

GRE Mathematics Practice Test 68

Upper-Intermediate Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

Intermediate reinforcement, combining standard applications with selected more demanding arguments. This is cycle 13 of 19, position 3 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Evaluate $\int_0^{\log 2} e^x (e^x + 14)^4 dx$.

- (A) $\frac{16^5 - 15^5}{5}$
(B) $-\frac{16^5 - 15^5}{5}$
(C) $16^4 - 15^4$
(D) $5 \left(\frac{16^5 - 15^5}{5} \right)$
(E) $\frac{16^5}{5}$

2. When $x^3 + 1$ is divided by $(x - 65)(x - 66)$, the remainder is $ux + v$. Find u .

- (A) 12872
(B) 12873
(C) 12870
(D) 12871
(E) 12869

3. For $x > 0$, $x^2 y'' + (1 - 18)xy' + 56y = 0$, $y(1) = 0$, $y'(1) = 14$. Find $y(e)$.

- (A) $-\frac{7(1 - e^{10})e^4}{5}$
(B) $-e^4 + e^{14}$
(C) $\frac{7(1 - e^{10})e^4}{5}$
(D) $e^4 + e^{14}$
(E) $\frac{e^{14}}{18}$

Questions 4–6

4. A box is chosen uniformly from two boxes. Box A contains 30 red and 32 blue balls; box B contains 32 red and 30 blue balls. Two balls are drawn without replacement and both are red. What is the conditional probability that A was chosen?

- (A) $\frac{435}{1862}$
- (B) $\frac{683}{931}$
- (C) $\frac{496}{931}$
- (D) $\frac{435}{931}$
- (E) $\frac{145}{931}$

5. If $y' = y(1 - y/4)$ and $y(0) = 2$, find $y(-\log 14)$.

- (A) $\frac{4}{15}$
- (B) $\frac{8}{15}$
- (C) $\frac{19}{15}$
- (D) $\frac{2}{15}$
- (E) $-\frac{4}{15}$

6. Let $P = \text{diag}(\underbrace{1, \dots, 1}_1, \underbrace{0, \dots, 0}_{70})$. What is the dimension of the space of 71×71 real matrices X satisfying $PX = X$ and $XP = 0$?

- (A) 70
- (B) 68
- (C) 72
- (D) 71
- (E) 69

Questions 7–9

7. Evaluate $\int_0^\infty x^4 e^{-14x} dx$.

- (A) $\frac{4!}{14^5}$
- (B) $\frac{1}{4} \left(\frac{4!}{14^5} \right)$
- (C) $\frac{1}{14} \left(\frac{4!}{14^5} \right)$
- (D) $14 \left(\frac{4!}{14^5} \right)$
- (E) 24

8. Let $X_1, \dots, X_{30}$ be independent uniform random variables on $[0, 1]$. What is $E[(\max_i X_i)^2]$?

- (A) $\frac{15}{32}$
- (B) $\frac{1}{16}$
- (C) $\frac{15}{16}$
- (D) $\frac{31}{32}$
- (E) $\frac{5}{16}$

9. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(14x) - \sum_{j=0}^2 (-1)^j (14x)^{2j+1} / (2j+1)!}{x^7}$.

- (A) $-\frac{(-1)^3 14^7}{7!}$
- (B) $7 \left(\frac{(-1)^3 14^7}{7!} \right)$
- (C) $14 \left(\frac{(-1)^3 14^7}{7!} \right)$
- (D) $\frac{1}{14} \left(\frac{(-1)^3 14^7}{7!} \right)$
- (E) $\frac{(-1)^3 14^7}{7!}$

Questions 10–12

10. A 3×3 matrix A has characteristic polynomial $(t - 64)(t + 1)(t - 2)$. What is $\text{tr}(A^3)$?

- (A) 262149
- (B) 262150
- (C) 262153
- (D) 262151
- (E) 262152

11. Evaluate $\int_0^{\pi/2} \sin^9 t \cos^{29} t \, dt$.

- (A) $\frac{1}{2} \left(\frac{4! 14!}{2 \cdot 19!} \right)$
- (B) $\frac{4! 14!}{2 \cdot 19!}$
- (C) $19 \left(\frac{4! 14!}{2 \cdot 19!} \right)$
- (D) $3060 \left(\frac{4! 14!}{2 \cdot 19!} \right)$
- (E) $2 \left(\frac{4! 14!}{2 \cdot 19!} \right)$

12. Independent trials have success probability $1/30$. Let T count trials up to and including the first success. Find $E[T]$.

- (A) 50
- (B) 30
- (C) 10
- (D) 20
- (E) 40

Questions 13–15

13. Evaluate $\iiint_B 14z^2(x^2 + y^2 + z^2)^4 dV$, where B is the unit ball.

- (A) $\frac{28\pi}{39}$
- (B) $\frac{112\pi}{39}$
- (C) $1 + \frac{56\pi}{39}$
- (D) $-\frac{56\pi}{39}$
- (E) $\frac{56\pi}{39}$

14. Let N be the 70×70 matrix with ones immediately above the diagonal and zeros elsewhere. Find $\dim(\ker N^{22} / \ker N^{11})$.

- (A) 13
- (B) 9
- (C) 11
- (D) 10
- (E) 12

15. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-14)^j}{j^1 (\log j)^m}$ converge absolutely at $x = 13$? Give the least such m , or 0 if none exists.

- (A) 1
- (B) 0
- (C) 3
- (D) 2
- (E) 4

Questions 16–18

16. A uniformly random subset of 30 objects is selected from a collection with 31 red and 32 blue objects. If X is the number of red objects selected, find $E[X]$.

- (A) $\frac{310}{21}$
- (B) $\frac{352}{21}$
- (C) $\frac{331}{21}$
- (D) $\frac{155}{21}$
- (E) $\frac{620}{21}$

17. The solution of $y' = 14x^3y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\frac{14^{\frac{3}{4}}}{14}$
- (B) $\frac{2^{\frac{3}{4}}\sqrt[4]{7}}{2}$
- (C) $\frac{2}{7}$
- (D) $\frac{7^{\frac{3}{4}}}{7}$
- (E) $\frac{\sqrt[4]{2} \cdot 7^{\frac{3}{4}}}{7}$

18. Let A be the 70×70 matrix whose diagonal entries are 67 and whose off-diagonal entries are 1. What is its largest eigenvalue?

- (A) 137
- (B) 138
- (C) 134
- (D) 135
- (E) 136

Questions 19–21

19. Find the coefficient of x^4 in $e^{3x} \log(1 + 14x)$.

- (A) 7238
- (B) -9604
- (C) -173712
- (D) -7238
- (E) -7224

20. X, Y are independent Poisson variables with means 30 and 31. Given $X + Y = 32$, find $\text{Var}(X \mid X + Y)$.

- (A) $\frac{14880}{3721}$
- (B) $\frac{33481}{3721}$
- (C) $\frac{3721}{37202}$
- (D) $\frac{3721}{29760}$
- (E) $\frac{59520}{3721}$

21. For the curve $x = t^4$, $y = t^5 + 14t$, find d^2y/dx^2 at $t = 1$.

- (A) $-\frac{37}{4}$
- (B) $-\frac{37}{16}$
- (C) $\frac{19}{4}$
- (D) $\frac{47}{16}$
- (E) $-\frac{7}{2}$

Questions 22–24

22. Positive real numbers x, y satisfy $\log_2 x + \log_2 y = \log_2 768$ and $x + y = 76$. Find $x^2 + y^2$.

- (A) 4241
- (B) 4242
- (C) 4239
- (D) 4240
- (E) 4238

23. Find the coefficient of x^6 in $\cos(14x)/(1 - 3x^2)$.

- (A) $\frac{292981}{45}$
- (B) $-\frac{292981}{45}$
- (C) $-\frac{293026}{45}$
- (D) -4687696
- (E) $-\frac{470596}{45}$

24. For which real a does $f_m(x) = m^a x^{25}(1 - x)^m$ converge uniformly to zero on $[0, 1]$?

- (A) $a < 25$
- (B) All real a
- (C) $a < 0$
- (D) $a > 25$
- (E) $a \leq 25$

Questions 25–27

25. If $y' = 28xy$ and $y(0) = 1$, find the coefficient of x^8 in the Maclaurin series of y .

- (A) $\frac{4802}{3}$
- (B) $-\frac{4802}{3}$
- (C) $\frac{2401}{3}$
- (D) $\frac{4805}{3}$
- (E) $\frac{9604}{3}$

26. A sequence satisfies $u_0 = 64$ and $u_{k+1} = 2u_k + 12$ for all $k \geq 0$. Find u_5 .

- (A) 2420
- (B) 2422
- (C) 2418
- (D) 2419
- (E) 2421

27. Evaluate $\int_0^1 \frac{x^3}{1 + 14x^4} dx$.

- (A) $\frac{\log(15)}{14}$
- (B) $-\frac{\log(15)}{56}$
- (C) $\log(15)$
- (D) $\frac{\log(15)}{56}$
- (E) $\frac{\log(15)}{4}$

Questions 28–30

28. For which real p does $\sum_{k=2}^{\infty} 1/[k(\log k)^{25p}]$ converge?

- (A) $p > 0$
- (B) $p \geq 1/25$
- (C) All real p
- (D) $p > 1/25$
- (E) $p < 1/25$

29. Evaluate $\lim_{x \rightarrow 0} \frac{e^{14x} - \sum_{j=0}^3 (14x)^j / j!}{x^4}$.

- (A) $24 \left(\frac{14^4}{4!} \right)$
- (B) $-\frac{14^4}{4!}$
- (C) $4 \left(\frac{14^4}{4!} \right)$
- (D) $\frac{1}{14} \left(\frac{14^4}{4!} \right)$
- (E) $\frac{14^4}{4!}$

30. Linear maps $T : \mathbb{R}^{71} \rightarrow \mathbb{R}^{71}$ and $U : \mathbb{R}^{71} \rightarrow \mathbb{R}^{71}$ have ranks 2 and 2. What is the largest possible rank of UT ?

- (A) 4
- (B) 2
- (C) 0
- (D) 3
- (E) 1

Questions 31–33

31. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 14z^2 = x + 4y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.
- (A) -28
(B) -4
(C) -112
(D) -56
(E) 112
32. How many accumulation points in $\mathbb{R}$ does $S = \{1/k : k \geq 1\} \cup \{25 + 1/k : k \geq 1\}$ have?
- (A) 4
(B) 1
(C) 2
(D) 3
(E) 0
33. The solution of $y' = 14x^4y^3$ has $y(0) = 1/15$. Find $y(1)$.
- (A) $\frac{\sqrt{5485}}{1097}$
(B) $\frac{\sqrt{5485}}{1097} + 1$
(C) $\frac{\sqrt{5485}}{2194}$
(D) $\frac{2\sqrt{5485}}{1097}$
(E) $-\frac{\sqrt{5485}}{1097}$

Questions 34–36

34. Fix a nonzero vector $u \in \mathbb{R}^{70}$. What is the dimension of the space of skew-symmetric 70×70 matrices A satisfying $Au = 0$?

- (A) 2345
- (B) 2344
- (C) 2346
- (D) 2347
- (E) 2348

35. Under $x = u + 14v$, $y = u - 4v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) 56
- (B) $\frac{1}{18}$
- (C) 10
- (D) 18
- (E) 324

36. How many connected components does $\mathbb{R} \setminus \{1, 2, \dots, 25\}$ have?

- (A) 42
- (B) 18
- (C) 34
- (D) 10
- (E) 26

Questions 37–39

37. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(14j)!}{(j!)^{14}} x^{4j}$?

- (A) $14^{-\frac{2}{7}}$
- (B) 14^{-14}
- (C) $14^{-\frac{7}{2}}$
- (D) $\frac{1}{14}$
- (E) $14^{\frac{7}{2}}$

38. Let $A = \begin{bmatrix} 1 & 64 & 0 \\ 0 & 1 & 12 \\ 0 & 0 & 1 \end{bmatrix}$. What is the $(1, 3)$ entry of A^2 ?

- (A) 770
- (B) 768
- (C) 766
- (D) 769
- (E) 767

39. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^{14} y^3 dV$.

- (A) $6 \left(\frac{14! 3!}{20!} \right)$
- (B) $\frac{14! 3!}{20!}$
- (C) $\frac{1}{60}$
- (D) $20 \left(\frac{14! 3!}{20!} \right)$
- (E) $\frac{1}{6} \left(\frac{14! 3!}{20!} \right)$

Questions 40–42

40. The circle $|z| = 2$ is traversed counterclockwise. What is $\frac{1}{2\pi i} \int_{|z|=2} \frac{e^z}{z^{21}} dz$?

- (A) $1/19!$
- (B) $1/20!$
- (C) $1/40!$
- (D) $20!$
- (E) $1/21!$

41. For $D = \{(x, y) : x^2 + y^2 \leq 14\}$, evaluate $\iint_D (x^2 + y^2)^3 dA$.

- (A) 38416π
- (B) 686π
- (C) 5488π
- (D) 9604π
- (E) 19208π

42. How many elements z of $\mathbb{F}_{5^{14}}$ satisfy $z^{25} = z$?

- (A) 26
- (B) 23
- (C) 27
- (D) 25
- (E) 24

Questions 43–45

43. Two solutions of $y'' + 14x^4y' + 4y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $e^{-\frac{14}{5}} + 1$
- (B) $-\frac{1}{e^{\frac{14}{5}}}$
- (C) $\frac{2}{e^{\frac{14}{5}}}$
- (D) $\frac{1}{2e^{\frac{14}{5}}}$
- (E) $e^{-\frac{14}{5}}$

44. Find the residue of $z^{20}/(z-1)^3$ at $z = 1$.

- (A) 190
- (B) 316
- (C) 253
- (D) 64
- (E) 127

45. The local inverse $y(x)$ of $x = 14y + y^3$ satisfies $y(0) = 0$. Find $y^{(3)}(0)$.

- (A) $-\frac{6}{14^4}$
- (B) $\frac{1}{6} \left(-\frac{6}{14^4} \right)$
- (C) $14 \left(-\frac{6}{14^4} \right)$
- (D) $\frac{6}{14^4}$
- (E) $\frac{1}{14} \left(-\frac{6}{14^4} \right)$

Questions 46–48

46. How many generators does the cyclic group of order 2825761 have?

- (A) $41^6(41 - 1)$
- (B) $41^5(41 - 1)$
- (C) $41^2(41 - 1)$
- (D) $41^4(41 - 1)$
- (E) $41^3(41 - 1)$

47. Let C be the unit circle $x^2 + y^2 = 1$, $z = 4$, counterclockwise viewed from above. Evaluate $\int_C -14y(x^2 + y^2)^3 dx + (14x(x^2 + y^2)^3) dy + 4z dz$.

- (A) 14π
- (B) 28π
- (C) 56π
- (D) -28π
- (E) $1 + 28\pi$

48. How many functions from a set of 30 elements onto a set of 3 elements exist?

- (A) $3^{30} - 3(2^{30})$
- (B) $3^{30} - 3(2^{30}) + 3$
- (C) $3^{30} - 3(2^{30}) - 3$
- (D) 3^{30}
- (E) $3^{30} - 2^{30}$

Questions 49–51

49. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 14x$ and has density $\rho(x, y) = x^2y$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{5}{3}$
- (B) $\frac{5}{6}$
- (C) $-\frac{5}{6}$
- (D) $\frac{5}{12}$
- (E) $\frac{11}{6}$

50. Let $R = (\mathbb{Z}/85\mathbb{Z})/([17])$, where $([17])$ is the ideal generated by $[17]$. How many units does R have?

- (A) 15
- (B) 18
- (C) 16
- (D) 17
- (E) 14

51. Evaluate $\int_0^1 x^{13}(\log x)^3 dx$.

- (A) $\frac{1372}{3} \left(\frac{(-1)^3 3!}{14^4} \right)$
- (B) $\frac{(-1)^3 3!}{14^4}$
- (C) $14 \left(\frac{(-1)^3 3!}{14^4} \right)$
- (D) $\frac{1}{4} \left(\frac{(-1)^3 3!}{14^4} \right)$
- (E) $-\frac{(-1)^3 3!}{14^4}$

Questions 52–54

- 52.** How many nonnegative integer triples sum to 60, if each coordinate is at most 30?
- (A) 331
 - (B) 166
 - (C) 496
 - (D) 661
 - (E) 826
- 53.** The region $0 \leq x \leq 1$, $0 \leq y \leq 14x^4$ is revolved about the line $x = -4$. Find its volume.
- (A) $1 + \frac{406\pi}{15}$
 - (B) $\frac{406\pi}{15}$
 - (C) $\frac{203\pi}{15}$
 - (D) $\frac{812\pi}{15}$
 - (E) $-\frac{406\pi}{15}$
- 54.** What is the largest integer k such that 3^k divides $73!$?
- (A) 32
 - (B) 36
 - (C) 35
 - (D) 33
 - (E) 34

Questions 55–57

55. If $y'' + 16y = 14$ and $y(0) = y'(0) = 0$, find $y(\pi/4)$.

- (A) $-\frac{7}{4}$
- (B) $\frac{7}{2}$
- (C) $-\frac{7}{4}$
- (D) $\frac{7}{8}$
- (E) $\frac{11}{4}$

56. A tree has 34 vertices. What is the sum of its vertex degrees?

- (A) 44
- (B) 88
- (C) 66
- (D) 22
- (E) 110

57. Let C be $x^2/16 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C \left(-\frac{14y^3}{3}\right) dx + \left(\frac{x^3}{3}\right) dy$.

- (A) 60π
- (B) 15π
- (C) -30π
- (D) $1 + 30\pi$
- (E) 30π

Questions 58–60

58. In the ring $\mathbb{Z}/408\mathbb{Z}$, how many elements satisfy $x^2 = 1$?

- (A) 17
- (B) 14
- (C) 18
- (D) 15
- (E) 16

59. Let $f(x, y) = x^2 + 28xy + (200)y^2$. What is the product of the eigenvalues of the Hessian of f ?

- (A) 8
- (B) 4
- (C) 16
- (D) 800
- (E) 200

60. A connected planar graph has 33 vertices and 61 edges. How many faces, including the unbounded face, are in a planar embedding?

- (A) 10
- (B) 20
- (C) 30
- (D) 50
- (E) 40

Questions 61–63

61. Let $\mathbf{F} = (14y^4, z^4, x^4)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (1, 1, 4)/\sqrt{18}$.

- (A) $-\frac{232\sqrt{2}}{3}$
- (B) $-\frac{58\sqrt{2}}{3}$
- (C) $-\frac{116\sqrt{2}}{3}$
- (D) $1 - \frac{116\sqrt{2}}{3}$
- (E) $\frac{116\sqrt{2}}{3}$

62. A permutation consists of disjoint cycles of lengths 5 and 5, together with a disjoint transposition when the sum of these two lengths is odd. It fixes all other points. What is its order?

- (A) 7
- (B) 5
- (C) 4
- (D) 6
- (E) 3

63. Evaluate $\int_C (4x^3 + 14y) dx + (14x + 4y^3) dy$, where C is $r(t) = (t, 4t^2)$, $0 \leq t \leq 1$.

- (A) $\frac{313}{2}$
- (B) 314
- (C) -313
- (D) 313
- (E) 626

Questions 64–66

64. How many integers in $\{1, \dots, 180\}$ are divisible by 2 or 3?

- (A) 200
- (B) 40
- (C) 160
- (D) 80
- (E) 120

65. Find the arc length of $r(t) = (14(t - t^3/3), 14t^2)$ for $0 \leq t \leq 1/4$.

- (A) $-\frac{343}{96}$
- (B) $\frac{343}{48}$
- (C) $\frac{343}{192}$
- (D) $\frac{439}{96}$
- (E) $\frac{343}{96}$

66. In $R = \mathbb{F}_5[x]/(x^{14})$, how many elements are units?

- (A) $5^{14} - 5^{13}$
- (B) $5^{16} - 5^{15}$
- (C) $5^{15} - 5^{14}$
- (D) $5^{14} - 5^0$
- (E) $5^{15} - 5^{13}$

Answer Key

Practice Test 68 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	A	23	B	45	A
2	D	24	A	46	E
3	A	25	A	47	B
4	D	26	A	48	B
5	A	27	D	49	B
6	A	28	D	50	C
7	A	29	E	51	B
8	C	30	B	52	C
9	E	31	C	53	B
10	D	32	C	54	E
11	B	33	A	55	C
12	B	34	C	56	C
13	E	35	D	57	E
14	C	36	E	58	E
15	D	37	C	59	C
16	A	38	B	60	C
17	E	39	B	61	C
18	E	40	B	62	B
19	D	41	D	63	D
20	D	42	D	64	E
21	B	43	E	65	E
22	D	44	A	66	A

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