

GRE Mathematics Practice Test 67

Advanced Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A demanding practice form with frequent multi-step arguments and a mix of computational and conceptual questions. This is cycle 13 of 19, position 2 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

zarimion.com/gre

100 practice tests, complete answer keys, and future updates.

Please retain the source attribution when sharing this material.



This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Evaluate $\int_0^1 x^{13}(\log x)^4 dx$.

- (A) $14 \left(\frac{(-1)^4 4!}{14^5} \right)$
(B) $-\frac{(-1)^4 4!}{14^5}$
(C) $\frac{1}{5} \left(\frac{(-1)^4 4!}{14^5} \right)$
(D) $\frac{4802}{3} \left(\frac{(-1)^4 4!}{14^5} \right)$
(E) $\frac{(-1)^4 4!}{14^5}$

2. What is the coefficient of x^7 in $(x + 13)^7(1 - x)^3$?

- (A) -66519
(B) -66520
(C) -66518
(D) -66522
(E) -66521

3. For $x > 0$, $x^2 y'' + (-32)xy' + 266y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

- (A) $-\frac{2(1 - e^5)e^{14}}{5}$
(B) $-\frac{(1 - e^5)e^{14}}{5}$
(C) $-\frac{(1 - e^5)e^{14}}{10}$
(D) $\frac{(1 - e^5)e^{14}}{5}$
(E) $-\frac{e^{14}}{5} + 1 + \frac{e^{19}}{5}$

Questions 4–6

4. A box is chosen uniformly from two boxes. Box A contains 29 red and 31 blue balls; box B contains 31 red and 29 blue balls. Two balls are drawn without replacement and both are red. What is the conditional probability that A was chosen?

- (A) $\frac{1277}{1742}$
- (B) $\frac{406}{2613}$
- (C) $\frac{465}{871}$
- (D) $\frac{406}{871}$
- (E) $\frac{203}{871}$

5. The solution of $y' = 14x^4y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\frac{14^{\frac{4}{5}}}{14}$
- (B) $\frac{\sqrt[5]{14} \cdot 5^{\frac{4}{5}}}{5}$
- (C) $\frac{14^{\frac{4}{5}} \sqrt[5]{5}}{14}$
- (D) $\frac{5}{14}$
- (E) $\frac{\sqrt[5]{96040}}{14}$

6. A matrix A satisfies $(A - 64I)(A - 66I) = 0$, and its minimal polynomial has degree two. If $A^5 = uA + vI$, find u .

- (A) $2^5 \cdot 31 \cdot 41 \cdot 4391$
- (B) $2^4 \cdot 31 \cdot 41 \cdot 4391$
- (C) $2^3 \cdot 31 \cdot 41 \cdot 4391$
- (D) $2^4 \cdot 3 \cdot 31 \cdot 41 \cdot 4391$
- (E) $2^4 \cdot 5 \cdot 31 \cdot 41 \cdot 4391$

Questions 7–9

7. Solve $y' + 14y = x^5 e^{-14x}$, $y(0) = 0$, and find $y(1)$.

- (A) $\frac{1}{5e^{14}}$
- (B) $\frac{e^{14}}{6}$
- (C) $\frac{1}{6e^{14}}$
- (D) $\frac{7}{3e^{14}}$
- (E) e^{-14}

8. Let $X_1, \dots, X_{29}$ be independent uniform random variables on $[0, 1]$. What is $E[(\max_i X_i)^2]$?

- (A) $\frac{29}{31}$
- (B) $\frac{29}{62}$
- (C) $\frac{29}{93}$
- (D) $\frac{2}{31}$
- (E) $\frac{30}{31}$

9. Let $f(x, y) = x^2 + 28xy + (201)y^2$. What is the product of the eigenvalues of the Hessian of f ?

- (A) 10
- (B) 5
- (C) 201
- (D) 20
- (E) 804

Questions 10–12

10. Let $V = \begin{bmatrix} 1 & 63 & 3969 \\ 1 & 65 & 4225 \\ 1 & 68 & 4624 \end{bmatrix}$. Find $\det(V^T V)$.

- (A) 899
- (B) 898
- (C) 901
- (D) 902
- (E) 900

11. Evaluate $\int_0^1 \frac{x^4}{1 + 14x^5} dx$.

- (A) $-\frac{\log(15)}{70}$
- (B) $\frac{\log(15)}{5}$
- (C) $\log(15)$
- (D) $\frac{\log(15)}{70}$
- (E) $\frac{\log(15)}{14}$

12. Independent trials have success probability $1/29$. Let T count trials up to and including the first success. Find $E[T^2]$.

- (A) 551
- (B) 1102
- (C) 1653
- (D) 2755
- (E) 2204

Questions 13–15

13. Let $f(x) = \log(1 + 14x)$. Find $f^{(5)}(0)$.

- (A) $\frac{1}{5} \left((-1)^4 4! 14^5 \right)$
- (B) $(-1)^4 4! 14^5$
- (C) $\frac{1}{14} \left((-1)^4 4! 14^5 \right)$
- (D) $-(-1)^4 4! 14^5$
- (E) $5 \left((-1)^4 4! 14^5 \right)$

14. Let $u = (1, 2, \dots, 69)^T$, $v = (1, 1, \dots, 1)^T$, and $B = I + uv^T$. Find $v^T B^{-1} u$.

- (A) $\frac{1207}{2416}$
- (B) $\frac{2415}{2416}$
- (C) $\frac{3623}{2416}$
- (D) $\frac{4831}{2416}$
- (E) $\frac{6039}{2416}$

15. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(14j)!}{(j!)^{14}} x^{5j}$?

- (A) 14^{-14}
- (B) $14^{-\frac{5}{14}}$
- (C) $14^{\frac{14}{5}}$
- (D) $\frac{1}{14}$
- (E) $14^{-\frac{14}{5}}$

Questions 16–18

16. A uniformly random subset of 29 objects is selected from a collection with 30 red and 31 blue objects. If X is the number of red objects selected, find $E[X]$.

- (A) $\frac{1740}{61}$
- (B) $\frac{61}{992}$
- (C) $\frac{61}{870}$
- (D) $\frac{931}{61}$
- (E) $\frac{435}{61}$

17. Find the curvature of the helix $r(t) = (14 \cos t, 14 \sin t, 5t)$.

- (A) $\frac{14}{221}$
- (B) $-\frac{14}{221}$
- (C) $\frac{28}{221}$
- (D) $\frac{235}{221}$
- (E) $\frac{7}{221}$

18. Let $D = \text{diag}(\underbrace{1, \dots, 1}_{12}, \underbrace{2, \dots, 2}_{57})$. Find the dimension of the space of real matrices X with $XD = DX$.

- (A) 3392
- (B) 3395
- (C) 3391
- (D) 3393
- (E) 3394

Questions 19–21

19. The region $0 \leq x \leq 1$, $0 \leq y \leq 14x^5$ is revolved about the line $x = -3$. Find its volume.

- (A) $1 + 18\pi$
- (B) 36π
- (C) 9π
- (D) 18π
- (E) -18π

20. X, Y are independent Poisson variables with means 29 and 30. Given $X + Y = 31$, find $\text{Var}(X \mid X + Y)$.

- (A) $\frac{26970}{3481}$
- (B) $\frac{13485}{3481}$
- (C) $\frac{33932}{3481}$
- (D) $\frac{30451}{3481}$
- (E) $\frac{53940}{3481}$

21. For $x > 0$, $x^2y'' + (1 - 19)xy' + 70y = 0$, $y(1) = 0$, $y'(1) = 14$. Find $y(e)$.

- (A) $-\frac{14(1 - e^9)e^5}{9}$
- (B) $e^5 + e^{14}$
- (C) $\frac{14(1 - e^9)e^5}{9}$
- (D) $-e^5 + e^{14}$
- (E) $\frac{e^{14}}{19}$

Questions 22–24

22. When $x^3 + 1$ is divided by $(x - 64)(x - 65)$, the remainder is $ux + v$. Find u .

- (A) 12483
- (B) 12479
- (C) 12482
- (D) 12481
- (E) 12480

23. The solution of $y' = 14x^5y^3$ has $y(0) = 1/15$. Find $y(1)$.

- (A) $\frac{\sqrt{1983}}{1322}$
- (B) $\frac{2\sqrt{1983}}{661}$
- (C) $-\frac{\sqrt{1983}}{661}$
- (D) $\frac{\sqrt{1983}}{661} + 1$
- (E) $\frac{\sqrt{1983}}{661}$

24. For which real a does $f_m(x) = m^a x^{24}(1 - x)^m$ converge uniformly to zero on $[0, 1]$?

- (A) $a \leq 24$
- (B) $a > 24$
- (C) $a < 24$
- (D) $a < 0$
- (E) All real a

Questions 25–27

25. The local inverse $y(x)$ of $x = 14y + y^3$ satisfies $y(0) = 0$. Find $y^{(5)}(0)$.

- (A) $\frac{1}{120} \left(\frac{360}{14^7} \right)$
- (B) $\frac{360}{14^7}$
- (C) $14 \left(\frac{360}{14^7} \right)$
- (D) $\frac{1}{14} \left(\frac{360}{14^7} \right)$
- (E) $-\frac{360}{14^7}$

26. Let $P = \text{diag}(\underbrace{1, \dots, 1}_4, \underbrace{0, \dots, 0}_{66})$. What is the dimension of the space of 70×70 real matrices X satisfying $PX = X$ and $XP = 0$?

- (A) 263
- (B) 262
- (C) 264
- (D) 265
- (E) 266

27. Let $\mathbf{F} = (14y^5, z^5, x^5)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (1, 1, 3)/\sqrt{11}$.

- (A) $1 - 20\sqrt{11}$
- (B) $-10\sqrt{11}$
- (C) $-40\sqrt{11}$
- (D) $20\sqrt{11}$
- (E) $-20\sqrt{11}$

Questions 28–30

28. For which real p does $\sum_{k=2}^{\infty} 1/[k(\log k)^{24p}]$ converge?

- (A) $p > 0$
- (B) $p \geq 1/24$
- (C) All real p
- (D) $p < 1/24$
- (E) $p > 1/24$

29. For $D = \{(x, y) : x^2 + y^2 \leq 14\}$, evaluate $\iint_D (x^2 + y^2)^4 dA$.

- (A) $\frac{1075648\pi}{5}$
- (B) $\frac{537824\pi}{9}$
- (C) $\frac{38416\pi}{5}$
- (D) $\frac{537824\pi}{5}$
- (E) 537824π

30. A 3×3 matrix A has characteristic polynomial $(t - 63)(t + 1)(t - 2)$. What is $\text{tr}(A^3 - 2A + I)$?

- (A) 249929
- (B) 249931
- (C) 249927
- (D) 249928
- (E) 249930

Questions 31–33

31. If $y' + 14y = 5x^5e^{-14x}$ and $y(0) = 3$, find $y(1)$.

- (A) $\frac{23}{6e^{14}}$
- (B) $\frac{23}{12e^{14}}$
- (C) $\frac{23}{6e^{14}} + 1$
- (D) $-\frac{23}{6e^{14}}$
- (E) $\frac{23}{3e^{14}}$

32. How many accumulation points in $\mathbb{R}$ does $S = \{1/k : k \geq 1\} \cup \{24 + 1/k : k \geq 1\}$ have?

- (A) 1
- (B) 4
- (C) 3
- (D) 2
- (E) 0

33. Evaluate $\iiint_B 14z^2(x^2 + y^2 + z^2)^5 dV$, where B is the unit ball.

- (A) $\frac{112\pi}{45}$
- (B) $\frac{56\pi}{45}$
- (C) $\frac{28\pi}{45}$
- (D) $-\frac{56\pi}{45}$
- (E) $1 + \frac{56\pi}{45}$

Questions 34–36

34. Let N be the 69×69 matrix with ones immediately above the diagonal and zeros elsewhere. Find $\dim(\ker N^{22} / \ker N^{11})$.

- (A) 9
- (B) 12
- (C) 13
- (D) 10
- (E) 11

35. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-14)^j}{j^{\frac{4}{3}} (\log j)^m}$ converge absolutely at $x = 13$? Give the least such m , or 0 if none exists.

- (A) 4
- (B) 0
- (C) 1
- (D) 3
- (E) 2

36. How many connected components does $\mathbb{R} \setminus \{1, 2, \dots, 24\}$ have?

- (A) 9
- (B) 33
- (C) 25
- (D) 17
- (E) 41

Questions 37–39

- 37.** Find the total outward flux of $\mathbf{F} = (14x^5y, y^5z, xz^5)$ through the boundary of $[0, 1]^3$.
- (A) 8
 - (B) 16
 - (C) -8
 - (D) 9
 - (E) 4
- 38.** Let A be the 69×69 matrix whose diagonal entries are 66 and whose off-diagonal entries are 1. What is its largest eigenvalue?
- (A) 135
 - (B) 136
 - (C) 133
 - (D) 134
 - (E) 132
- 39.** For $f(x, y) = x^2 + 14xy + 5y^2$, what is its maximum directional derivative at $(1, -1)$?
- (A) -12
 - (B) $4\sqrt{10}$
 - (C) 16
 - (D) 4
 - (E) 160

Questions 40–42

40. The circle $|z| = 2$ is traversed counterclockwise. What is $\frac{1}{2\pi i} \int_{|z|=2} \frac{e^z}{z^{20}} dz$?

- (A) $19!$
- (B) $1/20!$
- (C) $1/19!$
- (D) $1/38!$
- (E) $1/18!$

41. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{14t} \cos(5t)$, and $y = e^{14t} \sin(5t)$, find d^2f/dt^2 at $t = 1$.

- (A) 784
- (B) 28
- (C) 0
- (D) 442
- (E) 10

42. How many maximal proper subgroups does the cyclic group of order 804 have?

- (A) 3
- (B) 5
- (C) 2
- (D) 4
- (E) 1

Questions 43–45

43. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(14x) - \sum_{j=0}^3 (-1)^j (14x)^{2j+1} / (2j+1)!}{x^9}$.
- (A) $\frac{1}{14} \left(\frac{(-1)^4 14^9}{9!} \right)$
(B) $\frac{(-1)^4 14^9}{9!}$
(C) $9 \left(\frac{(-1)^4 14^9}{9!} \right)$
(D) $14 \left(\frac{(-1)^4 14^9}{9!} \right)$
(E) $-\frac{(-1)^4 14^9}{9!}$
44. Find the residue of $z^{19}/(z-1)^3$ at $z=1$.
- (A) 171
(B) 228
(C) 57
(D) 285
(E) 114
45. Two solutions of $y'' + 14x^5y' + 5y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.
- (A) $e^{-\frac{7}{3}}$
(B) $-\frac{1}{\frac{7}{e^{\frac{7}{3}}}}$
(C) $\frac{1}{\frac{7}{2e^{\frac{7}{3}}}}$
(D) $e^{-\frac{7}{3}} + 1$
(E) $\frac{2}{\frac{7}{e^{\frac{7}{3}}}}$

Questions 46–48

46. How many residues x modulo 102 satisfy $9x \equiv 27 \pmod{102}$ and $2x \equiv 0 \pmod{102}$?

- (A) 3
- (B) 1
- (C) 0
- (D) 4
- (E) 2

47. Let C be the unit circle $x^2 + y^2 = 1$, $z = 5$, counterclockwise viewed from above. Evaluate $\int_C -14y(x^2 + y^2)^4 dx + (14x(x^2 + y^2)^4) dy + 3z dz$.

- (A) 28π
- (B) 56π
- (C) -28π
- (D) 14π
- (E) $1 + 28\pi$

48. How many functions from a set of 29 elements onto a set of 3 elements exist?

- (A) $3^{29} - 2^{29}$
- (B) 3^{29}
- (C) $3^{29} - 3(2^{29}) - 3$
- (D) $3^{29} - 3(2^{29}) + 3$
- (E) $3^{29} - 3(2^{29})$

Questions 49–51

49. Under $x = u + 14v$, $y = u - 5v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) $\frac{1}{19}$
- (B) 19
- (C) 361
- (D) 70
- (E) 9

50. The field K has 7^{14} elements. Find the largest possible multiplicative order of a nonzero element of K .

- (A) 7^{14}
- (B) $7^{14} - 1$
- (C) $7^{15} - 1$
- (D) $7^{14} + 1$
- (E) $7^{13} - 1$

51. Evaluate $\int_C (5x^4 + 14y) dx + (14x + 5y^4) dy$, where C is $r(t) = (t, 3t^2)$, $0 \leq t \leq 1$.

- (A) 572
- (B) -286
- (C) 143
- (D) 287
- (E) 286

Questions 52–54

52. How many nonnegative integer triples sum to 58, if each coordinate is at most 29?

- (A) 310
- (B) 620
- (C) 775
- (D) 465
- (E) 155

53. Find the coefficient of x^5 in the Taylor series of $(1 - 14x)^{-2}$ at zero.

- (A) $\frac{5}{6} (6 \cdot 14^5)$
- (B) $-6 \cdot 14^5$
- (C) $\frac{1}{6} (6 \cdot 14^5)$
- (D) $6 \cdot 14^5$
- (E) $\frac{1}{14} (6 \cdot 14^5)$

54. Let $G = \mathbb{Z}/408\mathbb{Z} \times \mathbb{Z}/276\mathbb{Z}$. Find the order of G/H , where $H = \{g \in G : 12g = 0\}$.

- (A) 781
- (B) 783
- (C) 784
- (D) 780
- (E) 782

Questions 55–57

55. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^{14}y^4 dV$.

- (A) $\frac{1}{6} \left(\frac{14! 4!}{21!} \right)$
- (B) $\frac{14! 4!}{21!}$
- (C) $\frac{1}{75}$
- (D) $6 \left(\frac{14! 4!}{21!} \right)$
- (E) $21 \left(\frac{14! 4!}{21!} \right)$

56. A tree has 29 vertices of degree 3; every other vertex has degree 1. How many leaves does it have?

- (A) 21
- (B) 11
- (C) 51
- (D) 41
- (E) 31

57. Evaluate $\int_0^1 \int_0^x x^{14}y^5 dy dx$.

- (A) $\frac{1}{120}$
- (B) $\frac{1}{90}$
- (C) $\frac{1}{126}$
- (D) $\frac{1}{21}$
- (E) $\frac{1}{147}$

Questions 58–60

58. How many group homomorphisms $\mathbb{Z}/408\mathbb{Z} \rightarrow \mathbb{Z}/276\mathbb{Z}$ have image of order 12?

- (A) 2
- (B) 3
- (C) 5
- (D) 6
- (E) 4

59. Find the coefficient of x^6 in $\cos(14x)/(1 - 4x^2)$.

- (A) $-\frac{250156}{45}$
- (B) $-\frac{250201}{45}$
- (C) $-\frac{470596}{45}$
- (D) -4002496
- (E) $\frac{250156}{45}$

60. A connected planar graph has 32 vertices and 59 edges. How many faces, including the unbounded face, are in a planar embedding?

- (A) 47
- (B) 29
- (C) 20
- (D) 38
- (E) 11

Questions 61–63

61. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(14+5j)(19+5j)}$.

- (A) $\frac{1}{14}$
- (B) $\frac{5}{14}$
- (C) $\frac{1}{19}$
- (D) $\frac{1}{5}$
- (E) $\frac{1}{70}$

62. How many 2-dimensional subspaces does $\mathbb{F}_{41}^6$ have?

- (A) $\frac{(41^7 - 1)(41^7 - 41)}{(41^2 - 1)(41^2 - 41)}$
- (B) $\frac{(41^8 - 1)(41^8 - 41)}{(41^2 - 1)(41^2 - 41)}$
- (C) $\frac{(41^6 - 1)(41^6 - 41)}{(41^2 - 1)(41^2 - 41)}$
- (D) $\frac{(41^9 - 1)(41^9 - 41)}{(41^2 - 1)(41^2 - 41)}$
- (E) $\frac{(41^5 - 1)(41^5 - 41)}{(41^2 - 1)(41^2 - 41)}$

63. For the curve $x = t^5$, $y = t^6 + 14t$, find d^2y/dx^2 at $t = 1$.

- (A) $-\frac{14}{5}$
- (B) -10
- (C) 4
- (D) $\frac{62}{25}$
- (E) -2

Questions 64–66

64. How many integers in $\{1, \dots, 174\}$ are divisible by 2 or 3?

- (A) 116
- (B) 154
- (C) 78
- (D) 192
- (E) 40

65. Evaluate $\int_0^1 \int_x^1 14y^5 \, dy \, dx$.

- (A) $\frac{14}{5}$
- (B) 7
- (C) 2
- (D) $\frac{1}{3}$
- (E) $\frac{7}{3}$

66. How many monic irreducible polynomials of degree 4 are there over $\mathbb{F}_{41}$?

- (A) $\frac{3(41^4 - 41^2)}{4}$
- (B) $2(41^4 - 41^2)$
- (C) $41^4 - 41^2$
- (D) $\frac{41^4 - 41^2}{4}$
- (E) $\frac{2(41^4 - 41^2)}{4}$

Answer Key

Practice Test 67 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	E	23	E	45	A
2	B	24	C	46	C
3	B	25	B	47	A
4	D	26	C	48	D
5	C	27	E	49	B
6	B	28	E	50	B
7	C	29	D	51	E
8	A	30	A	52	D
9	D	31	A	53	D
10	E	32	D	54	E
11	D	33	B	55	B
12	C	34	E	56	E
13	B	35	C	57	C
14	B	36	C	58	E
15	E	37	A	59	A
16	C	38	D	60	B
17	A	39	B	61	E
18	D	40	C	62	C
19	D	41	C	63	E
20	A	42	A	64	A
21	A	43	B	65	C
22	D	44	A	66	D

zarimion.com/gre

100 practice tests, complete answer keys, and future updates.
Please retain the source attribution when sharing this material.

