

GRE Mathematics Practice Test 72

Advanced Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A demanding practice form with frequent multi-step arguments and a mix of computational and conceptual questions. This is cycle 14 of 19, position 2 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Find the directional derivative of $f(x, y, z) = x^2y + 5xz^2 + 15y^2z$ at $(1, 1, 1)$ in the direction $(1, 3, 0)$.

- (A) $1 + 10\sqrt{10}$
- (B) $-10\sqrt{10}$
- (C) $5\sqrt{10}$
- (D) $10\sqrt{10}$
- (E) $20\sqrt{10}$

2. A sequence satisfies $u_0 = 68$ and $u_{k+1} = 2u_k + 9$ for all $k \geq 0$. Find u_7 .

- (A) 9847
- (B) 9848
- (C) 9845
- (D) 9849
- (E) 9846

3. The solution of $y' = 15x^4y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\frac{15^{\frac{4}{5}}}{15}$
- (B) $\frac{6^{\frac{4}{5}}}{6}$
- (C) $\frac{1}{3}$
- (D) $\frac{3^{\frac{4}{5}}}{3}$
- (E) $\sqrt[5]{3}$

Questions 4–6

4. X, Y are independent, with variances 29 and 30. Find $\text{Cov}(X + 2Y, 3X - Y)$.

- (A) 18
- (B) 27
- (C) 36
- (D) 45
- (E) 9

5. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^{15}y^4 dV$.

- (A) $\frac{1}{80}$
- (B) $\frac{1}{6} \left(\frac{15! 4!}{22!} \right)$
- (C) $22 \left(\frac{15! 4!}{22!} \right)$
- (D) $\frac{15! 4!}{22!}$
- (E) $6 \left(\frac{15! 4!}{22!} \right)$

6. Linear maps $T : \mathbb{R}^{75} \rightarrow \mathbb{R}^{75}$ and $U : \mathbb{R}^{75} \rightarrow \mathbb{R}^{75}$ have ranks 3 and 4. What is the largest possible rank of UT ?

- (A) 5
- (B) 3
- (C) 1
- (D) 2
- (E) 4

Questions 7–9

7. Evaluate $\iiint_{x^2+y^2+z^2 \leq 15} (x^2 + y^2 + z^2)^4 dV$.

- (A) $11 \left(\frac{4\pi 15^{\frac{11}{2}}}{11} \right)$
- (B) $\frac{11}{135} \left(\frac{4\pi 15^{\frac{11}{2}}}{11} \right)$
- (C) $\frac{1}{3} \left(\frac{4\pi 15^{\frac{11}{2}}}{11} \right)$
- (D) $\frac{4\pi 15^{\frac{11}{2}}}{11}$
- (E) $\frac{1}{2} \left(\frac{4\pi 15^{\frac{11}{2}}}{11} \right)$

8. X has density cx^{29} on $0 < x < 1$ and zero elsewhere. Find $E[X^2]$.

- (A) $\frac{5}{16}$
- (B) $\frac{31}{32}$
- (C) $\frac{1}{16}$
- (D) $\frac{15}{32}$
- (E) $\frac{15}{16}$

9. Find the total outward flux of $\mathbf{F} = (15x^5y, y^5z, xz^5)$ through the boundary of $[0, 1]^3$.

- (A) $\frac{19}{2}$
- (B) $-\frac{17}{2}$
- (C) $\frac{17}{4}$
- (D) 17
- (E) $\frac{17}{2}$

Questions 10–12

10. Fix a nonzero vector $u \in \mathbb{R}^{74}$. What is the dimension of the space of skew-symmetric 74×74 matrices A satisfying $Au = 0$?

- (A) 2627
- (B) 2626
- (C) 2629
- (D) 2628
- (E) 2630

11. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(15j)!}{(j!)^{15}} x^{5j}$?

- (A) $15^{-\frac{1}{3}}$
- (B) 15^3
- (C) $225(15^{-3})$
- (D) 15^{-15}
- (E) 15^{-3}

12. Independent exponential waiting times X, Y have rates 29 and 31. Find $P(X < Y)$.

- (A) $\frac{29}{120}$
- (B) $\frac{89}{120}$
- (C) $\frac{29}{60}$
- (D) $\frac{31}{60}$
- (E) $\frac{29}{180}$

Questions 13–15

13. Let $F(x) = \int_0^{x^5} e^{15t} dt$. Find $F'(1)$.

- (A) $75e^{15}$
- (B) $\frac{e^{15}}{5}$
- (C) $5e^{15}$
- (D) e^{75}
- (E) e^{15}

14. Let $A = \begin{bmatrix} 1 & 68 & 0 \\ 0 & 1 & 9 \\ 0 & 0 & 1 \end{bmatrix}$. What is the $(1, 3)$ entry of A^2 ?

- (A) 611
- (B) 612
- (C) 610
- (D) 613
- (E) 614

15. Find the coefficient of x^4 in $e^{4x} \log(1 + 15x)$.

- (A) $-\frac{50625}{4}$
- (B) $-\frac{35525}{4}$
- (C) -213510
- (D) $\frac{35585}{4}$
- (E) $-\frac{35585}{4}$

Questions 16–18

16. $X_1, \dots, X_{29}$ are independent with common mean and variance 31. With $\bar{X}$ their sample mean, find $E[\sum_i (X_i - \bar{X})^2]$.

- (A) 1446
- (B) 290
- (C) 868
- (D) 579
- (E) 1157

17. For $x > 0$, $x^2 y'' + (-34)xy' + 300y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

- (A) $-\frac{2(1 - e^5)e^{15}}{5}$
- (B) $\frac{(1 - e^5)e^{15}}{5}$
- (C) $-\frac{(1 - e^5)e^{15}}{5}$
- (D) $-\frac{(1 - e^5)e^{15}}{10}$
- (E) $-\frac{e^{15}}{5} + 1 + \frac{e^{20}}{5}$

18. An invertible 74×74 matrix A has determinant 2. Find $\det(\text{adj } A)$.

- (A) 2^{74}
- (B) $2^{73} \cdot 5$
- (C) 2^{72}
- (D) $2^{73} \cdot 3$
- (E) 2^{73}

Questions 19–21

19. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 15x$ and has density $\rho(x, y) = x^3y$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{12}{7}$
- (B) $\frac{6}{7}$
- (C) $-\frac{6}{7}$
- (D) $\frac{13}{7}$
- (E) $\frac{3}{7}$

20. A permutation of 31 objects is chosen uniformly. Let Z be the number of fixed points. Find $E[Z^2]$.

- (A) 2
- (B) 4
- (C) 1
- (D) 3
- (E) 0

21. Find the coefficient of x^5 in the Taylor series of $(1 - 15x)^{-2}$ at zero.

- (A) $\frac{5}{6} (6 \cdot 15^5)$
- (B) $-6 \cdot 15^5$
- (C) $6 \cdot 15^5$
- (D) $\frac{1}{15} (6 \cdot 15^5)$
- (E) $\frac{1}{6} (6 \cdot 15^5)$

Questions 22–24

22. On P_{75} , the real polynomials of degree at most 75, define $T(p) = x^2p'' - 24xp' + 156p$. Find its rank.

- (A) 73
- (B) 74
- (C) 75
- (D) 72
- (E) 76

23. Evaluate $\lim_{x \rightarrow 0} \frac{(1 + 15x)^{1/(4x)} - e^{15/4}}{x}$.

- (A) $-\frac{225e^{\frac{15}{4}}}{8}$
- (B) $e^{\frac{15}{4}}$
- (C) $\frac{225e^{\frac{15}{4}}}{8}$
- (D) $-\frac{15e^{\frac{15}{4}}}{8}$
- (E) $-\frac{225e^{\frac{15}{4}}}{4}$

24. Which subset of $\mathbb{R}$ is compact, with 24 fixed as displayed?

- (A) $\{0\} \cup \{24/k : k \geq 1\}$
- (B) $(0, 24)$
- (C) $\mathbb{Q} \cap [0, 24]$
- (D) $24\mathbb{Z}$
- (E) $\{24/k : k \geq 1\}$

Questions 25–27

25. Evaluate $\lim_{x \rightarrow 0} \frac{e^{15x} - \sum_{j=0}^4 (15x)^j / j!}{x^5}$.

- (A) $\frac{15^5}{5!}$
- (B) $5 \left(\frac{15^5}{5!} \right)$
- (C) $-\frac{15^5}{5!}$
- (D) $120 \left(\frac{15^5}{5!} \right)$
- (E) $\frac{1}{15} \left(\frac{15^5}{5!} \right)$

26. Subspaces U, V of $\mathbb{R}^{76}$ have dimensions 5 and 6. What is the smallest possible value of $\dim(U \cap V)$?

- (A) 2
- (B) 4
- (C) 1
- (D) 3
- (E) 0

27. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{15t} \cos(5t)$, and $y = e^{15t} \sin(5t)$, find $d^2 f / dt^2$ at $t = 1$.

- (A) 30
- (B) 500
- (C) 0
- (D) 900
- (E) 10

Questions 28–30

28. For $a_k = (-1)^k + 24/k$, find $\limsup_{k \rightarrow \infty} a_k - \liminf_{k \rightarrow \infty} a_k$.

- (A) 3
- (B) 2
- (C) 1
- (D) 4
- (E) 0

29. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(15+5j)(20+5j)}$.

- (A) $\frac{1}{20}$
- (B) $\frac{1}{15}$
- (C) $\frac{1}{3}$
- (D) $\frac{1}{75}$
- (E) $\frac{1}{5}$

30. Let V consist of real polynomials of degree at most 9 that have both value and first derivative zero at 68, 69, and 71. Find $\dim V$.

- (A) 3
- (B) 2
- (C) 5
- (D) 6
- (E) 4

Questions 31–33

31. Let $f(x) = \log(1 + 15x)$. Find $f^{(5)}(0)$.

- (A) $(-1)^4 4! 15^5$
- (B) $\frac{1}{15} \left((-1)^4 4! 15^5 \right)$
- (C) $\frac{1}{5} \left((-1)^4 4! 15^5 \right)$
- (D) $-(-1)^4 4! 15^5$
- (E) $5 \left((-1)^4 4! 15^5 \right)$

32. For which real a is $f(x) = x^a$ in $L^{24}(0, 1)$?

- (A) $a \geq -1/24$
- (B) $a > -1/24$
- (C) $a < 1/24$
- (D) All real a
- (E) $a > 0$

33. If $y'' + 25y = 15$ and $y(0) = y'(0) = 0$, find $y(\pi/5)$.

- (A) $\frac{12}{5}$
- (B) $\frac{6}{5}$
- (C) $-\frac{6}{5}$
- (D) $\frac{3}{5}$
- (E) $\frac{11}{5}$

Questions 34–36

34. The roots r, s of $x^2 - 77x + 612 = 0$ are counted with multiplicity. Find $r^3 + s^3$.

- (A) 315159
- (B) 315160
- (C) 315161
- (D) 315163
- (E) 315162

35. If $y' + 15y = 5x^5e^{-15x}$ and $y(0) = 3$, find $y(1)$.

- (A) $-\frac{23}{6e^{15}}$
- (B) $\frac{23}{6e^{15}}$
- (C) $\frac{23}{12e^{15}}$
- (D) $\frac{23}{6e^{15}} + 1$
- (E) $\frac{23}{3e^{15}}$

36. $T : \mathbb{R} \rightarrow \mathbb{R}$ has Lipschitz constant at most $1/24$. If $x_1 = T(x_0)$ and $|x_1 - x_0| \leq 1$, what is the sharp upper bound on $|T^3(x_0) - x_*|$, where x_* is its fixed point?

- (A) $\frac{13249}{26496}$
- (B) $\frac{1}{26496}$
- (C) $\frac{13247}{13248}$
- (D) $\frac{1}{13248}$
- (E) $\frac{1}{39744}$

Questions 37–39

37. Evaluate $\int_0^1 \frac{x^4}{1+15x^5} dx$.

- (A) $\frac{4 \log(2)}{75}$
- (B) $-\frac{4 \log(2)}{75}$
- (C) $\log(16)$
- (D) $\frac{4 \log(2)}{5}$
- (E) $\frac{4 \log(2)}{15}$

38. For real t , let $M(t) = \begin{pmatrix} t-68 & 9 & 0 \\ 0 & t-68 & 1 \\ 0 & 0 & t-68 \end{pmatrix}$. At $t = 68$, what is $\dim \ker(M(t)^2) - \dim \ker M(t)$?

- (A) 1
- (B) 2
- (C) 4
- (D) 3
- (E) 0

39. Two solutions of $y'' + 15x^5y' + 5y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $\frac{1}{2e^{\frac{5}{2}}}$
- (B) $-\frac{1}{e^{\frac{5}{2}}}$
- (C) $\frac{2}{e^{\frac{5}{2}}}$
- (D) $e^{-\frac{5}{2}}$
- (E) $e^{-\frac{5}{2}} + 1$

Questions 40–42

40. How many zeros, counted with multiplicity, does $z^{19} + 22z + 1$ have in $|z| < 1$?

- (A) 4
- (B) 0
- (C) 1
- (D) 3
- (E) 2

41. For $x > 0$, $x^2y'' + (1 - 20)xy' + 75y = 0$, $y(1) = 0$, $y'(1) = 15$. Find $y(e)$.

- (A) $e^5 + e^{15}$
- (B) $\frac{3(1 - e^{10})e^5}{2}$
- (C) $\frac{e^{15}}{20}$
- (D) $-e^5 + e^{15}$
- (E) $-\frac{3(1 - e^{10})e^5}{2}$

42. Let $G = \mathbb{Z}/438\mathbb{Z} \times \mathbb{Z}/296\mathbb{Z}$. Find the order of G/H , where $H = \{g \in G : 2g = 0\}$.

- (A) 32412
- (B) 32413
- (C) 32411
- (D) 32414
- (E) 32410

Questions 43–45

43. If $y' = 30xy$ and $y(0) = 1$, find the coefficient of x^{10} in the Maclaurin series of y .
- (A) $\frac{50625}{4}$
(B) $-\frac{50625}{8}$
(C) $\frac{50625}{8}$
(D) $\frac{50633}{8}$
(E) $\frac{50625}{16}$
44. A meromorphic function has 21 zeros and 2 poles inside a positively oriented simple closed contour, counted with multiplicity, and no zeros or poles on it. Find $\frac{1}{2\pi i} \int f'(z)/f(z) dz$.
- (A) 25
(B) 31
(C) 7
(D) 19
(E) 13
45. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 15z^2 = x + 5y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.
- (A) -75
(B) -150
(C) -30
(D) -5
(E) 150

Questions 46–48

46. How many group homomorphisms $\mathbb{Z}/438\mathbb{Z} \rightarrow \mathbb{Z}/296\mathbb{Z}$ have image of order 2?

- (A) 3
- (B) 4
- (C) 2
- (D) 1
- (E) 0

47. If $y'' - 30y' + 225y = 5e^{15x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) $5e^{15}$
- (B) $\frac{5e^{15}}{2}$
- (C) $\frac{5e^{15}}{4}$
- (D) $1 + \frac{5e^{15}}{2}$
- (E) $-\frac{5e^{15}}{2}$

48. Let $F_0 = 0$, $F_1 = 1$, and $F_{k+2} = F_{k+1} + F_k$. How many binary strings of length 29 have no adjacent 1s?

- (A) F_{33}
- (B) F_{29}
- (C) F_{31}
- (D) F_{32}
- (E) F_{30}

Questions 49–51

49. Evaluate $\int_0^{\log 2} e^x(e^x + 15)^5 dx$.
- (A) $6 \left(\frac{17^6 - 16^6}{6} \right)$
(B) $\frac{17^6 - 16^6}{6}$
(C) $17^5 - 16^5$
(D) $\frac{17^6}{6}$
(E) $-\frac{17^6 - 16^6}{6}$
50. How many 2-dimensional subspaces does $\mathbb{F}_{43}^6$ have?
- (A) $\frac{(43^6 - 1)(43^6 - 43)}{(43^2 - 1)(43^2 - 43)}$
(B) $\frac{(43^7 - 1)(43^7 - 43)}{(43^2 - 1)(43^2 - 43)}$
(C) $\frac{(43^5 - 1)(43^5 - 43)}{(43^2 - 1)(43^2 - 43)}$
(D) $\frac{(43^9 - 1)(43^9 - 43)}{(43^2 - 1)(43^2 - 43)}$
(E) $\frac{(43^8 - 1)(43^8 - 43)}{(43^2 - 1)(43^2 - 43)}$
51. For $\mathbf{F} = (15x^5y, y^5z, xz^5)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.
- (A) $\frac{235}{2}$
(B) 470
(C) -235
(D) 236
(E) 235

Questions 52–54

52. For Newton iteration on $f(x) = x^2 - 841$ near the root $r = 29$, find $\lim_{k \rightarrow \infty} (x_{k+1} - r)/(x_k - r)^2$ for nonterminating convergent iterates.

- (A) $\frac{1}{116}$
- (B) $\frac{1}{174}$
- (C) $\frac{57}{58}$
- (D) $\frac{59}{116}$
- (E) $\frac{1}{58}$

53. Evaluate $\iiint_B 15z^2(x^2 + y^2 + z^2)^5 dV$, where B is the unit ball.

- (A) $\frac{8\pi}{3}$
- (B) $-\frac{4\pi}{3}$
- (C) $1 + \frac{4\pi}{3}$
- (D) $\frac{2\pi}{3}$
- (E) $\frac{4\pi}{3}$

54. How many monic irreducible polynomials of degree 4 are there over $\mathbb{F}_{43}$?

- (A) $\frac{3(43^4 - 43^2)}{4}$
- (B) $\frac{2(43^4 - 43^2)}{4}$
- (C) $2(43^4 - 43^2)$
- (D) $\frac{43^4 - 43^2}{4}$
- (E) $43^4 - 43^2$

Questions 55–57

55. Evaluate $\int_0^1 x^{14}(\log x)^4 dx$.

- (A) $\frac{(-1)^4 4!}{15^5}$
(B) $-\frac{(-1)^4 4!}{15^5}$
(C) $\frac{16875}{8} \left(\frac{(-1)^4 4!}{15^5} \right)$
(D) $15 \left(\frac{(-1)^4 4!}{15^5} \right)$
(E) $\frac{1}{5} \left(\frac{(-1)^4 4!}{15^5} \right)$

56. Bisection starts with an interval of length 2^{29} . After 31 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{1}{24}$
(B) $\frac{7}{8}$
(C) $\frac{1}{8}$
(D) $\frac{1}{16}$
(E) $\frac{9}{16}$

57. The region $0 \leq x \leq 1$, $0 \leq y \leq 15x^5$ is revolved about the line $x = -3$. Find its volume.

- (A) $\frac{135\pi}{7}$
(B) $-\frac{135\pi}{7}$
(C) $\frac{270\pi}{7}$
(D) $1 + \frac{135\pi}{7}$
(E) $\frac{135\pi}{14}$

Questions 58–60

58. How many elements z of $\mathbb{F}_{7^{15}}$ satisfy $z^{49} = z$?

- (A) 8
- (B) 5
- (C) 7
- (D) 9
- (E) 6

59. For $D = \{(x, y) : x^2 + y^2 \leq 15\}$, evaluate $\iint_D (x^2 + y^2)^4 dA$.

- (A) 84375π
- (B) 759375π
- (C) 10125π
- (D) 151875π
- (E) 303750π

60. What is the least positive multiplicative inverse of 2 modulo 59?

- (A) 50
- (B) 10
- (C) 30
- (D) 20
- (E) 40

Questions 61–63

- 61.** Let $f(x, y) = x^2 + 30xy + (230)y^2$. What is the product of the eigenvalues of the Hessian of f ?
- (A) 5
(B) 20
(C) 230
(D) 10
(E) 920
- 62.** How many generators does the cyclic group of order 147008443 have?
- (A) $43^5(43 - 1)$
(B) $43^7(43 - 1)$
(C) $43^6(43 - 1)$
(D) $43^3(43 - 1)$
(E) $43^4(43 - 1)$
- 63.** Under $x = u + 15v$, $y = u - 5v$, what is the area of the image of $0 \leq u, v \leq 1$?
- (A) 20
(B) $\frac{1}{20}$
(C) 10
(D) 75
(E) 400

Questions 64–66

64. How many truth assignments to $p, q, r_1, \dots, r_{27}$ make $p \Rightarrow q$ false?

- (A) 2^{30}
- (B) 2^{29}
- (C) 2^{28}
- (D) 2^{26}
- (E) 2^{27}

65. The solution of $y' = 15x^5y^3$ has $y(0) = 1/16$. Find $y(1)$.

- (A) $\frac{2\sqrt{251}}{251}$
- (B) $-\frac{\sqrt{251}}{251}$
- (C) $\frac{\sqrt{251}}{251} + 1$
- (D) $\frac{\sqrt{251}}{502}$
- (E) $\frac{\sqrt{251}}{251}$

66. Let $R = (\mathbb{Z}/51\mathbb{Z})/([17])$, where $([17])$ is the ideal generated by $[17]$. How many units does R have?

- (A) 15
- (B) 17
- (C) 14
- (D) 16
- (E) 18

Answer Key

Practice Test 72 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	D	23	A	45	B
2	A	24	A	46	D
3	D	25	A	47	B
4	B	26	E	48	C
5	D	27	C	49	B
6	B	28	B	50	A
7	D	29	D	51	E
8	E	30	E	52	E
9	E	31	A	53	E
10	D	32	B	54	D
11	E	33	B	55	A
12	C	34	C	56	C
13	C	35	B	57	A
14	B	36	D	58	C
15	E	37	A	59	D
16	C	38	A	60	C
17	C	39	D	61	B
18	E	40	C	62	E
19	B	41	E	63	A
20	A	42	A	64	E
21	C	43	C	65	E
22	B	44	D	66	D

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