\frac{4}{3}$
- (D) $\frac{1}{3}$
- (E) $-\frac{2}{3}$

54. How many permutations in S_{67} consist of one 4-cycle and 63 fixed points?

- (A) $2^6 \cdot 3 \cdot 5 \cdot 11 \cdot 13 \cdot 67$
- (B) $2^5 \cdot 3 \cdot 5 \cdot 11 \cdot 13 \cdot 67$
- (C) $2^4 \cdot 3 \cdot 5 \cdot 11 \cdot 13 \cdot 67$
- (D) $2^5 \cdot 3 \cdot 5^2 \cdot 11 \cdot 13 \cdot 67$
- (E) $2^5 \cdot 3^2 \cdot 5 \cdot 11 \cdot 13 \cdot 67$

Questions 55–57

55. Evaluate $\iiint_B 13(x^2 + y^2 + z^2)^1 dV$, where B is the unit ball.

- (A) $\frac{52\pi}{5}$
- (B) $-\frac{52\pi}{5}$
- (C) $1 + \frac{52\pi}{5}$
- (D) $\frac{26\pi}{5}$
- (E) $\frac{104\pi}{5}$

56. How many truth assignments to $p, q, r_1, \dots, r_{25}$ make $p \Rightarrow q$ false?

- (A) 2^{25}
- (B) 2^{26}
- (C) 2^{27}
- (D) 2^{28}
- (E) 2^{24}

57. What is the maximum of x^2y^{13} over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\left(\frac{2}{15}\right)^2 \left(\frac{13}{15}\right)^{13}$
- (B) $\left(\frac{2}{15}\right)^{13} \left(\frac{13}{15}\right)^2$
- (C) $15 \left(\left(\frac{2}{15}\right)^2 \left(\frac{13}{15}\right)^{13} \right)$
- (D) $\frac{1}{2^{15}}$
- (E) $\frac{26}{225}$

Questions 58–60

58. How many subgroups does the cyclic group of order 780 have?

- (A) 23
- (B) 24
- (C) 22
- (D) 25
- (E) 26

59. Let C be the unit circle $x^2 + y^2 = 1$, $z = 2$, counterclockwise viewed from above. Evaluate $\int_C -13y(x^2 + y^2) dx + (13x(x^2 + y^2)) dy$.

- (A) $1 + 26\pi$
- (B) 13π
- (C) -26π
- (D) 52π
- (E) 26π

60. What is the coefficient of x^2 in $(1 + 2x)^{27}$?

- (A) 936
- (B) 1404
- (C) 468
- (D) 1872
- (E) 2340

Questions 61–63

61. Evaluate $\int_0^1 \int_x^1 13y^2 \, dy \, dx$.

- (A) $\frac{13}{2}$
- (B) $\frac{13}{4}$
- (C) $\frac{13}{12}$
- (D) $\frac{13}{8}$
- (E) $\frac{13}{3}$

62. How many residues x modulo 84 satisfy $6x \equiv 42 \pmod{84}$?

- (A) 7
- (B) 8
- (C) 5
- (D) 4
- (E) 6

63. If $y' = 13y$, $y(0) = 1$, what is the coefficient of x^2 in the Maclaurin series of y ?

- (A) $\frac{169}{2}$
- (B) $\frac{169}{4}$
- (C) $\frac{171}{2}$
- (D) $-\frac{169}{2}$
- (E) 169

Questions 64–66

64. How many edges does the complete bipartite graph $K_{27,29}$ have?

- (A) 261
- (B) 1044
- (C) 783
- (D) 522
- (E) 1305

65. For $f(x, y) = x^2 + 13xy + 2y^2$, what is its maximum directional derivative at $(1, -1)$?

- (A) 202
- (B) -11
- (C) $\sqrt{202}$
- (D) 9
- (E) 20

66. The field K has 2^{13} elements. Find $[K : \mathbb{F}_2]$.

- (A) 14
- (B) 15
- (C) 12
- (D) 13
- (E) 11

Answer Key

Practice Test 65 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	E	23	C	45	E
2	C	24	B	46	E
3	C	25	B	47	C
4	A	26	D	48	E
5	E	27	A	49	A
6	C	28	E	50	A
7	E	29	E	51	C
8	B	30	D	52	C
9	E	31	D	53	B
10	A	32	D	54	B
11	D	33	E	55	A
12	B	34	C	56	A
13	A	35	B	57	A
14	C	36	B	58	B
15	B	37	B	59	E
16	D	38	D	60	B
17	A	39	C	61	B
18	C	40	D	62	E
19	D	41	C	63	A
20	A	42	D	64	C
21	D	43	D	65	C
22	A	44	E	66	D

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