

GRE Mathematics Practice Test 73

Upper-Intermediate Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

Intermediate reinforcement, combining standard applications with selected more demanding arguments. This is cycle 14 of 19, position 3 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Evaluate $\int_0^1 \frac{x^3}{1+15x^4} dx$.

- (A) $\log(2)$
- (B) $\log(16)$
- (C) $\frac{4\log(2)}{15}$
- (D) $\frac{\log(2)}{15}$
- (E) $-\frac{\log(2)}{15}$

2. On P_{76} , the real polynomials of degree at most 76, define $T(p) = x^2p'' - 132p$. Find its rank.

- (A) 75
- (B) 77
- (C) 76
- (D) 78
- (E) 74

3. Evaluate $\int_0^{\log 2} e^x(e^x + 15)^4 dx$.

- (A) $17^4 - 16^4$
- (B) $\frac{17^5}{5}$
- (C) $-\frac{17^5 - 16^5}{5}$
- (D) $\frac{17^5 - 16^5}{5}$
- (E) $5 \left(\frac{17^5 - 16^5}{5} \right)$

Questions 4–6

4. X, Y are independent, with variances 30 and 31. Find $\text{Cov}(X + 2Y, 3X - Y)$.

- (A) 10
- (B) 37
- (C) 28
- (D) 46
- (E) 19

5. The solution of $y' = 15x^4y^3$ has $y(0) = 1/16$. Find $y(1)$.

- (A) $\frac{\sqrt{10}}{100}$
- (B) $-\frac{\sqrt{10}}{50}$
- (C) $\frac{\sqrt{10}}{50}$
- (D) $\frac{\sqrt{10}}{25}$
- (E) $\frac{\sqrt{10}}{50} + 1$

6. Subspaces U, V of $\mathbb{R}^{77}$ have dimensions 4 and 5. What is the smallest possible value of $\dim(U \cap V)$?

- (A) 4
- (B) 3
- (C) 0
- (D) 2
- (E) 1

Questions 7–9

7. Find the arc length of $r(t) = (15(t - t^3/3), 15t^2)$ for $0 \leq t \leq 1/4$.

- (A) $\frac{245}{32}$
- (B) $-\frac{245}{64}$
- (C) $\frac{309}{64}$
- (D) $\frac{245}{128}$
- (E) $\frac{245}{64}$

8. X has density cx^{30} on $0 < x < 1$ and zero elsewhere. Find the normalizing constant c .

- (A) 11
- (B) 41
- (C) 21
- (D) 31
- (E) 51

9. Evaluate $\int_0^1 \int_x^1 15y^4 dy dx$.

- (A) $\frac{15}{4}$
- (B) $\frac{15}{2}$
- (C) $\frac{1}{2}$
- (D) 3
- (E) $\frac{5}{2}$

Questions 10–12

10. Let V consist of real polynomials of degree at most 8 that vanish at 69, 70, and 72. Find $\dim V$.

- (A) 7
- (B) 6
- (C) 8
- (D) 5
- (E) 4

11. The local inverse $y(x)$ of $x = 15y + y^3$ satisfies $y(0) = 0$. Find $y^{(3)}(0)$.

- (A) $\frac{1}{15} \left(-\frac{6}{15^4} \right)$
- (B) $15 \left(-\frac{6}{15^4} \right)$
- (C) $-\frac{6}{15^4}$
- (D) $\frac{1}{6} \left(-\frac{6}{15^4} \right)$
- (E) $\frac{6}{15^4}$

12. Independent exponential waiting times X, Y have rates 30 and 32. Find $P(X < Y)$.

- (A) $\frac{23}{31}$
- (B) $\frac{5}{31}$
- (C) $\frac{15}{31}$
- (D) $\frac{16}{31}$
- (E) $\frac{15}{62}$

Questions 13–15

13. For $f(x, y) = x^2 + 15xy + 4y^2$, what is its maximum directional derivative at $(1, -1)$?

- (A) -13
- (B) 20
- (C) $\sqrt{218}$
- (D) 218
- (E) 7

14. The roots r, s of $x^2 - 81x + 828 = 0$ are counted with multiplicity. Find $r^3 + s^3$.

- (A) 330237
- (B) 330235
- (C) 330236
- (D) 330239
- (E) 330238

15. Let $F(x) = \int_0^x (x - t)^3 e^{15t} dt$. Find $F^{(4)}(0)$.

- (A) 90
- (B) 24
- (C) $\frac{1}{6}$
- (D) 6
- (E) 0

Questions 16–18

16. $X_1, \dots, X_{30}$ are independent with common mean and variance 32. With $\bar{X}$ their sample mean, find $E[\sum_i (X_i - \bar{X})^2]$.

- (A) 1237
- (B) 310
- (C) 1546
- (D) 619
- (E) 928

17. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 15x$ and has density $\rho(x, y) = x^2y$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{5}{3}$
- (B) $-\frac{5}{6}$
- (C) $\frac{11}{6}$
- (D) $\frac{5}{12}$
- (E) $\frac{5}{6}$

18. For real t , let $M(t) = \begin{pmatrix} t-69 & 12 & 0 \\ 0 & t-69 & 1 \\ 0 & 0 & t-69 \end{pmatrix}$. At $t = 69$, what is $\dim \ker(M(t)^2) - \dim \ker M(t)$?

- (A) 2
- (B) 0
- (C) 4
- (D) 1
- (E) 3

Questions 19–21

19. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-15)^j}{j^1 (\log j)^m}$ converge absolutely at $x = 14$? Give the least such m , or 0 if none exists.

- (A) 1
- (B) 4
- (C) 0
- (D) 2
- (E) 3

20. A permutation of 32 objects is chosen uniformly. Let Z be the number of fixed points. Find $E[Z^2]$.

- (A) 1
- (B) 3
- (C) 0
- (D) 4
- (E) 2

21. Evaluate $\lim_{x \rightarrow 0} \frac{e^{15x} - \sum_{j=0}^3 (15x)^j / j!}{x^4}$.

- (A) $\frac{15^4}{4!}$
- (B) $-\frac{15^4}{4!}$
- (C) $24 \left(\frac{15^4}{4!} \right)$
- (D) $4 \left(\frac{15^4}{4!} \right)$
- (E) $\frac{1}{15} \left(\frac{15^4}{4!} \right)$

Questions 22–24

22. Let $u = (14, 4)^T$ and $v = (1, 0)^T$. Find the squared distance from $(14, 0)^T$ to $\text{span}(u)$.

- (A) $\frac{1515}{106}$
- (B) $\frac{784}{53}$
- (C) $\frac{1621}{106}$
- (D) $\frac{731}{53}$
- (E) $\frac{837}{53}$

23. Let $f(x) = \log(1 + 15x)$. Find $f^{(4)}(0)$.

- (A) $\frac{1}{15} \left((-1)^3 3! 15^4 \right)$
- (B) $(-1)^3 3! 15^4$
- (C) $\frac{1}{4} \left((-1)^3 3! 15^4 \right)$
- (D) $-(-1)^3 3! 15^4$
- (E) $4 \left((-1)^3 3! 15^4 \right)$

24. Which subset of $\mathbb{R}$ is compact, with 25 fixed as displayed?

- (A) $25\mathbb{Z}$
- (B) $\mathbb{Q} \cap [0, 25]$
- (C) $\{0\} \cup \{25/k : k \geq 1\}$
- (D) $(0, 25)$
- (E) $\{25/k : k \geq 1\}$

Questions 25–27

25. For $f(x) = x^4 e^{15x}$, find $f^{(6)}(0)$.

- (A) $\frac{6! 15^2}{2}$
- (B) $\frac{1}{360} \left(\frac{6! 15^2}{2} \right)$
- (C) $\frac{2}{15} \left(\frac{6! 15^2}{2} \right)$
- (D) $2 \left(\frac{6! 15^2}{2} \right)$
- (E) $\frac{1}{30} \left(\frac{6! 15^2}{2} \right)$

26. An idempotent 75×75 real matrix P has rank 2. What is $\det(I + P)$?

- (A) 2
- (B) 5
- (C) 3
- (D) 6
- (E) 4

27. Evaluate $\iiint_{x^2+y^2+z^2 \leq 15} (x^2 + y^2 + z^2)^3 dV$.

- (A) $\frac{1}{3} \left(\frac{4\pi 15^{\frac{9}{2}}}{9} \right)$
- (B) $\frac{3}{35} \left(\frac{4\pi 15^{\frac{9}{2}}}{9} \right)$
- (C) $\frac{1}{2} \left(\frac{4\pi 15^{\frac{9}{2}}}{9} \right)$
- (D) $9 \left(\frac{4\pi 15^{\frac{9}{2}}}{9} \right)$
- (E) $\frac{4\pi 15^{\frac{9}{2}}}{9}$

Questions 28–30

28. For $a_k = (-1)^k + 25/k$, find $\limsup_{k \rightarrow \infty} a_k - \liminf_{k \rightarrow \infty} a_k$.

- (A) 2
- (B) 1
- (C) 0
- (D) 3
- (E) 4

29. Evaluate $\int_0^1 x^{14}(\log x)^3 dx$.

- (A) $\frac{(-1)^3 3!}{15^4}$
- (B) $-\frac{(-1)^3 3!}{15^4}$
- (C) $15 \left(\frac{(-1)^3 3!}{15^4} \right)$
- (D) $\frac{1}{4} \left(\frac{(-1)^3 3!}{15^4} \right)$
- (E) $\frac{1125}{2} \left(\frac{(-1)^3 3!}{15^4} \right)$

30. For $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} 69 \\ 12 \\ 81 \end{bmatrix}$, let x minimize $\|Ax - b\|^2$. Find x_2 .

- (A) 5
- (B) 8
- (C) 6
- (D) 4
- (E) 7

Questions 31–33

31. What is the maximum of x^4y^{15} over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\frac{1}{2^{19}}$
- (B) $\left(\frac{4}{19}\right)^4 \left(\frac{15}{19}\right)^{15}$
- (C) $\left(\frac{4}{19}\right)^{15} \left(\frac{15}{19}\right)^4$
- (D) $19 \left(\left(\frac{4}{19}\right)^4 \left(\frac{15}{19}\right)^{15} \right)$
- (E) $\frac{60}{361}$

32. For which real a is $f(x) = x^a$ in $L^{25}(0, 1)$?

- (A) $a > 0$
- (B) $a > -1/25$
- (C) $a \geq -1/25$
- (D) All real a
- (E) $a < 1/25$

33. Evaluate $\int_0^1 x^{15}(1-x)^3 dx$.

- (A) $\frac{1}{4} \left(\frac{15! 3!}{19!} \right)$
- (B) $\frac{15! 3!}{19!}$
- (C) $19 \left(\frac{15! 3!}{19!} \right)$
- (D) $\frac{969}{4} \left(\frac{15! 3!}{19!} \right)$
- (E) $16 \left(\frac{15! 3!}{19!} \right)$

Questions 34–36

34. What is the coefficient of x^7 in $(x + 14)^6(1 - x)^3$?

- (A) -2692
- (B) -2690
- (C) -2693
- (D) -2691
- (E) -2689

35. Find the directional derivative of $f(x, y, z) = x^2y + 4xz^2 + 15y^2z$ at $(1, 1, 1)$ in the direction $(1, 4, 0)$.

- (A) $-\frac{130\sqrt{17}}{17}$
- (B) $\frac{130\sqrt{17}}{17}$
- (C) $1 + \frac{130\sqrt{17}}{17}$
- (D) $\frac{260\sqrt{17}}{17}$
- (E) $\frac{65\sqrt{17}}{17}$

36. $T : \mathbb{R} \rightarrow \mathbb{R}$ has Lipschitz constant at most $1/25$. If $x_1 = T(x_0)$ and $|x_1 - x_0| \leq 1$, what is the sharp upper bound on $|T^3(x_0) - x_*|$, where x_* is its fixed point?

- (A) $\frac{1}{30000}$
- (B) $\frac{15001}{30000}$
- (C) $\frac{1}{45000}$
- (D) $\frac{14999}{15000}$
- (E) $\frac{1}{15000}$

Questions 37–39

37. Find the coefficient of x^4 in the Taylor series of $(1 - 15x)^{-2}$ at zero.

- (A) $\frac{4}{5} (5 \cdot 15^4)$
- (B) $\frac{1}{15} (5 \cdot 15^4)$
- (C) $\frac{1}{5} (5 \cdot 15^4)$
- (D) $5 \cdot 15^4$
- (E) $2 (5 \cdot 15^4)$

38. A matrix A satisfies $(A - 70I)(A - 72I) = 0$, and its minimal polynomial has degree two. If $A^4 = uA + vI$, find u .

- (A) $2^3 \cdot 71 \cdot 2521$
- (B) $2^3 \cdot 5 \cdot 71 \cdot 2521$
- (C) $2^3 \cdot 3 \cdot 71 \cdot 2521$
- (D) 715964
- (E) $2^4 \cdot 71 \cdot 2521$

39. Two solutions of $y'' + 15x^4y' + 4y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) e^{-3}
- (B) $\frac{2}{e^3}$
- (C) $\frac{1}{2e^3}$
- (D) $-\frac{1}{e^3}$
- (E) $e^{-3} + 1$

Questions 40–42

40. How many zeros, counted with multiplicity, does $z^{20} + 23z + 1$ have in $|z| < 1$?

- (A) 4
- (B) 2
- (C) 3
- (D) 0
- (E) 1

41. Evaluate $\lim_{x \rightarrow 0} \frac{(1 + 15x)^{1/(3x)} - e^{15/3}}{x}$.

- (A) $-\frac{75e^5}{2}$
- (B) $-\frac{5e^5}{2}$
- (C) e^5
- (D) $-75e^5$
- (E) $\frac{75e^5}{2}$

42. What is the largest integer k such that 3^k divides $78!$?

- (A) 37
- (B) 34
- (C) 38
- (D) 35
- (E) 36

Questions 43–45

43. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(15j)!}{(j!)^{15}} x^{4j}$?
- (A) $\frac{1}{15}$
(B) 15^{-15}
(C) $15^{-\frac{15}{4}}$
(D) $15^{-\frac{4}{15}}$
(E) $15^{\frac{15}{4}}$
44. A meromorphic function has 22 zeros and 2 poles inside a positively oriented simple closed contour, counted with multiplicity, and no zeros or poles on it. Find $\frac{1}{2\pi i} \int f'(z)/f(z) dz$.
- (A) 8
(B) 32
(C) 14
(D) 26
(E) 20
45. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^{15} y^3 dV$.
- (A) $\frac{15! 3!}{21!}$
(B) $\frac{1}{64}$
(C) $21 \left(\frac{15! 3!}{21!} \right)$
(D) $6 \left(\frac{15! 3!}{21!} \right)$
(E) $\frac{1}{6} \left(\frac{15! 3!}{21!} \right)$

Questions 46–48

46. In the ring $\mathbb{Z}/438\mathbb{Z}$, how many elements satisfy $x^2 = 1$?

- (A) 6
- (B) 4
- (C) 3
- (D) 2
- (E) 5

47. If $y'' + 16y = 15$ and $y(0) = y'(0) = 0$, find $y(\pi/4)$.

- (A) $\frac{23}{8}$
- (B) $\frac{15}{8}$
- (C) $\frac{16}{15}$
- (D) $-\frac{15}{8}$
- (E) $\frac{15}{4}$

48. Let $F_0 = 0$, $F_1 = 1$, and $F_{k+2} = F_{k+1} + F_k$. How many binary strings of length 30 have no adjacent 1s?

- (A) F_{34}
- (B) F_{32}
- (C) F_{30}
- (D) F_{33}
- (E) F_{31}

Questions 49–51

49. Let C be $x^2/16 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C (-5y^3) dx + (\frac{x^3}{3}) dy$.
- (A) 62π
(B) $1 + 31\pi$
(C) -31π
(D) 31π
(E) $\frac{31\pi}{2}$
50. A permutation consists of disjoint cycles of lengths 4 and 3, together with a disjoint transposition when the sum of these two lengths is odd. It fixes all other points. What is its order?
- (A) 13
(B) 11
(C) 10
(D) 14
(E) 12
51. Let C be the unit circle $x^2 + y^2 = 1$, $z = 4$, counterclockwise viewed from above. Evaluate $\int_C -15y(x^2 + y^2)^3 dx + (15x(x^2 + y^2)^3) dy + 4z dz$.
- (A) 30π
(B) 15π
(C) $1 + 30\pi$
(D) -30π
(E) 60π

Questions 52–54

52. Apply one Newton iteration to solve $x^2 - 30 = 0$, starting at $x_0 = 1$. What is x_1 ?

- (A) $\frac{33}{2}$
- (B) $\frac{31}{2}$
- (C) $\frac{31}{4}$
- (D) $\frac{35}{2}$
- (E) 31

53. Let $F(x) = \int_0^{x^4} e^{15t} dt$. Find $F'(1)$.

- (A) $4e^{15}$
- (B) $\frac{e^{15}}{4}$
- (C) $60e^{15}$
- (D) e^{15}
- (E) e^{60}

54. In $R = \mathbb{F}_5[x]/(x^{15})$, how many elements are units?

- (A) $5^{16} - 5^{14}$
- (B) $5^{16} - 5^{15}$
- (C) $5^{15} - 5^0$
- (D) $5^{15} - 5^{14}$
- (E) $5^{17} - 5^{16}$

Questions 55–57

55. Evaluate $\int_0^1 \int_0^x x^{15} y^4 dy dx$.

- (A) $\frac{1}{126}$
- (B) $\frac{1}{80}$
- (C) $\frac{1}{100}$
- (D) $\frac{1}{21}$
- (E) $\frac{1}{105}$

56. Bisection starts with an interval of length 2^{30} . After 32 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{9}{16}$
- (B) $\frac{7}{8}$
- (C) $\frac{1}{24}$
- (D) $\frac{1}{8}$
- (E) $\frac{1}{16}$

57. If $y' = 30xy$ and $y(0) = 1$, find the coefficient of x^8 in the Maclaurin series of y .

- (A) $\frac{16875}{16}$
- (B) $\frac{16875}{8}$
- (C) $-\frac{16875}{8}$
- (D) $\frac{16875}{4}$
- (E) $\frac{16883}{8}$

Questions 58–60

58. Integers x satisfy $x \equiv 2 \pmod{7}$ and $x \equiv 1 \pmod{9}$. Find the least nonnegative solution.

- (A) 38
- (B) 39
- (C) 37
- (D) 35
- (E) 36

59. For $x > 0$, $x^2 y'' + (1 - 19)xy' + 60y = 0$, $y(1) = 0$, $y'(1) = 15$. Find $y(e)$.

- (A) $e^4 + e^{15}$
- (B) $\frac{e^{15}}{19}$
- (C) $\frac{15(1 - e^{11})e^4}{11}$
- (D) $-\frac{15(1 - e^{11})e^4}{11}$
- (E) $-e^4 + e^{15}$

60. What is the least positive multiplicative inverse of 2 modulo 61?

- (A) 31
- (B) 41
- (C) 51
- (D) 11
- (E) 21

Questions 61–63

61. The solution of $y' = 15x^3y^2$, $y(0) = 1$, has its first positive singularity at which x ?
- (A) $\frac{15^{\frac{3}{4}}\sqrt[4]{2}}{15}$
(B) $\frac{\sqrt[4]{15}\sqrt{2}}{2}$
(C) $\frac{15^{\frac{3}{4}}\sqrt{2}}{15}$
(D) $\frac{4}{15}$
(E) $\frac{15^{\frac{3}{4}}}{15}$
62. The symmetric group S_{75} acts on the two-element subsets of $\{1, \dots, 75\}$. What is the index of the setwise stabilizer of $\{1, 2\}$?
- (A) 2774
(B) 2773
(C) 2777
(D) 2775
(E) 2776
63. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(15x) - \sum_{j=0}^2 (-1)^j (15x)^{2j+1} / (2j+1)!}{x^7}$.
- (A) $15 \left(\frac{(-1)^3 15^7}{7!} \right)$
(B) $\frac{(-1)^3 15^7}{7!}$
(C) $-\frac{(-1)^3 15^7}{7!}$
(D) $\frac{1}{15} \left(\frac{(-1)^3 15^7}{7!} \right)$
(E) $7 \left(\frac{(-1)^3 15^7}{7!} \right)$

Questions 64–66

64. How many truth assignments to $p, q, r_1, \dots, r_{28}$ make $p \Rightarrow q$ false?

- (A) 2^{28}
- (B) 2^{30}
- (C) 2^{29}
- (D) 2^{31}
- (E) 2^{27}

65. Find the curvature of the helix $r(t) = (15 \cos t, 15 \sin t, 4t)$.

- (A) $\frac{256}{241}$
- (B) $\frac{15}{482}$
- (C) $\frac{30}{241}$
- (D) $\frac{15}{241}$
- (E) $-\frac{15}{241}$

66. In $\mathbb{Z}/2238\mathbb{Z}$, how many residues are not idempotent ($x^2 \neq x$)?

- (A) 2232
- (B) 2230
- (C) 2229
- (D) 2228
- (E) 2231

Answer Key

Practice Test 73 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	D	23	B	45	A
2	C	24	C	46	B
3	D	25	A	47	C
4	C	26	E	48	B
5	C	27	E	49	D
6	C	28	A	50	E
7	E	29	A	51	A
8	D	30	C	52	B
9	E	31	B	53	A
10	B	32	B	54	D
11	C	33	B	55	E
12	C	34	D	56	D
13	C	35	B	57	B
14	A	36	E	58	C
15	D	37	D	59	D
16	E	38	A	60	A
17	E	39	A	61	C
18	D	40	E	62	D
19	D	41	A	63	B
20	E	42	E	64	A
21	A	43	C	65	D
22	B	44	E	66	B

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