

# GRE Mathematics Practice Test 70

## Standard-Level Target

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

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### Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural,  $\mathbb{N} = \{1, 2, 3, \dots\}$ , and distances in  $\mathbb{R}^n$  are Euclidean.  $I_n$  denotes the  $n \times n$  identity matrix. Graphs are finite and simple unless stated otherwise.

### Difficulty target

A mixed standard-level target, intended to approximate public GRE Mathematics practice in scope and overall demand. This is cycle 13 of 19, position 5 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

### Coverage

|                 |                                                                                                                           |
|-----------------|---------------------------------------------------------------------------------------------------------------------------|
| Calculus / 33   | Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations. |
| Algebra / 17    | Elementary and linear algebra, groups, rings, fields, and number theory.                                                  |
| Additional / 16 | Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.              |

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

**Questions 1–3**

1. For  $y' = 14y(1 - y/2)$ , what is the derivative of the right-hand side with respect to  $y$  at the positive equilibrium?

- (A) 14
- (B)  $-7$
- (C)  $-13$
- (D)  $-28$
- (E)  $-14$

2. An invertible  $72 \times 72$  matrix  $A$  has determinant 3. Find  $\det(A^T A)$ .

- (A) 11
- (B) 10
- (C) 9
- (D) 8
- (E) 7

3. Evaluate  $\int_0^1 x^{14}(1 - x)^1 dx$ .

- (A)  $16 \left( \frac{14! 1!}{16!} \right)$
- (B)  $\frac{1}{2} \left( \frac{14! 1!}{16!} \right)$
- (C)  $\frac{14! 1!}{16!}$
- (D)  $8 \left( \frac{14! 1!}{16!} \right)$
- (E)  $15 \left( \frac{14! 1!}{16!} \right)$

## Questions 4–6

4. A box is chosen uniformly from two boxes. Box A contains 32 red and 34 blue balls; box B contains 34 red and 32 blue balls. One ball is drawn and is red. What is the conditional probability that A was chosen?

- (A)  $\frac{17}{33}$
- (B)  $\frac{16}{99}$
- (C)  $\frac{8}{33}$
- (D)  $\frac{16}{33}$
- (E)  $\frac{49}{66}$

5. Evaluate  $\int_0^1 x^{13}(\log x)^1 dx$ .

- (A)  $\frac{(-1)^1 1!}{14^2}$
- (B)  $2 \left( \frac{(-1)^1 1!}{14^2} \right)$
- (C)  $14 \left( \frac{(-1)^1 1!}{14^2} \right)$
- (D)  $-\frac{(-1)^1 1!}{14^2}$
- (E)  $\frac{1}{2} \left( \frac{(-1)^1 1!}{14^2} \right)$

6. On the space  $P_{73}$  of real polynomials of degree at most 73, define  $T(p) = p'$ . What is the rank of  $T$ ?

- (A) 73
- (B) 72
- (C) 74
- (D) 75
- (E) 71

## Questions 7–9

7. For the curve  $x = t^2$ ,  $y = t^3 + 14t$ , find  $d^2y/dx^2$  at  $t = 1$ .

- (A)  $-7$
- (B)  $\frac{17}{2}$
- (C)  $\frac{17}{4}$
- (D)  $-\frac{11}{4}$
- (E)  $-\frac{11}{2}$

8. Let  $X_1, \dots, X_{32}$  be independent uniform random variables on  $[0, 1]$ . What is  $E[\max_i X_i]$ ?

- (A)  $\frac{32}{33}$
- (B)  $\frac{16}{33}$
- (C)  $\frac{65}{66}$
- (D)  $\frac{1}{33}$
- (E)  $\frac{32}{99}$

9. Evaluate  $\int_C (2x + 14y) dx + (14x + 2y) dy$ , where  $C$  is the line segment from  $(0, 0)$  to  $(6, 1)$ .

- (A)  $-121$
- (B)  $242$
- (C)  $\frac{121}{2}$
- (D)  $122$
- (E)  $121$

## Questions 10–12

10. Subspaces  $U, V$  of  $\mathbb{R}^{74}$  have dimensions 2 and 3. What is the largest possible value of  $\dim(U \cap V)$ ?

- (A) 0
- (B) 1
- (C) 4
- (D) 2
- (E) 3

11. For  $f(x) = x^2 e^{14x}$ , find  $f^{(4)}(0)$ .

- (A)  $\frac{1}{7} \left( \frac{4! 14^2}{2} \right)$
- (B)  $\frac{4! 14^2}{2}$
- (C)  $\frac{1}{12} \left( \frac{4! 14^2}{2} \right)$
- (D)  $\frac{1}{2} \left( \frac{4! 14^2}{2} \right)$
- (E)  $2 \left( \frac{4! 14^2}{2} \right)$

12. Independent trials have success probability  $1/32$ . Let  $T$  count trials up to and including the first success. Find  $E[T]$ .

- (A) 32
- (B) 22
- (C) 12
- (D) 42
- (E) 52

**Questions 13–15**

**13.** For  $D = \{(x, y) : x^2 + y^2 \leq 14\}$ , evaluate  $\iint_D (x^2 + y^2)^1 dA$ .

- (A)  $7\pi$
- (B)  $\frac{196\pi}{3}$
- (C)  $98\pi$
- (D)  $196\pi$
- (E)  $49\pi$

**14.** Let  $V$  consist of real polynomials of degree at most 5 that vanish at 66, 67, and 69. Find  $\dim V$ .

- (A) 4
- (B) 2
- (C) 1
- (D) 3
- (E) 5

**15.** For  $f(x, y) = x^2 + 14xy + 2y^2$ , what is its maximum directional derivative at  $(1, -1)$ ?

- (A) 244
- (B)  $2\sqrt{61}$
- (C)  $-12$
- (D) 10
- (E) 22

**Questions 16–18**

**16.** A uniformly random subset of 32 objects is selected from a collection with 33 red and 34 blue objects. If  $X$  is the number of red objects selected, find  $E[X]$ .

- (A)  $\frac{528}{67}$
- (B)  $\frac{1123}{67}$
- (C)  $\frac{1056}{67}$
- (D)  $\frac{2112}{67}$
- (E)  $\frac{1190}{67}$

**17.** Let  $F(x) = \int_0^x (x - t)^{14} e^{14t} dt$ . Find  $F^{(2)}(0)$ .

- (A) 2
- (B) 14
- (C) 0
- (D)  $\frac{1}{2}$
- (E) 1

**18.** The roots  $r, s$  of  $x^2 - 74x + 528 = 0$  are counted with multiplicity. Find  $r^2 + s^2$ .

- (A) 4422
- (B) 4419
- (C) 4421
- (D) 4418
- (E) 4420

## Questions 19–21

19. Find the directional derivative of  $f(x, y, z) = x^2 + 14y^2 + 2z^2$  at  $(1, 1, 1)$  in the direction  $(1, 6, 0)$ .

- (A)  $\frac{85\sqrt{37}}{37}$
- (B)  $\frac{340\sqrt{37}}{37}$
- (C)  $\frac{170\sqrt{37}}{37}$
- (D)  $1 + \frac{170\sqrt{37}}{37}$
- (E)  $-\frac{170\sqrt{37}}{37}$

20.  $X, Y$  are independent Poisson variables with means 32 and 33. Given  $X + Y = 34$ , find  $E[X \mid X + Y]$ .

- (A)  $\frac{544}{65}$
- (B)  $\frac{1088}{65}$
- (C)  $\frac{1218}{65}$
- (D)  $\frac{1153}{65}$
- (E)  $\frac{2176}{65}$

21. Find the arc length of  $r(t) = (14t, 2t)$  for  $0 \leq t \leq 1$ .

- (A)  $-10\sqrt{2}$
- (B)  $1 + 10\sqrt{2}$
- (C)  $10\sqrt{2}$
- (D)  $5\sqrt{2}$
- (E)  $20\sqrt{2}$

## Questions 22–24

22. For real  $t$ , let  $M(t) = \begin{pmatrix} t-66 & 8 & 0 \\ 0 & t-66 & 1 \\ 0 & 0 & t-66 \end{pmatrix}$ . At  $t = 66$ , what is  $\text{rank } M(t)$ ?

- (A) 4
- (B) 2
- (C) 3
- (D) 1
- (E) 0

23. Evaluate  $\iiint_{x^2+y^2+z^2 \leq 14} (x^2 + y^2 + z^2)^1 dV$ .

- (A)  $\frac{1}{2} \left( \frac{4\pi 14^{\frac{5}{2}}}{5} \right)$
- (B)  $5 \left( \frac{4\pi 14^{\frac{5}{2}}}{5} \right)$
- (C)  $\frac{1}{3} \left( \frac{4\pi 14^{\frac{5}{2}}}{5} \right)$
- (D)  $\frac{5}{42} \left( \frac{4\pi 14^{\frac{5}{2}}}{5} \right)$
- (E)  $\frac{4\pi 14^{\frac{5}{2}}}{5}$

24. For which real  $a$  does  $f_m(x) = x^{27m}/m^a$  converge uniformly to zero on  $[0, 1]$ ?

- (A) All real  $a$
- (B)  $a > 0$
- (C)  $a < 0$
- (D)  $a > 1$
- (E)  $a = 0$

**Questions 25–27**

- 25.** Find the coefficient of  $x^2$  in  $e^{1x} \log(1 + 14x)$ .
- (A)  $-70$   
(B)  $-84$   
(C)  $-98$   
(D)  $-168$   
(E)  $84$
- 26.** Let  $u = (13, 2)^T$  and  $v = (1, 0)^T$ . Find the squared norm of the orthogonal projection of  $v$  onto  $\text{span}(u)$ .
- (A)  $\frac{342}{173}$   
(B)  $\frac{857}{346}$   
(C)  $\frac{165}{346}$   
(D)  $\frac{511}{346}$   
(E)  $\frac{169}{173}$
- 27.** If  $y'' + 4y = 0$ ,  $y(0) = 14$ , and  $y'(0) = 6$ , find  $y(\pi/4)$ .
- (A)  $3$   
(B)  $6$   
(C)  $4$   
(D)  $\frac{3}{2}$   
(E)  $-3$

**Questions 28–30**

**28.** For which real  $p$  does  $\sum_{k=2}^{\infty} 1/[k(\log k)^{27p}]$  converge?

- (A)  $p < 1/27$
- (B)  $p > 0$
- (C)  $p > 1/27$
- (D)  $p \geq 1/27$
- (E) All real  $p$

**29.** If  $f(x, y) = \log(x^2 + y^2)$ ,  $x = e^{14t} \cos(2t)$ , and  $y = e^{14t} \sin(2t)$ , find  $d^2f/dt^2$  at  $t = 1$ .

- (A) 400
- (B) 4
- (C) 28
- (D) 0
- (E) 784

**30.** An idempotent  $72 \times 72$  real matrix  $P$  has rank 3. What is  $\text{tr } P$ ?

- (A) 3
- (B) 5
- (C) 1
- (D) 4
- (E) 2

## Questions 31–33

31. The solution of  $y' = 14x^1y^2$ ,  $y(0) = 1$ , has its first positive singularity at which  $x$ ?
- (A)  $\frac{1}{7}$   
(B)  $\frac{\sqrt{7}}{7}$   
(C)  $\frac{2\sqrt{7}}{7}$   
(D)  $\frac{\sqrt{14}}{14}$   
(E)  $\sqrt{7}$
32. How many accumulation points in  $\mathbb{R}$  does  $S = \{1/k : k \geq 1\} \cup \{27 + 1/k : k \geq 1\}$  have?
- (A) 3  
(B) 0  
(C) 2  
(D) 1  
(E) 4
33. For which positive integer  $m$  does  $\sum_{j=2}^{\infty} \frac{(-1)^j(x-14)^j}{j^{\frac{1}{3}}(\log j)^m}$  converge absolutely at  $x = 13$ ? Give the least such  $m$ , or 0 if none exists.
- (A) 0  
(B) 2  
(C) 1  
(D) 3  
(E) 4

## Questions 34–36

34. For  $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$  and  $b = \begin{bmatrix} 66 \\ 8 \\ 74 \end{bmatrix}$ , let  $x$  minimize  $\|Ax - b\|^2$ . Find  $x_2$ .

- (A) 7
- (B) 9
- (C) 8
- (D) 6
- (E) 10

35. Let  $\mathbf{F} = (14y^2, z^2, x^2)$ . Find  $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (0, 0, 1)$ .

- (A)  $-28$
- (B)  $-56$
- (C)  $28$
- (D)  $-14$
- (E)  $-27$

36. How many connected components does  $\mathbb{R} \setminus \{1, 2, \dots, 27\}$  have?

- (A) 37
- (B) 10
- (C) 19
- (D) 28
- (E) 46

## Questions 37–39

37. Two solutions of  $y'' + 14x^1y' + 2y = 0$  have Wronskian  $W(0) = 1$ . Find  $W(1)$ .

- (A)  $-\frac{1}{e^7}$
- (B)  $e^{-7}$
- (C)  $\frac{2}{e^7}$
- (D)  $\frac{1}{2e^7}$
- (E)  $e^{-7} + 1$

38. What is the coefficient of  $x^2$  in  $(x + 13)^4$ ?

- (A) 1014
- (B) 1016
- (C) 1015
- (D) 1013
- (E) 1012

39. Let  $C$  be the unit circle  $x^2 + y^2 = 1$ ,  $z = 2$ , counterclockwise viewed from above. Evaluate  $\int_C -14y(x^2 + y^2) dx + (14x(x^2 + y^2)) dy$ .

- (A)  $-28\pi$
- (B)  $28\pi$
- (C)  $14\pi$
- (D)  $1 + 28\pi$
- (E)  $56\pi$

## Questions 40–42

40. The circle  $|z| = 2$  is traversed counterclockwise. What is  $\frac{1}{2\pi i} \int_{|z|=2} \frac{e^z}{z^{23}} dz$ ?

- (A)  $1/21!$
- (B)  $1/23!$
- (C)  $1/44!$
- (D)  $22!$
- (E)  $1/22!$

41. Find the curvature at  $t = 0$  of  $r(t) = (2t, 14t^2)$ .

- (A)  $-7$
- (B)  $14$
- (C)  $\frac{7}{2}$
- (D)  $8$
- (E)  $7$

42. A matrix  $A$  satisfies  $(A - 67I)(A - 69I) = 0$ , and its minimal polynomial has degree two. If  $A^2 = uA + vI$ , find  $u$ .

- (A) 136
- (B) 138
- (C) 134
- (D) 135
- (E) 137

## Questions 43–45

43. Near  $(0, 0)$ ,  $z = z(x, y)$  satisfies  $z + 14z^2 = x + 2y$  and  $z(0, 0) = 0$ . Find  $z_{xy}(0, 0)$ .

- (A) 56
- (B)  $-28$
- (C)  $-2$
- (D)  $-56$
- (E)  $-112$

44. Find the residue of  $z^{22}/(z - 1)^2$  at  $z = 1$ .

- (A) 22
- (B) 8
- (C) 36
- (D) 29
- (E) 15

45. The solution of  $y' = 14x^2y^2$  has  $y(0) = 1/15$ . Find  $y(1)$ .

- (A)  $-\frac{3}{31}$
- (B)  $\frac{3}{62}$
- (C)  $\frac{6}{31}$
- (D)  $\frac{34}{31}$
- (E)  $\frac{3}{31}$

## Questions 46–48

46. Let  $V = \begin{bmatrix} 1 & 66 & 4356 \\ 1 & 68 & 4624 \\ 1 & 71 & 5041 \end{bmatrix}$ . Find  $\det V$ .

- (A) 31
- (B) 29
- (C) 28
- (D) 30
- (E) 32

47. The function  $y(x)$  satisfies  $y^3 + 14y = 1x$  and  $y(0) = 0$ . Find  $y'(0)$ .

- (A) 0
- (B)  $-\frac{1}{14}$
- (C)  $\frac{1}{14}$
- (D) 14
- (E)  $\frac{1}{17}$

48. How many functions from a set of 32 elements onto a set of 2 elements exist?

- (A)  $2^{32} - 1$
- (B)  $2^{32}$
- (C)  $2^{31}$
- (D)  $2^{32} - 3$
- (E)  $2^{32} - 2$

## Questions 49–51

49. Evaluate  $\sum_{j=0}^{\infty} \frac{1}{(14+2j)(16+2j)}$ .

- (A)  $\frac{1}{2}$
- (B)  $\frac{1}{7}$
- (C)  $\frac{1}{14}$
- (D)  $\frac{1}{28}$
- (E)  $\frac{1}{16}$

50. How many residues  $x$  modulo 96 satisfy  $4x \equiv 24 \pmod{96}$ ?

- (A) 3
- (B) 5
- (C) 4
- (D) 2
- (E) 6

51. For  $x > 0$ ,  $x^2 y'' + (1 - 16)x y' + 28y = 0$ ,  $y(1) = 0$ ,  $y'(1) = 14$ . Find  $y(e)$ .

- (A)  $e^2 + e^{14}$
- (B)  $\frac{7(1 - e^{12})e^2}{6}$
- (C)  $-\frac{7(1 - e^{12})e^2}{6}$
- (D)  $\frac{e^{14}}{16}$
- (E)  $-e^2 + e^{14}$

## Questions 52–54

52. How many nonnegative integer triples  $(a, b, c)$  satisfy  $a + b + c = 32$ ?

- (A) 935
- (B) 187
- (C) 374
- (D) 748
- (E) 561

53. For  $x > 0$ ,  $xy' = 14y$  and  $y(1) = 1$ . Find  $y(e^2)$ .

- (A)  $e^{28}$
- (B)  $1 + e^{28}$
- (C)  $\frac{e^{28}}{2}$
- (D)  $-e^{28}$
- (E)  $2e^{28}$

54. The field  $K$  has  $2^{14}$  elements. Find  $[K : \mathbb{F}_2]$ .

- (A) 13
- (B) 12
- (C) 15
- (D) 14
- (E) 16

## Questions 55–57

55. Evaluate  $\lim_{x \rightarrow 0} \frac{e^{14x} - \sum_{j=0}^1 (14x)^j / j!}{x^2}$ .

- (A)  $\frac{14^2}{2!}$
- (B)  $\frac{1}{14} \left( \frac{14^2}{2!} \right)$
- (C)  $\frac{1}{2} \left( \frac{14^2}{2!} \right)$
- (D)  $-\frac{14^2}{2!}$
- (E)  $2 \left( \frac{14^2}{2!} \right)$

56. A tree has 36 vertices. What is the sum of its vertex degrees?

- (A) 93
- (B) 70
- (C) 116
- (D) 47
- (E) 24

57. Evaluate  $\int_0^1 \int_0^x x^{14} y^2 dy dx$ .

- (A)  $\frac{1}{18}$
- (B)  $\frac{1}{54}$
- (C)  $\frac{1}{72}$
- (D)  $\frac{1}{51}$
- (E)  $\frac{1}{45}$

## Questions 58–60

58. Let  $G = \mathbb{Z}/426\mathbb{Z} \times \mathbb{Z}/288\mathbb{Z}$ . Find the order of  $(1, 1)$ .

- (A) 20449
- (B) 20446
- (C) 20447
- (D) 20448
- (E) 20450

59. Evaluate  $\int_0^{\log 2} e^x (e^x + 14)^2 dx$ .

- (A)  $\frac{16^3 - 15^3}{3}$
- (B)  $-\frac{16^3 - 15^3}{3}$
- (C)  $\frac{16^3}{3}$
- (D)  $16^2 - 15^2$
- (E)  $3 \left( \frac{16^3 - 15^3}{3} \right)$

60. A connected planar graph has 35 vertices and 65 edges. How many faces, including the unbounded face, are in a planar embedding?

- (A) 12
- (B) 32
- (C) 52
- (D) 22
- (E) 42

## Questions 61–63

61. Let  $C$  be  $x^2/4 + y^2 = 1$ , oriented counterclockwise. Evaluate  $\int_C (-y) dx + (14x) dy$ .

- (A)  $-30\pi$
- (B)  $60\pi$
- (C)  $15\pi$
- (D)  $30\pi$
- (E)  $1 + 30\pi$

62. How many group homomorphisms  $\mathbb{Z}/426\mathbb{Z} \rightarrow \mathbb{Z}/288\mathbb{Z}$  are there?

- (A) 4
- (B) 8
- (C) 6
- (D) 7
- (E) 5

63. If  $y' + 14y = 2e^{-14x}$  and  $y(0) = 6$ , find  $y(1)$ .

- (A)  $\frac{16}{e^{14}}$
- (B)  $\frac{8}{e^{14}} + 1$
- (C)  $-\frac{8}{e^{14}}$
- (D)  $\frac{4}{e^{14}}$
- (E)  $\frac{8}{e^{14}}$

## Questions 64–66

64. How many integers in  $\{1, \dots, 192\}$  are divisible by 2 or 3?

- (A) 86
- (B) 170
- (C) 44
- (D) 128
- (E) 212

65. Evaluate  $\lim_{x \rightarrow 0} \frac{\sin(14x) - \sum_{j=0}^0 (-1)^j (14x)^{2j+1} / (2j+1)!}{x^3}$ .

- (A)  $-\frac{(-1)^1 14^3}{3!}$
- (B)  $\frac{1}{14} \left( \frac{(-1)^1 14^3}{3!} \right)$
- (C)  $3 \left( \frac{(-1)^1 14^3}{3!} \right)$
- (D)  $14 \left( \frac{(-1)^1 14^3}{3!} \right)$
- (E)  $\frac{(-1)^1 14^3}{3!}$

66. How many 1-dimensional subspaces does  $\mathbb{F}_{41}^3$  have?

- (A)  $\frac{41^5 - 1}{41 - 1}$
- (B)  $\frac{41^3 - 1}{41 - 1}$
- (C)  $\frac{41^2 - 1}{41 - 1}$
- (D)  $\frac{41^4 - 1}{41 - 1}$
- (E)  $\frac{41^6 - 1}{41 - 1}$

# Answer Key

Practice Test 70 / Questions 1–66

| Question | Answer | Question | Answer | Question | Answer |
|----------|--------|----------|--------|----------|--------|
| 1        | E      | 23       | E      | 45       | E      |
| 2        | C      | 24       | B      | 46       | D      |
| 3        | C      | 25       | B      | 47       | C      |
| 4        | D      | 26       | E      | 48       | E      |
| 5        | A      | 27       | A      | 49       | D      |
| 6        | A      | 28       | C      | 50       | C      |
| 7        | D      | 29       | D      | 51       | C      |
| 8        | A      | 30       | A      | 52       | E      |
| 9        | E      | 31       | B      | 53       | A      |
| 10       | D      | 32       | C      | 54       | D      |
| 11       | B      | 33       | A      | 55       | A      |
| 12       | A      | 34       | C      | 56       | B      |
| 13       | C      | 35       | A      | 57       | B      |
| 14       | D      | 36       | D      | 58       | D      |
| 15       | B      | 37       | B      | 59       | A      |
| 16       | C      | 38       | A      | 60       | B      |
| 17       | E      | 39       | B      | 61       | D      |
| 18       | E      | 40       | E      | 62       | C      |
| 19       | C      | 41       | E      | 63       | E      |
| 20       | B      | 42       | A      | 64       | D      |
| 21       | C      | 43       | D      | 65       | E      |
| 22       | B      | 44       | A      | 66       | B      |

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