

GRE Mathematics Practice Test 75

Standard-Level Target

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A mixed standard-level target, intended to approximate public GRE Mathematics practice in scope and overall demand. This is cycle 14 of 19, position 5 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Two solutions of $y'' + 15x^1y' + 2y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $\frac{1}{2e^{\frac{15}{2}}}$
- (B) $-\frac{1}{e^{\frac{15}{2}}}$
- (C) $\frac{2}{e^{\frac{15}{2}}}$
- (D) $e^{-\frac{15}{2}}$
- (E) $e^{-\frac{15}{2}} + 1$

2. Let $V = \begin{bmatrix} 1 & 71 & 5041 \\ 1 & 73 & 5329 \\ 1 & 76 & 5776 \end{bmatrix}$. Find $\det V$.

- (A) 32
- (B) 31
- (C) 28
- (D) 30
- (E) 29

3. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j(x-15)^j}{j^{\frac{1}{3}}(\log j)^m}$ converge absolutely at $x = 14$? Give the least such m , or 0 if none exists.

- (A) 4
- (B) 0
- (C) 3
- (D) 1
- (E) 2

Questions 4–6

4. X, Y are independent, with variances 32 and 33. Find $\text{Cov}(X + 2Y, 3X - Y)$.

- (A) 50
- (B) 30
- (C) 10
- (D) 20
- (E) 40

5. For $\mathbf{F} = (15x^2, y^2, z^2)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) 36
- (B) 18
- (C) 72
- (D) -36
- (E) 37

6. Let $u = (1, 2, \dots, 77)^T$, $v = (1, 1, \dots, 1)^T$, and $B = I + uv^T$. Find $\det B$.

- (A) 3006
- (B) 3005
- (C) 3002
- (D) 3003
- (E) 3004

Questions 7–9

7. For $y' = 15y(1 - y/2)$, what is the derivative of the right-hand side with respect to y at the positive equilibrium?

- (A) -14
- (B) 15
- (C) $-\frac{15}{2}$
- (D) -30
- (E) -15

8. X has density cx^{32} on $0 < x < 1$ and zero elsewhere. Find the normalizing constant c .

- (A) 33
- (B) 55
- (C) 22
- (D) 11
- (E) 44

9. Find the coefficient of x^4 in $\cos(15x)/(1 - 1x^2)$.

- (A) $-\frac{15983}{8}$
- (B) $\frac{15975}{8}$
- (C) $\frac{16875}{8}$
- (D) 47949
- (E) $\frac{15983}{8}$

Questions 10–12

10. Let D be a diagonal 77×77 real matrix with distinct diagonal entries. What is the dimension of the space of matrices X satisfying $XD = DX$?

- (A) 79
- (B) 78
- (C) 76
- (D) 77
- (E) 75

11. Let $f(x, y) = e^{15x+2y}$. Find $\partial^2 f / \partial x^1 \partial y$ at $(0, 0)$.

- (A) $2(2 \cdot 15^1)$
- (B) $\frac{2}{15}(2 \cdot 15^1)$
- (C) $2 \cdot 15^1$
- (D) $\frac{1}{2}(2 \cdot 15^1)$
- (E) $\frac{15}{2}(2 \cdot 15^1)$

12. Independent exponential waiting times X, Y have rates 32 and 34. Find $P(X < Y)$.

- (A) $\frac{49}{66}$
- (B) $\frac{8}{33}$
- (C) $\frac{16}{33}$
- (D) $\frac{17}{33}$
- (E) $\frac{16}{99}$

Questions 13–15

13. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 15z^2 = x + 2y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.

- (A) -30
- (B) -120
- (C) 60
- (D) -60
- (E) -2

14. What is the remainder when $x^2 + 1$ is divided by $x - 72$?

- (A) 5183
- (B) 5187
- (C) 5184
- (D) 5185
- (E) 5186

15. Find the arc length of $r(t) = (15t, 2t)$ for $0 \leq t \leq 1$.

- (A) $-\sqrt{229}$
- (B) $2\sqrt{229}$
- (C) $\sqrt{229}$
- (D) $1 + \sqrt{229}$
- (E) $\frac{\sqrt{229}}{2}$

Questions 16–18

16. $X_1, \dots, X_{32}$ are independent with common mean and variance 34. With $\bar{X}$ their sample mean, find $E[\sum_i (X_i - \bar{X})^2]$.

- (A) 703
- (B) 1405
- (C) 1054
- (D) 1756
- (E) 352

17. The region $0 \leq x \leq 1$, $0 \leq y \leq 15x^2$ is revolved about the x -axis. Find its volume.

- (A) 90π
- (B) $\frac{45\pi}{2}$
- (C) 45π
- (D) $1 + 45\pi$
- (E) -45π

18. Let $P = \text{diag}(\underbrace{1, \dots, 1}_1, \underbrace{0, \dots, 0}_{77})$. What is the dimension of the space of 78×78 real matrices X satisfying $PX = X$?

- (A) 80
- (B) 77
- (C) 76
- (D) 78
- (E) 79

Questions 19–21

19. If $y' + 15y = 2e^{-15x}$ and $y(0) = 6$, find $y(1)$.

- (A) $\frac{16}{e^{15}}$
- (B) $\frac{8}{e^{15}}$
- (C) $-\frac{8}{e^{15}}$
- (D) $\frac{8}{e^{15}} + 1$
- (E) $\frac{4}{e^{15}}$

20. A permutation of 34 objects is chosen uniformly. Let Z be the number of fixed points. Find $E[Z]$.

- (A) 3
- (B) 2
- (C) 4
- (D) 0
- (E) 1

21. What is the maximum of x^2y^{15} over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\frac{30}{289}$
- (B) $\left(\frac{2}{17}\right)^{15} \left(\frac{15}{17}\right)^2$
- (C) $17 \left(\left(\frac{2}{17}\right)^2 \left(\frac{15}{17}\right)^{15} \right)$
- (D) $\frac{1}{2^{17}}$
- (E) $\left(\frac{2}{17}\right)^2 \left(\frac{15}{17}\right)^{15}$

Questions 22–24

22. A 3×3 matrix A has characteristic polynomial $(t - 71)(t + 1)(t - 2)$. What is $\text{tr}(A^2)$?

- (A) 5048
- (B) 5046
- (C) 5047
- (D) 5045
- (E) 5044

23. Evaluate $\int_0^{\log 2} e^x (e^x + 15)^2 dx$.

- (A) $\frac{17^3 - 16^3}{3}$
- (B) $17^2 - 16^2$
- (C) $-\frac{17^3 - 16^3}{3}$
- (D) $\frac{17^3}{3}$
- (E) $3 \left(\frac{17^3 - 16^3}{3} \right)$

24. Which subset of $\mathbb{R}$ is compact, with 27 fixed as displayed?

- (A) $27\mathbb{Z}$
- (B) $\{0\} \cup \{27/k : k \geq 1\}$
- (C) $\{27/k : k \geq 1\}$
- (D) $\mathbb{Q} \cap [0, 27]$
- (E) $(0, 27)$

Questions 25–27

25. Let $\mathbf{F} = (15y^2, z^2, x^2)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (0, 0, 1)$.

- (A) -29
- (B) -60
- (C) 30
- (D) -15
- (E) -30

26. Let N be the 77×77 matrix with ones immediately above the diagonal and zeros elsewhere. Find $\text{rank } N^{12}$.

- (A) 65
- (B) 64
- (C) 66
- (D) 67
- (E) 63

27. If $y' = 15y$, $y(0) = 1$, what is the coefficient of x^2 in the Maclaurin series of y ?

- (A) 225
- (B) $\frac{225}{4}$
- (C) $\frac{227}{2}$
- (D) $\frac{225}{2}$
- (E) $-\frac{225}{2}$

Questions 28–30

28. For $a_k = (-1)^k + 27/k$, find $\limsup_{k \rightarrow \infty} a_k - \liminf_{k \rightarrow \infty} a_k$.

- (A) 0
- (B) 1
- (C) 3
- (D) 4
- (E) 2

29. Let C be the unit circle $x^2 + y^2 = 1$, $z = 2$, counterclockwise viewed from above. Evaluate $\int_C -15y(x^2 + y^2) dx + (15x(x^2 + y^2)) dy$.

- (A) 15π
- (B) 60π
- (C) $1 + 30\pi$
- (D) 30π
- (E) -30π

30. Let A be the 77×77 matrix whose diagonal entries are 74 and whose off-diagonal entries are 1. What is its smallest eigenvalue?

- (A) 72
- (B) 73
- (C) 71
- (D) 74
- (E) 75

Questions 31–33

31. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(15+2j)(17+2j)}$.

- (A) $\frac{1}{15}$
- (B) $\frac{1}{17}$
- (C) $\frac{1}{2}$
- (D) $\frac{2}{15}$
- (E) $\frac{1}{30}$

32. For which real a is $f(x) = x^a$ in $L^{27}(0, 1)$?

- (A) All real a
- (B) $a \geq -1/27$
- (C) $a < 1/27$
- (D) $a > -1/27$
- (E) $a > 0$

33. The solution of $y' = 15x^1y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\frac{\sqrt{30}}{15}$
- (B) $\frac{2}{15}$
- (C) $\frac{2\sqrt{30}}{15}$
- (D) $\frac{\sqrt{30}}{2}$
- (E) $\frac{\sqrt{15}}{15}$

Questions 34–36

34. Positive real numbers x, y satisfy $\log_2 x + \log_2 y = \log_2 568$ and $x + y = 79$. Find xy .

- (A) 569
- (B) 566
- (C) 570
- (D) 568
- (E) 567

35. Evaluate $\iiint_B 15(x^2 + y^2 + z^2)^1 dV$, where B is the unit ball.

- (A) -12π
- (B) 24π
- (C) 6π
- (D) $1 + 12\pi$
- (E) 12π

36. A map $T : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $|T(x) - T(y)| \leq |x - y|/27$. If $T(x_*) = x_*$ and $|x_0 - x_*| \leq 1$, what is the smallest guaranteed upper bound for $|T^3(x_0) - x_*|$?

- (A) $\frac{19682}{19683}$
- (B) $\frac{1}{39366}$
- (C) $\frac{1}{19683}$
- (D) $\frac{1}{59049}$
- (E) $\frac{9842}{19683}$

Questions 37–39

37. For the curve $x = t^2$, $y = t^3 + 15t$, find d^2y/dx^2 at $t = 1$.

- (A) $-\frac{15}{2}$
- (B) -3
- (C) 9
- (D) $\frac{9}{2}$
- (E) -6

38. A sequence satisfies $u_0 = 71$ and $u_{k+1} = 1u_k + 8$ for all $k \geq 0$. Find u_4 .

- (A) 104
- (B) 105
- (C) 103
- (D) 102
- (E) 101

39. If $y'' + 4y = 0$, $y(0) = 15$, and $y'(0) = 6$, find $y(\pi/4)$.

- (A) $\frac{3}{2}$
- (B) -3
- (C) 3
- (D) 6
- (E) 4

Questions 40–42

40. How many zeros, counted with multiplicity, does $z^{22} + 25z + 1$ have in $|z| < 1$?

- (A) 4
- (B) 0
- (C) 1
- (D) 3
- (E) 2

41. Under $x = u + 15v$, $y = u - 2v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) 30
- (B) 13
- (C) $\frac{1}{17}$
- (D) 289
- (E) 17

42. A linear map $T : \mathbb{R}^{78} \rightarrow \mathbb{R}^{80}$ has rank 4. What is its nullity?

- (A) 75
- (B) 74
- (C) 73
- (D) 72
- (E) 76

Questions 43–45

43. Evaluate $\lim_{x \rightarrow 0} (1 + 15x)^{1/(1x)}$.

- (A) $e^{\frac{1}{15}}$
- (B) e^{16}
- (C) e^{225}
- (D) e^{15}
- (E) 15

44. A meromorphic function has 24 zeros and 2 poles inside a positively oriented simple closed contour, counted with multiplicity, and no zeros or poles on it. Find $\frac{1}{2\pi i} \int f'(z)/f(z) dz$.

- (A) 8
- (B) 22
- (C) 15
- (D) 29
- (E) 36

45. Evaluate $\int_0^1 \int_0^x x^{15} y^2 dy dx$.

- (A) $\frac{1}{19}$
- (B) $\frac{1}{57}$
- (C) $\frac{1}{48}$
- (D) $\frac{1}{54}$
- (E) $\frac{1}{76}$

Questions 46–48

46. What is the dimension of the real vector space of 77×77 symmetric matrices?

- (A) 3004
- (B) 3001
- (C) 3002
- (D) 3005
- (E) 3003

47. Solve $y' + 15y = x^2e^{-15x}$, $y(0) = 0$, and find $y(1)$.

- (A) $\frac{5}{e^{15}}$
- (B) $\frac{1}{2e^{15}}$
- (C) e^{-15}
- (D) $\frac{1}{3e^{15}}$
- (E) $\frac{e^{15}}{3}$

48. Let $F_0 = 0$, $F_1 = 1$, and $F_{k+2} = F_{k+1} + F_k$. How many binary strings of length 32 have no adjacent 1s?

- (A) F_{34}
- (B) F_{33}
- (C) F_{32}
- (D) F_{35}
- (E) F_{36}

Questions 49–51

49. Find the coefficient of x^2 in the Taylor series of $(1 - 15x)^{-2}$ at zero.

- (A) $\frac{1}{15} (3 \cdot 15^2)$
- (B) $3 \cdot 15^2$
- (C) $\frac{1}{3} (3 \cdot 15^2)$
- (D) $\frac{2}{3} (3 \cdot 15^2)$
- (E) $2 (3 \cdot 15^2)$

50. How many group homomorphisms $\mathbb{Z}/456\mathbb{Z} \rightarrow \mathbb{Z}/308\mathbb{Z}$ are there?

- (A) 5
- (B) 3
- (C) 4
- (D) 2
- (E) 6

51. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 15x$ and has density $\rho(x, y) = 1$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{2}{3}$
- (B) $\frac{4}{3}$
- (C) $-\frac{2}{3}$
- (D) $\frac{5}{3}$
- (E) $\frac{1}{3}$

Questions 52–54

52. Apply one Newton iteration to solve $x^2 - 32 = 0$, starting at $x_0 = 1$. What is x_1 ?

- (A) $\frac{33}{2}$
- (B) $\frac{33}{4}$
- (C) $\frac{35}{2}$
- (D) $\frac{37}{4}$
- (E) $\frac{37}{2}$

53. Evaluate $\int_0^1 \int_x^1 15y^2 \, dy \, dx$.

- (A) 5
- (B) $\frac{15}{4}$
- (C) $\frac{15}{2}$
- (D) $\frac{5}{4}$
- (E) $\frac{15}{8}$

54. How many 1-dimensional subspaces does $\mathbb{F}_{43}^3$ have?

- (A) $\frac{43^3 - 1}{43 - 1}$
- (B) $\frac{43^4 - 1}{43 - 1}$
- (C) $\frac{43^5 - 1}{43 - 1}$
- (D) $\frac{43^6 - 1}{43 - 1}$
- (E) $\frac{43^2 - 1}{43 - 1}$

Questions 55–57

55. Find the curvature at $t = 0$ of $r(t) = (2t, 15t^2)$.

- (A) $\frac{15}{2}$
- (B) $\frac{15}{4}$
- (C) $\frac{17}{2}$
- (D) 15
- (E) $-\frac{15}{2}$

56. Bisection starts with an interval of length 2^{32} . After 34 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{1}{8}$
- (B) $\frac{9}{16}$
- (C) $\frac{7}{8}$
- (D) $\frac{1}{24}$
- (E) $\frac{1}{16}$

57. Evaluate $\int_0^1 x^{15}(1-x)^1 dx$.

- (A) $\frac{1}{2} \left(\frac{15! 1!}{17!} \right)$
- (B) $17 \left(\frac{15! 1!}{17!} \right)$
- (C) $\frac{15! 1!}{17!}$
- (D) $\frac{17}{2} \left(\frac{15! 1!}{17!} \right)$
- (E) $16 \left(\frac{15! 1!}{17!} \right)$

Questions 58–60

58. How many monic irreducible polynomials of degree 2 are there over $\mathbb{F}_{43}$?

- (A) 902
- (B) 905
- (C) 904
- (D) 901
- (E) 903

59. Let $f(x) = \log(1 + 15x)$. Find $f^{(2)}(0)$.

- (A) $\frac{1}{15} \left((-1)^1 1! 15^2 \right)$
- (B) $(-1)^1 1! 15^2$
- (C) $2 \left((-1)^1 1! 15^2 \right)$
- (D) $\frac{1}{2} \left((-1)^1 1! 15^2 \right)$
- (E) $-(-1)^1 1! 15^2$

60. What is the least positive multiplicative inverse of 2 modulo 65?

- (A) 33
- (B) 11
- (C) 44
- (D) 55
- (E) 22

Questions 61–63

61. Find the upward flux of $\mathbf{F} = (0, 0, z)$ through $z = 15x^2 + 2y^2$, $0 \leq x, y \leq 1$.

- (A) $\frac{17}{3}$
- (B) $\frac{34}{3}$
- (C) $\frac{20}{3}$
- (D) $-\frac{17}{3}$
- (E) $\frac{17}{6}$

62. How many elements z of $\mathbb{F}_{2^{15}}$ satisfy $z^2 = z$?

- (A) 4
- (B) 1
- (C) 0
- (D) 2
- (E) 3

63. Find the directional derivative of $f(x, y, z) = x^2 + 15y^2 + 2z^2$ at $(1, 1, 1)$ in the direction $(1, 6, 0)$.

- (A) $1 + \frac{182\sqrt{37}}{37}$
- (B) $-\frac{182\sqrt{37}}{37}$
- (C) $\frac{182\sqrt{37}}{37}$
- (D) $\frac{91\sqrt{37}}{37}$
- (E) $\frac{364\sqrt{37}}{37}$

Questions 64–66

64. How many truth assignments to $p, q, r_1, \dots, r_{30}$ make $p \Rightarrow q$ false?

- (A) 2^{33}
- (B) 2^{30}
- (C) 2^{29}
- (D) 2^{31}
- (E) 2^{32}

65. Evaluate $\int_0^{\pi/2} \sin^5 t \cos^{31} t \, dt$.

- (A) $2 \left(\frac{2! 15!}{2 \cdot 18!} \right)$
- (B) $136 \left(\frac{2! 15!}{2 \cdot 18!} \right)$
- (C) $18 \left(\frac{2! 15!}{2 \cdot 18!} \right)$
- (D) $\frac{1}{2} \left(\frac{2! 15!}{2 \cdot 18!} \right)$
- (E) $\frac{2! 15!}{2 \cdot 18!}$

66. How many generators does the cyclic group of order 43 have?

- (A) 41
- (B) 43
- (C) 42
- (D) 44
- (E) 40

Answer Key

Practice Test 75 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	D	23	A	45	B
2	D	24	B	46	E
3	B	25	E	47	D
4	B	26	A	48	A
5	A	27	D	49	B
6	E	28	E	50	C
7	E	29	D	51	A
8	A	30	B	52	A
9	E	31	E	53	B
10	D	32	D	54	A
11	C	33	A	55	A
12	C	34	D	56	A
13	D	35	E	57	C
14	D	36	C	58	E
15	C	37	B	59	B
16	C	38	C	60	A
17	C	39	C	61	A
18	D	40	C	62	D
19	B	41	E	63	C
20	E	42	B	64	B
21	E	43	D	65	E
22	B	44	B	66	C

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