

GRE Mathematics Practice Test 69

Bridge to Standard

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A bridge toward standard practice, with routine multi-step questions and a smaller number of challenging concepts. This is cycle 13 of 19, position 4 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. The solution of $y' = 14x^2y^2$, $y(0) = 1$, has its first positive singularity at which x ?
- (A) $\frac{\sqrt[3]{14} \cdot 3^{\frac{2}{3}}}{3}$
(B) $\frac{3}{14}$
(C) $\frac{14^{\frac{2}{3}}}{14}$
(D) $\frac{\sqrt[3]{294}}{14}$
(E) $\frac{14^{\frac{2}{3}} \sqrt[3]{3}}{14}$
2. Positive real numbers x, y satisfy $\log_2 x + \log_2 y = \log_2 650$ and $x + y = 75$. Find xy .
- (A) 649
(B) 651
(C) 648
(D) 652
(E) 650
3. Evaluate $\int_0^1 x^{14}(1-x)^2 dx$.
- (A) $15 \left(\frac{14! 2!}{17!} \right)$
(B) $\frac{1}{3} \left(\frac{14! 2!}{17!} \right)$
(C) $\frac{14! 2!}{17!}$
(D) $17 \left(\frac{14! 2!}{17!} \right)$
(E) $\frac{136}{3} \left(\frac{14! 2!}{17!} \right)$

Questions 4–6

4. A box is chosen uniformly from two boxes. Box A contains 31 red and 33 blue balls; box B contains 33 red and 31 blue balls. Two balls are drawn without replacement and both are red. What is the conditional probability that A was chosen?

- (A) $\frac{155}{331}$
- (B) $\frac{155}{662}$
- (C) $\frac{993}{155}$
- (D) $\frac{243}{331}$
- (E) $\frac{176}{331}$

5. The region $0 \leq x \leq 1$, $0 \leq y \leq 14x^3$ is revolved about the x -axis. Find its volume.

- (A) 14π
- (B) 56π
- (C) $1 + 28\pi$
- (D) 28π
- (E) -28π

6. A sequence satisfies $u_0 = 65$ and $u_{k+1} = 1u_k + 10$ for all $k \geq 0$. Find u_4 .

- (A) 107
- (B) 106
- (C) 104
- (D) 105
- (E) 103

Questions 7–9

7. Evaluate $\lim_{x \rightarrow 0} (1 + 14x)^{1/(2x)}$.

- (A) e^{196}
- (B) e^7
- (C) $e^{\frac{1}{7}}$
- (D) 14
- (E) e^{15}

8. Let $X_1, \dots, X_{31}$ be independent uniform random variables on $[0, 1]$. What is $E[\max_i X_i]$?

- (A) $\frac{31}{32}$
- (B) $\frac{31}{96}$
- (C) $\frac{63}{64}$
- (D) $\frac{31}{64}$
- (E) $\frac{1}{32}$

9. Let $\mathbf{F} = (14y^3, z^3, x^3)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (0, 0, 1)$.

- (A) 42
- (B) -41
- (C) -42
- (D) -84
- (E) -21

Questions 10–12

10. A linear map $T : \mathbb{R}^{72} \rightarrow \mathbb{R}^{74}$ has rank 3. What is its nullity?

- (A) 71
- (B) 69
- (C) 68
- (D) 70
- (E) 67

11. The solution of $y' = 14x^3y^2$ has $y(0) = 1/15$. Find $y(1)$.

- (A) $\frac{25}{23}$
- (B) $-\frac{2}{23}$
- (C) $\frac{2}{23}$
- (D) $\frac{1}{23}$
- (E) $\frac{4}{23}$

12. Independent trials have success probability $1/31$. Let T count trials up to and including the first success. Find $E[T]$.

- (A) 41
- (B) 51
- (C) 21
- (D) 11
- (E) 31

Questions 13–15

13. Under $x = u + 14v$, $y = u - 3v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) 17
- (B) $\frac{1}{17}$
- (C) 289
- (D) 42
- (E) 11

14. What is the dimension of the real vector space of 71×71 symmetric matrices?

- (A) 2556
- (B) 2558
- (C) 2557
- (D) 2555
- (E) 2554

15. Evaluate $\int_0^1 \int_0^x x^{14} y^3 dy dx$.

- (A) $\frac{1}{72}$
- (B) $\frac{1}{60}$
- (C) $\frac{1}{76}$
- (D) $\frac{1}{95}$
- (E) $\frac{1}{19}$

Questions 16–18

16. A uniformly random subset of 31 objects is selected from a collection with 32 red and 33 blue objects. If X is the number of red objects selected, find $E[X]$.

- (A) $\frac{496}{65}$
- (B) $\frac{1057}{65}$
- (C) $\frac{1984}{65}$
- (D) $\frac{1122}{65}$
- (E) $\frac{992}{65}$

17. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(14x) - \sum_{j=0}^1 (-1)^j (14x)^{2j+1} / (2j+1)!}{x^5}$.

- (A) $\frac{(-1)^2 14^5}{5!}$
- (B) $14 \left(\frac{(-1)^2 14^5}{5!} \right)$
- (C) $-\frac{(-1)^2 14^5}{5!}$
- (D) $5 \left(\frac{(-1)^2 14^5}{5!} \right)$
- (E) $\frac{1}{14} \left(\frac{(-1)^2 14^5}{5!} \right)$

18. Let $A = \begin{bmatrix} 1 & 65 & 0 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{bmatrix}$. What is the $(1, 2)$ entry of A^3 ?

- (A) 195
- (B) 196
- (C) 194
- (D) 197
- (E) 193

Questions 19–21

19. Evaluate $\int_0^{\log 2} e^x(e^x + 14)^3 dx$.

- (A) $16^3 - 15^3$
- (B) $-\frac{16^4 - 15^4}{4}$
- (C) $4\left(\frac{16^4 - 15^4}{4}\right)$
- (D) $\frac{16^4}{4}$
- (E) $\frac{16^4 - 15^4}{4}$

20. X, Y are independent Poisson variables with means 31 and 32. Given $X + Y = 33$, find $\text{Var}(X | X + Y)$.

- (A) $\frac{21824}{1323}$
- (B) $\frac{12235}{1323}$
- (C) $\frac{5456}{1323}$
- (D) $\frac{10912}{1323}$
- (E) $\frac{13558}{1323}$

21. Evaluate $\lim_{x \rightarrow 0} \frac{e^{14x} - \sum_{j=0}^2 (14x)^j / j!}{x^3}$.

- (A) $\frac{1}{14} \left(\frac{14^3}{3!} \right)$
- (B) $-\frac{14^3}{3!}$
- (C) $3 \left(\frac{14^3}{3!} \right)$
- (D) $6 \left(\frac{14^3}{3!} \right)$
- (E) $\frac{14^3}{3!}$

Questions 22–24

22. An invertible 71×71 matrix A has determinant 2. Find $\det(A^T A)$.

- (A) 6
- (B) 2
- (C) 3
- (D) 5
- (E) 4

23. For $f(x) = x^3 e^{14x}$, find $f^{(5)}(0)$.

- (A) $\frac{1}{20} \left(\frac{5! 14^2}{2} \right)$
- (B) $\frac{5! 14^2}{2}$
- (C) $2 \left(\frac{5! 14^2}{2} \right)$
- (D) $\frac{1}{60} \left(\frac{5! 14^2}{2} \right)$
- (E) $\frac{1}{7} \left(\frac{5! 14^2}{2} \right)$

24. For which real a does $f_m(x) = x^{26m}/m^a$ converge uniformly to zero on $[0, 1]$?

- (A) $a > 0$
- (B) All real a
- (C) $a = 0$
- (D) $a < 0$
- (E) $a > 1$

Questions 25–27

25. For $x > 0$, $x^2y'' + (1 - 17)xy' + 42y = 0$, $y(1) = 0$, $y'(1) = 14$. Find $y(e)$.

- (A) $e^3 + e^{14}$
- (B) $-\frac{14(1 - e^{11})e^3}{11}$
- (C) $\frac{14(1 - e^{11})e^3}{11}$
- (D) $\frac{e^{14}}{17}$
- (E) $-e^3 + e^{14}$

26. On the space P_{72} of real polynomials of degree at most 72, define $T(p) = p'$. What is the rank of T ?

- (A) 71
- (B) 70
- (C) 72
- (D) 74
- (E) 73

27. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(14 + 3j)(17 + 3j)}$.

- (A) $\frac{3}{14}$
- (B) $\frac{1}{3}$
- (C) $\frac{1}{17}$
- (D) $\frac{1}{42}$
- (E) $\frac{1}{14}$

Questions 28–30

28. For which real p does $\sum_{k=2}^{\infty} 1/[k(\log k)^{26p}]$ converge?

- (A) $p \geq 1/26$
- (B) All real p
- (C) $p < 1/26$
- (D) $p > 1/26$
- (E) $p > 0$

29. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(14j)!}{(j!)^{14}} x^{3j}$?

- (A) $14^{-\frac{3}{14}}$
- (B) $14^{\frac{14}{3}}$
- (C) 14^{-14}
- (D) $\frac{1}{14}$
- (E) $14^{-\frac{14}{3}}$

30. Subspaces U, V of $\mathbb{R}^{73}$ have dimensions 3 and 4. What is the largest possible value of $\dim(U \cap V)$?

- (A) 3
- (B) 2
- (C) 5
- (D) 4
- (E) 1

Questions 31–33

31. For the curve $x = t^3$, $y = t^4 + 14t$, find d^2y/dx^2 at $t = 1$.

- (A) $-\frac{14}{3}$
- (B) $-\frac{8}{3}$
- (C) -8
- (D) $\frac{32}{9}$
- (E) 6

32. How many accumulation points in $\mathbb{R}$ does $S = \{1/k : k \geq 1\} \cup \{26 + 1/k : k \geq 1\}$ have?

- (A) 4
- (B) 2
- (C) 0
- (D) 1
- (E) 3

33. Solve $y' + 14y = x^3e^{-14x}$, $y(0) = 0$, and find $y(1)$.

- (A) $\frac{1}{4e^{14}}$
- (B) e^{-14}
- (C) $\frac{1}{3e^{14}}$
- (D) $\frac{e^{14}}{4}$
- (E) $\frac{7}{2e^{14}}$

Questions 34–36

34. Let V consist of real polynomials of degree at most 6 that vanish at 65, 66, and 68. Find $\dim V$.

- (A) 2
- (B) 6
- (C) 3
- (D) 5
- (E) 4

35. Let $F(x) = \int_0^x (x-t)^2 e^{14t} dt$. Find $F^{(3)}(0)$.

- (A) 0
- (B) 6
- (C) $\frac{1}{2}$
- (D) 2
- (E) 28

36. How many connected components does $\mathbb{R} \setminus \{1, 2, \dots, 26\}$ have?

- (A) 36
- (B) 45
- (C) 18
- (D) 27
- (E) 9

Questions 37–39

37. Let $f(x, y) = e^{14x+3y}$. Find $\partial^3 f / \partial x^2 \partial y$ at $(0, 0)$.

- (A) $\frac{3}{14} (3 \cdot 14^2)$
- (B) $3 \cdot 14^2$
- (C) $\frac{9}{196} (3 \cdot 14^2)$
- (D) $\frac{1}{3} (3 \cdot 14^2)$
- (E) $\frac{14}{3} (3 \cdot 14^2)$

38. The roots r, s of $x^2 - 75x + 650 = 0$ are counted with multiplicity. Find $r^2 + s^2$.

- (A) 4323
- (B) 4326
- (C) 4325
- (D) 4324
- (E) 4327

39. Find the curvature at $t = 0$ of $r(t) = (3t, 14t^2)$.

- (A) $\frac{56}{9}$
- (B) $\frac{37}{9}$
- (C) $\frac{28}{9}$
- (D) $-\frac{28}{9}$
- (E) $\frac{14}{9}$

Questions 40–42

40. The circle $|z| = 2$ is traversed counterclockwise. What is $\frac{1}{2\pi i} \int_{|z|=2} \frac{e^z}{z^{22}} dz$?

- (A) $1/42!$
- (B) $1/22!$
- (C) $1/21!$
- (D) $1/20!$
- (E) $21!$

41. If $y'' + 9y = 0$, $y(0) = 14$, and $y'(0) = 5$, find $y(\pi/6)$.

- (A) $-\frac{5}{3}$
- (B) $\frac{10}{3}$
- (C) $\frac{8}{3}$
- (D) $\frac{3}{5}$
- (E) $\frac{5}{6}$

42. For real t , let $M(t) = \begin{pmatrix} t-65 & 10 & 0 \\ 0 & t-65 & 1 \\ 0 & 0 & t-65 \end{pmatrix}$. At $t = 65$, what is $\text{rank } M(t)$?

- (A) 3
- (B) 4
- (C) 2
- (D) 1
- (E) 0

Questions 43–45

43. For $f(x, y) = x^2 + 14xy + 3y^2$, what is its maximum directional derivative at $(1, -1)$?

- (A) -12
- (B) $4\sqrt{13}$
- (C) 208
- (D) 8
- (E) 20

44. Find the residue of $z^{21}/(z - 1)^2$ at $z = 1$.

- (A) 28
- (B) 7
- (C) 21
- (D) 35
- (E) 14

45. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{14t} \cos(3t)$, and $y = e^{14t} \sin(3t)$, find d^2f/dt^2 at $t = 1$.

- (A) 6
- (B) 0
- (C) 784
- (D) 28
- (E) 410

Questions 46–48

46. Integers x satisfy $x \equiv 1 \pmod{5}$ and $x \equiv 7 \pmod{8}$. Find the least nonnegative solution.

- (A) 29
- (B) 30
- (C) 31
- (D) 32
- (E) 33

47. Let C be $x^2/9 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C (-y) dx + (14x) dy$.

- (A) -45π
- (B) 45π
- (C) $1 + 45\pi$
- (D) 90π
- (E) $\frac{45\pi}{2}$

48. How many functions from a set of 31 elements onto a set of 3 elements exist?

- (A) $3^{31} - 2^{31}$
- (B) $3^{31} - 3(2^{31}) - 3$
- (C) $3^{31} - 3(2^{31}) + 3$
- (D) 3^{31}
- (E) $3^{31} - 3(2^{31})$

Questions 49–51

49. Find the total outward flux of $\mathbf{F} = (14x^3, y^3, z^3)$ through the boundary of $[0, 1]^3$.
- (A) -16
(B) 17
(C) 16
(D) 8
(E) 32
50. The symmetric group S_{71} acts on $\{1, \dots, 71\}$. What is the index of the stabilizer of 1?
- (A) 69
(B) 71
(C) 73
(D) 72
(E) 70
51. Two solutions of $y'' + 14x^1y' + 3y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.
- (A) $\frac{1}{2e^7}$
(B) e^{-7}
(C) $\frac{2}{e^7}$
(D) $-\frac{1}{e^7}$
(E) $e^{-7} + 1$

Questions 52–54

52. How many nonnegative integer triples (a, b, c) satisfy $a + b + c = 31$?

- (A) 704
- (B) 176
- (C) 880
- (D) 528
- (E) 352

53. Find the arc length of $r(t) = (14t, 3t)$ for $0 \leq t \leq 1$.

- (A) $-\sqrt{205}$
- (B) $1 + \sqrt{205}$
- (C) $\frac{\sqrt{205}}{2}$
- (D) $\sqrt{205}$
- (E) $2\sqrt{205}$

54. In $\mathbb{Z}/2094\mathbb{Z}$, how many residues are idempotent ($x^2 = x$)?

- (A) 7
- (B) 10
- (C) 6
- (D) 8
- (E) 9

Questions 55–57

55. Find the upward flux of $\mathbf{F} = (0, 0, z)$ through $z = 14x^2 + 3y^2$, $0 \leq x, y \leq 1$.

- (A) $\frac{17}{3}$
- (B) $-\frac{17}{3}$
- (C) $\frac{20}{3}$
- (D) $\frac{34}{3}$
- (E) $\frac{17}{6}$

56. A tree has 35 vertices. What is the sum of its vertex degrees?

- (A) 46
- (B) 90
- (C) 112
- (D) 24
- (E) 68

57. For $y' = 14y(1 - y/3)$, what is the derivative of the right-hand side with respect to y at the positive equilibrium?

- (A) -28
- (B) -13
- (C) -14
- (D) 14
- (E) -7

Questions 58–60

58. In the additive group $\mathbb{Z}/90\mathbb{Z}$, find the order of $[2]$.

- (A) 45
- (B) 43
- (C) 47
- (D) 44
- (E) 46

59. Find the coefficient of x^6 in $\cos(14x)/(1 - 2x^2)$.

- (A) -5501056
- (B) $-\frac{343816}{45}$
- (C) $-\frac{343861}{45}$
- (D) $-\frac{470596}{45}$
- (E) $\frac{343816}{45}$

60. A connected planar graph has 34 vertices and 63 edges. How many faces, including the unbounded face, are in a planar embedding?

- (A) 21
- (B) 41
- (C) 11
- (D) 51
- (E) 31

Questions 61–63

61. Find the coefficient of x^3 in the Taylor series of $(1 - 14x)^{-2}$ at zero.

- (A) $\frac{1}{4} (4 \cdot 14^3)$
- (B) $\frac{3}{4} (4 \cdot 14^3)$
- (C) $-4 \cdot 14^3$
- (D) $\frac{1}{14} (4 \cdot 14^3)$
- (E) $4 \cdot 14^3$

62. How many invertible 2×2 matrices over $\mathbb{F}_{41}$ are there with determinant 1?

- (A) $\frac{(41^2 - 41^1)(41^2 - 41^2)}{40}$
- (B) $\frac{(41^2 - 41^0)}{40}$
- (C) $\frac{(41^2 - 41^0)(41^2 - 41^0)}{40}$
- (D) $\frac{(41^2 - 41^0)(41^2 - 41^1)}{40}$
- (E) $\frac{41^2 41^2}{40}$

63. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 14x$ and has density $\rho(x, y) = x$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{3}{4}$
- (B) $\frac{3}{8}$
- (C) $\frac{3}{2}$
- (D) $\frac{7}{4}$
- (E) $-\frac{3}{4}$

Questions 64–66

64. How many integers in $\{1, \dots, 186\}$ are divisible by 2 or 3?
- (A) 83
(B) 42
(C) 206
(D) 124
(E) 165
65. Solve $y'' - 31y' + 238y = 0$ with $y(0) = 1$, $y'(0) = 0$. Find $y(1)$.
- (A) $\frac{(17 - 14e^3) e^{14}}{6}$
(B) $\frac{2(17 - 14e^3) e^{14}}{3}$
(C) $\frac{(17 - 14e^3) e^{14}}{3} + 1$
(D) $\frac{(-17 + 14e^3) e^{14}}{3}$
(E) $\frac{(17 - 14e^3) e^{14}}{3}$
66. How many permutations in S_{71} consist of one 3-cycle and 68 fixed points?
- (A) 114310
(B) 114308
(C) 114311
(D) 114309
(E) 114312

Answer Key

Practice Test 69 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	E	23	B	45	B
2	E	24	A	46	C
3	C	25	B	47	B
4	A	26	C	48	C
5	D	27	D	49	C
6	D	28	D	50	B
7	B	29	E	51	B
8	A	30	A	52	D
9	C	31	B	53	D
10	B	32	B	54	D
11	C	33	A	55	A
12	E	34	E	56	E
13	A	35	D	57	C
14	A	36	D	58	A
15	C	37	B	59	B
16	E	38	C	60	E
17	A	39	C	61	E
18	A	40	C	62	D
19	E	41	D	63	A
20	D	42	C	64	D
21	E	43	B	65	E
22	E	44	C	66	A

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