

GRE Mathematics Practice Test 71

Advanced Challenge

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

The most demanding form in this five-test cycle. Emphasis on connected ideas, non-routine reasoning, and careful conceptual distinctions. This is cycle 14 of 19, position 1 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. For $D = \{(x, y) : x^2 + y^2 \leq 15\}$, evaluate $\iint_D (x^2 + y^2)^5 dA$.
- (A) $\frac{11390625\pi}{11}$
(B) $\frac{3796875\pi}{2}$
(C) 3796875π
(D) $\frac{253125\pi}{2}$
(E) 11390625π
2. Let $P = \text{diag}(\underbrace{1, \dots, 1}_1, \underbrace{0, \dots, 0}_{73})$. What is the dimension of the space of 74×74 real matrices X satisfying $PX = X$ and $XP = 0$?
- (A) 74
(B) 72
(C) 75
(D) 73
(E) 71
3. Evaluate $\int_0^\infty x^6 e^{-15x} dx$.
- (A) $15 \left(\frac{6!}{15^7} \right)$
(B) $\frac{6!}{15^7}$
(C) 720
(D) $\frac{1}{6} \left(\frac{6!}{15^7} \right)$
(E) $\frac{1}{15} \left(\frac{6!}{15^7} \right)$

Questions 4–6

4. X, Y are independent, with variances 28 and 29. Find $\text{Cov}(X + 2Y, 3X - Y)$.

- (A) 18
- (B) 10
- (C) 26
- (D) 34
- (E) 42

5. Let $f(x) = \log(1 + 15x)$. Find $f^{(6)}(0)$.

- (A) $6 \left((-1)^5 5! 15^6 \right)$
- (B) $-(-1)^5 5! 15^6$
- (C) $(-1)^5 5! 15^6$
- (D) $\frac{1}{6} \left((-1)^5 5! 15^6 \right)$
- (E) $\frac{1}{15} \left((-1)^5 5! 15^6 \right)$

6. A 3×3 matrix A has characteristic polynomial $(t - 67)(t + 1)(t - 2)$. What is $\text{tr}(A^3 - 2A + I)$?

- (A) 300636
- (B) 300637
- (C) 300635
- (D) 300639
- (E) 300638

Questions 7–9

7. Let C be $x^2/36 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C (-5y^3) dx + (\frac{x^3}{3}) dy$.

- (A) 153π
- (B) $-\frac{153\pi}{2}$
- (C) $\frac{153\pi}{4}$
- (D) $1 + \frac{153\pi}{2}$
- (E) $\frac{153\pi}{2}$

8. X has density cx^{28} on $0 < x < 1$ and zero elsewhere. Find $E[X^2]$.

- (A) $\frac{29}{31}$
- (B) $\frac{29}{62}$
- (C) $\frac{2}{31}$
- (D) $\frac{30}{31}$
- (E) $\frac{29}{93}$

9. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{15t} \cos(6t)$, and $y = e^{15t} \sin(6t)$, find d^2f/dt^2 at $t = 1$.

- (A) 900
- (B) 12
- (C) 522
- (D) 0
- (E) 30

Questions 10–12

10. Let N be the 73×73 matrix with ones immediately above the diagonal and zeros elsewhere. Find $\dim(\ker N^{24} / \ker N^{12})$.

- (A) 12
- (B) 13
- (C) 10
- (D) 11
- (E) 14

11. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 15x$ and has density $\rho(x, y) = x^4y$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{15}{8}$
- (B) $-\frac{7}{8}$
- (C) $\frac{7}{4}$
- (D) $\frac{7}{8}$
- (E) $\frac{7}{16}$

12. Independent exponential waiting times X, Y have rates 28 and 30. Find $E[\min(X, Y)]$.

- (A) $\frac{57}{58}$
- (B) $\frac{1}{174}$
- (C) $\frac{1}{58}$
- (D) $\frac{1}{116}$
- (E) $\frac{59}{116}$

Questions 13–15

13. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(15j)!}{(j!)^{15}} x^{6j}$?

- (A) $15^{-\frac{5}{2}}$
- (B) 15^{-15}
- (C) $15^{\frac{5}{2}}$
- (D) $\frac{1}{15}$
- (E) $15^{-\frac{2}{5}}$

14. Let A be the 73×73 matrix whose diagonal entries are 70 and whose off-diagonal entries are 1. What is its largest eigenvalue?

- (A) 143
- (B) 141
- (C) 140
- (D) 144
- (E) 142

15. Evaluate $\int_C (6x^5 + 15y) dx + (15x + 6y^5) dy$, where C is $r(t) = (t, 2t^2)$, $0 \leq t \leq 1$.

- (A) 190
- (B) 96
- (C) -95
- (D) $\frac{95}{2}$
- (E) 95

Questions 16–18

16. $X_1, \dots, X_{28}$ are independent with common mean and variance 30. With $\bar{X}$ their sample mean, find $E[\sum_i (X_i - \bar{X})^2]$.

- (A) 1350
- (B) 270
- (C) 810
- (D) 1080
- (E) 540

17. The region $0 \leq x \leq 1$, $0 \leq y \leq 15x^6$ is revolved about the line $x = -2$. Find its volume.

- (A) $\frac{345\pi}{14}$
- (B) $-\frac{345\pi}{28}$
- (C) $1 + \frac{345\pi}{28}$
- (D) $\frac{345\pi}{56}$
- (E) $\frac{345\pi}{28}$

18. Positive real numbers x, y satisfy $\log_2 x + \log_2 y = \log_2 737$ and $x + y = 78$. Find $x^2 + y^2$.

- (A) 4610
- (B) 4609
- (C) 4611
- (D) 4612
- (E) 4608

Questions 19–21

19. Evaluate $\int_0^1 \frac{x^5}{1+15x^6} dx$.

- (A) $\log(16)$
- (B) $\frac{4\log(2)}{15}$
- (C) $-\frac{2\log(2)}{45}$
- (D) $\frac{2\log(2)}{45}$
- (E) $\frac{2\log(2)}{3}$

20. A permutation of 30 objects is chosen uniformly. Let Z be the number of fixed points. Find $E[Z^2]$.

- (A) 3
- (B) 2
- (C) 1
- (D) 0
- (E) 4

21. Let $\mathbf{F} = (15y^6, z^6, x^6)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (1, 1, 2)/\sqrt{6}$.

- (A) $1 - 32\sqrt{6}$
- (B) $-32\sqrt{6}$
- (C) $-64\sqrt{6}$
- (D) $-16\sqrt{6}$
- (E) $32\sqrt{6}$

Questions 22–24

22. A sequence satisfies $u_0 = 67$ and $u_{k+1} = 2u_k + 11$ for all $k \geq 0$. Find u_8 .

- (A) 19957
- (B) 19955
- (C) 19958
- (D) 19959
- (E) 19956

23. If $y' = 30xy$ and $y(0) = 1$, find the coefficient of x^{12} in the Maclaurin series of y .

- (A) $\frac{253125}{16}$
- (B) $\frac{253125}{8}$
- (C) $-\frac{253125}{16}$
- (D) $\frac{253125}{32}$
- (E) $\frac{253141}{16}$

24. Which subset of $\mathbb{R}$ is compact, with 23 fixed as displayed?

- (A) $23\mathbb{Z}$
- (B) $\{0\} \cup \{23/k : k \geq 1\}$
- (C) $\mathbb{Q} \cap [0, 23]$
- (D) $\{23/k : k \geq 1\}$
- (E) $(0, 23)$

Questions 25–27

25. For $f(x) = x^6 e^{15x}$, find $f^{(8)}(0)$.

- (A) $\frac{1}{56} \left(\frac{8! 15^2}{2} \right)$
- (B) $\frac{2}{15} \left(\frac{8! 15^2}{2} \right)$
- (C) $2 \left(\frac{8! 15^2}{2} \right)$
- (D) $\frac{8! 15^2}{2}$
- (E) $\frac{1}{20160} \left(\frac{8! 15^2}{2} \right)$

26. Linear maps $T : \mathbb{R}^{74} \rightarrow \mathbb{R}^{74}$ and $U : \mathbb{R}^{74} \rightarrow \mathbb{R}^{74}$ have ranks 4 and 4. What is the largest possible rank of UT ?

- (A) 2
- (B) 3
- (C) 5
- (D) 4
- (E) 6

27. What is the maximum of $x^6 y^{15}$ over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $21 \left(\left(\frac{6}{21} \right)^6 \left(\frac{15}{21} \right)^{15} \right)$
- (B) $\left(\frac{6}{21} \right)^{15} \left(\frac{15}{21} \right)^6$
- (C) $\frac{10}{49}$
- (D) $\frac{1}{2^{21}}$
- (E) $\left(\frac{6}{21} \right)^6 \left(\frac{15}{21} \right)^{15}$

Questions 28–30

28. For $a_k = (-1)^k + 23/k$, find $\limsup_{k \rightarrow \infty} a_k - \liminf_{k \rightarrow \infty} a_k$.
- (A) 0
(B) 4
(C) 2
(D) 1
(E) 3
29. Evaluate $\int_0^1 \int_x^1 15y^6 \, dy \, dx$.
- (A) $\frac{15}{2}$
(B) $\frac{15}{56}$
(C) $\frac{5}{2}$
(D) $\frac{15}{7}$
(E) $\frac{15}{8}$
30. Fix a nonzero vector $u \in \mathbb{R}^{73}$. What is the dimension of the space of skew-symmetric 73×73 matrices A satisfying $Au = 0$?
- (A) 2555
(B) 2554
(C) 2558
(D) 2556
(E) 2557

Questions 31–33

31. Evaluate $\int_0^1 \int_0^x x^{15} y^6 dy dx$.

- (A) $\frac{1}{112}$
- (B) $\frac{1}{23}$
- (C) $\frac{1}{161}$
- (D) $\frac{1}{184}$
- (E) $\frac{1}{154}$

32. For which real a is $f(x) = x^a$ in $L^{23}(0, 1)$?

- (A) $a < 1/23$
- (B) All real a
- (C) $a > -1/23$
- (D) $a > 0$
- (E) $a \geq -1/23$

33. For $\mathbf{F} = (15x^6y, y^6z, xz^6)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) -378
- (B) 189
- (C) 756
- (D) 378
- (E) 379

Questions 34–36

34. Let $A = \begin{bmatrix} 1 & 67 & 0 \\ 0 & 1 & 11 \\ 0 & 0 & 1 \end{bmatrix}$. What is the $(1, 3)$ entry of A^6 ?

- (A) 11054
- (B) 11055
- (C) 11057
- (D) 11053
- (E) 11056

35. Solve $y' + 15y = x^6 e^{-15x}$, $y(0) = 0$, and find $y(1)$.

- (A) e^{-15}
- (B) $\frac{e^{15}}{7}$
- (C) $\frac{1}{6e^{15}}$
- (D) $\frac{15}{7e^{15}}$
- (E) $\frac{1}{7e^{15}}$

36. $T : \mathbb{R} \rightarrow \mathbb{R}$ has Lipschitz constant at most $1/23$. If $x_1 = T(x_0)$ and $|x_1 - x_0| \leq 1$, what is the sharp upper bound on $|T^3(x_0) - x_*|$, where x_* is its fixed point?

- (A) $\frac{11639}{23276}$
- (B) $\frac{1}{34914}$
- (C) $\frac{11637}{11638}$
- (D) $\frac{1}{11638}$
- (E) $\frac{1}{23276}$

Questions 37–39

37. The solution of $y' = 15x^5y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\frac{15^{\frac{5}{6}}}{15}$
- (B) $\frac{5^{\frac{5}{6}}}{5}$
- (C) $\frac{\sqrt[6]{2} \cdot 5^{\frac{5}{6}}}{5}$
- (D) $\frac{2^{\frac{5}{6}}\sqrt[6]{5}}{2}$
- (E) $\frac{2}{5}$

38. An invertible 73×73 matrix A has determinant 4. Find $\det(\text{adj } A)$.

- (A) 2^{143}
- (B) $2^{144} \cdot 3$
- (C) $2^{144} \cdot 5$
- (D) 2^{145}
- (E) 2^{144}

39. If $y'' + 36y = 15$ and $y(0) = y'(0) = 0$, find $y(\pi/6)$.

- (A) $\frac{5}{3}$
- (B) $\frac{11}{6}$
- (C) $\frac{5}{12}$
- (D) $\frac{5}{6}$
- (E) $-\frac{5}{6}$

Questions 40–42

40. How many zeros, counted with multiplicity, does $z^{18} + 21z + 1$ have in $|z| < 1$?
- (A) 1
(B) 2
(C) 3
(D) 0
(E) 4
41. Find the total outward flux of $\mathbf{F} = (15x^6y, y^6z, xz^6)$ through the boundary of $[0, 1]^3$.
- (A) $\frac{17}{2}$
(B) 17
(C) $\frac{19}{2}$
(D) $\frac{17}{4}$
(E) $-\frac{17}{2}$
42. In $\mathbb{Z}/2154\mathbb{Z}$, how many residues are not idempotent ($x^2 \neq x$)?
- (A) 2147
(B) 2148
(C) 2146
(D) 2144
(E) 2145

Questions 43–45

43. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^{15}y^5 dV$.

- (A) $\frac{1}{6} \left(\frac{15!5!}{23!} \right)$
- (B) $6 \left(\frac{15!5!}{23!} \right)$
- (C) $\frac{15!5!}{23!}$
- (D) $23 \left(\frac{15!5!}{23!} \right)$
- (E) $\frac{1}{96}$

44. A meromorphic function has 20 zeros and 2 poles inside a positively oriented simple closed contour, counted with multiplicity, and no zeros or poles on it. Find $\frac{1}{2\pi i} \int f'(z)/f(z) dz$.

- (A) 24
- (B) 6
- (C) 12
- (D) 30
- (E) 18

45. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 15z^2 = x + 6y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.

- (A) -6
- (B) -90
- (C) -180
- (D) -30
- (E) 180

Questions 46–48

46. In the additive group $\mathbb{Z}/108\mathbb{Z}$, find the index of the subgroup generated by $[2]$.

- (A) 1
- (B) 2
- (C) 3
- (D) 4
- (E) 0

47. Let $f(x, y) = x^2 + 30xy + (231)y^2$. What is the product of the eigenvalues of the Hessian of f ?

- (A) 24
- (B) 12
- (C) 924
- (D) 231
- (E) 6

48. Let $F_0 = 0$, $F_1 = 1$, and $F_{k+2} = F_{k+1} + F_k$. How many binary strings of length 28 have no adjacent 1s?

- (A) F_{30}
- (B) F_{29}
- (C) F_{32}
- (D) F_{28}
- (E) F_{31}

Questions 49–51

49. Let $F(x) = \int_0^x (x-t)^5 e^{15t} dt$. Find $F^{(6)}(0)$.

- (A) 720
- (B) 0
- (C) $\frac{1}{120}$
- (D) 1800
- (E) 120

50. How many invertible 4×4 matrices over $\mathbb{F}_{43}$ are there with determinant 1?

- (A) $\frac{(43^4 - 43^1)(43^4 - 43^2)(43^4 - 43^3)(43^4 - 43^4)}{42}$
- (B) $\frac{(43^4 - 43^0)(43^4 - 43^1)(43^4 - 43^2)(43^4 - 43^3)}{42}$
- (C) $\frac{43^4 43^4 43^4 43^4}{42}$
- (D) $\frac{(43^4 - 43^0)(43^4 - 43^1)(43^4 - 43^2)}{42}$
- (E) $\frac{(43^4 - 43^0)(43^4 - 43^0)(43^4 - 43^0)(43^4 - 43^0)}{42}$

51. Evaluate $\lim_{x \rightarrow 0} \frac{e^{15x} - \sum_{j=0}^5 (15x)^j / j!}{x^6}$.

- (A) $6 \left(\frac{15^6}{6!} \right)$
- (B) $720 \left(\frac{15^6}{6!} \right)$
- (C) $-\frac{15^6}{6!}$
- (D) $\frac{1}{15} \left(\frac{15^6}{6!} \right)$
- (E) $\frac{15^6}{6!}$

Questions 52–54

52. For Newton iteration on $f(x) = x^2 - 784$ near the root $r = 28$, find $\lim_{k \rightarrow \infty} (x_{k+1} - r)/(x_k - r)^2$ for nonterminating convergent iterates.

- (A) $\frac{1}{112}$
- (B) $\frac{55}{56}$
- (C) $\frac{1}{56}$
- (D) $\frac{1}{168}$
- (E) $\frac{57}{112}$

53. For $x > 0$, $x^2 y'' + (-35)xy' + 315y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

- (A) $-\frac{(1 - e^6) e^{15}}{3}$
- (B) $-\frac{e^{15}}{6} + 1 + \frac{e^{21}}{6}$
- (C) $\frac{(1 - e^6) e^{15}}{6}$
- (D) $-\frac{(1 - e^6) e^{15}}{6}$
- (E) $-\frac{(1 - e^6) e^{15}}{12}$

54. How many permutations in S_{73} consist of one 5-cycle and 68 fixed points?

- (A) $2^3 \cdot 3^3 \cdot 7 \cdot 23 \cdot 71 \cdot 73$
- (B) $2^4 \cdot 3^3 \cdot 5 \cdot 7 \cdot 23 \cdot 71 \cdot 73$
- (C) $2^5 \cdot 3^3 \cdot 7 \cdot 23 \cdot 71 \cdot 73$
- (D) $2^4 \cdot 3^3 \cdot 7 \cdot 23 \cdot 71 \cdot 73$
- (E) $2^4 \cdot 3^4 \cdot 7 \cdot 23 \cdot 71 \cdot 73$

Questions 55–57

55. For $x > 0$, $x^2 y'' + (1 - 21)xy' + 90y = 0$, $y(1) = 0$, $y'(1) = 15$. Find $y(e)$.

- (A) $\frac{e^{15}}{21}$
- (B) $-\frac{5(1 - e^9)e^6}{3}$
- (C) $e^6 + e^{15}$
- (D) $\frac{5(1 - e^9)e^6}{3}$
- (E) $-e^6 + e^{15}$

56. Bisection starts with an interval of length 2^{28} . After 30 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{1}{24}$
- (B) $\frac{9}{16}$
- (C) $\frac{1}{8}$
- (D) $\frac{7}{8}$
- (E) $\frac{1}{16}$

57. The local inverse $y(x)$ of $x = 15y + y^3$ satisfies $y(0) = 0$. Find $y^{(7)}(0)$.

- (A) $-\frac{60480}{15^{10}}$
- (B) $\frac{60480}{15^{10}}$
- (C) $\frac{1}{15} \left(-\frac{60480}{15^{10}} \right)$
- (D) $15 \left(-\frac{60480}{15^{10}} \right)$
- (E) $\frac{1}{5040} \left(-\frac{60480}{15^{10}} \right)$

Questions 58–60

58. How many maximal proper subgroups does the cyclic group of order 852 have?

- (A) 1
- (B) 2
- (C) 4
- (D) 5
- (E) 3

59. Find the coefficient of x^4 in $e^{5x} \log(1 + 15x)$.

- (A) $-\frac{50625}{4}$
- (B) -8110
- (C) -195000
- (D) -8125
- (E) 8125

60. What is the least positive multiplicative inverse of 2 modulo 57?

- (A) 29
- (B) 20
- (C) 47
- (D) 11
- (E) 38

Questions 61–63

61. Evaluate $\int_0^1 x^{15}(1-x)^5 dx$.

- (A) $16 \left(\frac{15! 5!}{21!} \right)$
- (B) $\frac{1}{6} \left(\frac{15! 5!}{21!} \right)$
- (C) $21 \left(\frac{15! 5!}{21!} \right)$
- (D) $\frac{6783}{2} \left(\frac{15! 5!}{21!} \right)$
- (E) $\frac{15! 5!}{21!}$

62. How many residues x modulo 108 satisfy $6x \equiv 6 \pmod{108}$ and $2x \equiv 0 \pmod{108}$?

- (A) 1
- (B) 0
- (C) 3
- (D) 4
- (E) 2

63. Let $F(x) = \int_0^{x^6} e^{15t} dt$. Find $F'(1)$.

- (A) $6e^{15}$
- (B) $\frac{e^{15}}{6}$
- (C) e^{15}
- (D) e^{90}
- (E) $90e^{15}$

Questions 64–66

64. How many truth assignments to $p, q, r_1, \dots, r_{26}$ make $p \Rightarrow q$ false?

- (A) 2^{26}
- (B) 2^{29}
- (C) 2^{28}
- (D) 2^{25}
- (E) 2^{27}

65. The solution of $y' = 15x^6y^3$ has $y(0) = 1/16$. Find $y(1)$.

- (A) $\frac{\sqrt{12334}}{881}$
- (B) $\frac{\sqrt{12334}}{1762}$
- (C) $-\frac{\sqrt{12334}}{1762}$
- (D) $\frac{\sqrt{12334}}{3524}$
- (E) $\frac{\sqrt{12334}}{1762} + 1$

66. The field K has 11^{15} elements. Find the largest possible multiplicative order of a nonzero element of K .

- (A) $11^{15} + 1$
- (B) $11^{15} - 1$
- (C) 11^{15}
- (D) $11^{16} - 1$
- (E) $11^{14} - 1$

Answer Key

Practice Test 71 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	B	23	A	45	C
2	D	24	B	46	B
3	B	25	D	47	A
4	C	26	D	48	A
5	C	27	E	49	E
6	B	28	C	50	B
7	E	29	E	51	E
8	A	30	D	52	C
9	D	31	C	53	D
10	A	32	C	54	D
11	D	33	D	55	B
12	C	34	B	56	C
13	A	35	E	57	A
14	E	36	D	58	E
15	E	37	C	59	D
16	C	38	E	60	A
17	E	39	D	61	E
18	A	40	A	62	B
19	D	41	A	63	A
20	B	42	C	64	A
21	B	43	C	65	B
22	A	44	E	66	B

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