

GRE Mathematics Practice Test 77

Advanced Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A demanding practice form with frequent multi-step arguments and a mix of computational and conceptual questions. This is cycle 15 of 19, position 2 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Evaluate $\int_0^1 \frac{x^4}{1 + 16x^5} dx$.

- (A) $\frac{\log(17)}{16}$
- (B) $\frac{\log(17)}{80}$
- (C) $\frac{\log(17)}{5}$
- (D) $\log(17)$
- (E) $-\frac{\log(17)}{80}$

2. An idempotent 79×79 real matrix P has rank 2. What is $\det(I + P)$?

- (A) 4
- (B) 2
- (C) 6
- (D) 5
- (E) 3

3. If $y'' - 32y' + 256y = 5e^{16x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) $\frac{5e^{16}}{2}$
- (B) $5e^{16}$
- (C) $1 + \frac{5e^{16}}{2}$
- (D) $\frac{5e^{16}}{4}$
- (E) $-\frac{5e^{16}}{2}$

Questions 4–6

4. A coin success probability p has uniform prior on $[0, 1]$. After observing 29 successes and 2 failures in independent tosses conditional on p , what is the predictive probability of success on the next toss?

- (A) $\frac{10}{11}$
- (B) $\frac{1}{11}$
- (C) $\frac{10}{33}$
- (D) $\frac{5}{11}$
- (E) $\frac{21}{22}$

5. For $f(x, y) = x^2 + 16xy + 5y^2$, what is its maximum directional derivative at $(1, -1)$?

- (A) 6
- (B) $2\sqrt{58}$
- (C) -14
- (D) 20
- (E) 232

6. For $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} 73 \\ 10 \\ 83 \end{bmatrix}$, let x minimize $\|Ax - b\|^2$. Find the minimum squared residual.

- (A) $\frac{9251}{3}$
- (B) $\frac{9248}{3}$
- (C) $\frac{18493}{6}$
- (D) $\frac{18499}{6}$
- (E) $\frac{9245}{3}$

Questions 7–9

7. Evaluate $\int_0^1 \int_0^x x^{16} y^5 dy dx$.

- (A) $\frac{1}{102}$
- (B) $\frac{1}{132}$
- (C) $\frac{1}{161}$
- (D) $\frac{1}{23}$
- (E) $\frac{1}{138}$

8. A random variable has mean 0 and variance 29. What upper bound does the Chebyshev inequality give for $P(|X| \geq 58)$?

- (A) $\frac{1}{232}$
- (B) $\frac{1}{115}$
- (C) $\frac{1}{116}$
- (D) $\frac{348}{117}$
- (E) $\frac{1}{232}$

9. For the curve $x = t^5$, $y = t^6 + 16t$, find d^2y/dx^2 at $t = 1$.

- (A) $-\frac{58}{5}$
- (B) $\frac{14}{5}$
- (C) $-\frac{16}{5}$
- (D) $-\frac{58}{25}$
- (E) $\frac{22}{5}$

Questions 10–12

10. What is the coefficient of x^7 in $(x + 15)^7(1 - x)^3$?

- (A) -104265
- (B) -104266
- (C) -104264
- (D) -104263
- (E) -104262

11. If $y' = y(1 - y/5)$ and $y(0) = 5/2$, find $y(-\log 16)$.

- (A) $\frac{5}{34}$
- (B) $\frac{10}{17}$
- (C) $\frac{5}{17}$
- (D) $\frac{22}{17}$
- (E) $-\frac{5}{17}$

12. A box is chosen uniformly from two boxes. Box A contains 34 red and 36 blue balls; box B contains 36 red and 34 blue balls. Two balls are drawn without replacement and both are red. What is the conditional probability that A was chosen?

- (A) $\frac{187}{794}$
- (B) $\frac{210}{397}$
- (C) $\frac{187}{397}$
- (D) $\frac{187}{1191}$
- (E) $\frac{292}{397}$

Questions 13–15

13. Evaluate $\iiint_B 16z^2(x^2 + y^2 + z^2)^5 dV$, where B is the unit ball.

- (A) $-\frac{64\pi}{45}$
- (B) $\frac{128\pi}{45}$
- (C) $1 + \frac{64\pi}{45}$
- (D) $\frac{64\pi}{45}$
- (E) $\frac{32\pi}{45}$

14. A matrix A satisfies $(A - 74I)(A - 76I) = 0$, and its minimal polynomial has degree two. If $A^5 = uA + vI$, find u .

- (A) $2^4 \cdot 5 \cdot 11 \cdot 61 \cdot 14741$
- (B) $2^3 \cdot 11 \cdot 61 \cdot 14741$
- (C) $2^5 \cdot 11 \cdot 61 \cdot 14741$
- (D) $2^4 \cdot 11 \cdot 61 \cdot 14741$
- (E) $2^4 \cdot 3 \cdot 11 \cdot 61 \cdot 14741$

15. Find the directional derivative of $f(x, y, z) = x^2y + 5xz^2 + 16y^2z$ at $(1, 1, 1)$ in the direction $(1, 3, 0)$.

- (A) $-\frac{53\sqrt{10}}{5}$
- (B) $\frac{106\sqrt{10}}{5}$
- (C) $\frac{53\sqrt{10}}{10}$
- (D) $\frac{53\sqrt{10}}{5}$
- (E) $1 + \frac{53\sqrt{10}}{5}$

Questions 16–18

16. Let $X_1, \dots, X_{34}$ be independent uniform random variables on $[0, 1]$. What is $E[(\max_i X_i)^2]$?

- (A) $\frac{35}{36}$
- (B) $\frac{1}{18}$
- (C) $\frac{17}{18}$
- (D) $\frac{17}{54}$
- (E) $\frac{17}{36}$

17. Evaluate $\int_0^{\pi/2} \sin^{11} t \cos^{33} t \, dt$.

- (A) $20349 \left(\frac{5! 16!}{2 \cdot 22!} \right)$
- (B) $2 \left(\frac{5! 16!}{2 \cdot 22!} \right)$
- (C) $\frac{5! 16!}{2 \cdot 22!}$
- (D) $\frac{1}{2} \left(\frac{5! 16!}{2 \cdot 22!} \right)$
- (E) $22 \left(\frac{5! 16!}{2 \cdot 22!} \right)$

18. Let $V = \begin{bmatrix} 1 & 73 & 5329 \\ 1 & 75 & 5625 \\ 1 & 78 & 6084 \end{bmatrix}$. Find $\det(V^T V)$.

- (A) 900
- (B) 898
- (C) 901
- (D) 902
- (E) 899

Questions 19–21

19. Evaluate $\lim_{x \rightarrow 0} \frac{(1 + 16x)^{1/(4x)} - e^{16/4}}{x}$.

- (A) $-2e^4$
- (B) $-32e^4$
- (C) $-64e^4$
- (D) $32e^4$
- (E) e^4

20. Independent trials have success probability $1/34$. Let T count trials up to and including the first success. Find $E[T^2]$.

- (A) 3796
- (B) 2278
- (C) 3037
- (D) 760
- (E) 1519

21. The solution of $y' = 16x^5y^3$ has $y(0) = 1/17$. Find $y(1)$.

- (A) $-\frac{\sqrt{2553}}{851}$
- (B) $\frac{\sqrt{2553}}{851} + 1$
- (C) $\frac{\sqrt{2553}}{1702}$
- (D) $\frac{2\sqrt{2553}}{851}$
- (E) $\frac{\sqrt{2553}}{851}$

Questions 22–24

22. Let $u = (1, 2, \dots, 79)^T$, $v = (1, 1, \dots, 1)^T$, and $B = I + uv^T$. Find $v^T B^{-1}u$.

- (A) $\frac{3160}{3161}$
- (B) $\frac{3161}{6321}$
- (C) $\frac{3161}{15803}$
- (D) $\frac{6322}{9481}$
- (E) $\frac{3159}{6322}$

23. Find the arc length of $r(t) = (16(t - t^3/3), 16t^2)$ for $0 \leq t \leq 1/5$.

- (A) $\frac{608}{375}$
- (B) $\frac{2432}{375}$
- (C) $\frac{1216}{375}$
- (D) $-\frac{1216}{375}$
- (E) $\frac{1591}{375}$

24. A continuous $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(q) = q^{24}$ for every rational q . Find $f(2)$.

- (A) 2^{23}
- (B) 2^{25}
- (C) 2^{26}
- (D) 2^{27}
- (E) 2^{24}

Questions 25–27

25. Evaluate $\lim_{x \rightarrow 0} \frac{e^{16x} - \sum_{j=0}^4 (16x)^j / j!}{x^5}$.

- (A) $120 \left(\frac{16^5}{5!} \right)$
- (B) $\frac{16^5}{5!}$
- (C) $-\frac{16^5}{5!}$
- (D) $\frac{1}{16} \left(\frac{16^5}{5!} \right)$
- (E) $5 \left(\frac{16^5}{5!} \right)$

26. Let $D = \text{diag}(\underbrace{1, \dots, 1}_{14}, \underbrace{2, \dots, 2}_{65})$. Find the dimension of the space of real matrices X with $XD = DX$.

- (A) 4423
- (B) 4422
- (C) 4420
- (D) 4421
- (E) 4419

27. Evaluate $\int_C (5x^4 + 16y) dx + (16x + 5y^4) dy$, where C is $r(t) = (t, 3t^2)$, $0 \leq t \leq 1$.

- (A) 292
- (B) 146
- (C) 584
- (D) -292
- (E) 293

Questions 28–30

28. Find the radius of convergence of $\sum_{k=1}^{\infty} k^{24}x^k/24^k$.

- (A) 16
- (B) 32
- (C) 24
- (D) 8
- (E) 40

29. Solve $y' + 16y = x^5e^{-16x}$, $y(0) = 0$, and find $y(1)$.

- (A) $\frac{1}{5e^{16}}$
- (B) e^{-16}
- (C) $\frac{e^{16}}{6}$
- (D) $\frac{1}{6e^{16}}$
- (E) $\frac{8}{3e^{16}}$

30. When $x^3 + 1$ is divided by $(x - 74)(x - 75)$, the remainder is $ux + v$. Find u .

- (A) 16650
- (B) 16651
- (C) 16652
- (D) 16649
- (E) 16653

Questions 31–33

31. Let $F(x) = \int_0^x (x-t)^4 e^{16t} dt$. Find $F^{(5)}(0)$.

- (A) 384
- (B) 24
- (C) 120
- (D) 0
- (E) $\frac{1}{24}$

32. On $[0, 1]$, f is 24 on rationals and $1/24$ on irrationals. Find its upper Darboux integral minus its lower Darboux integral.

- (A) $\frac{599}{24}$
- (B) $\frac{575}{48}$
- (C) $\frac{24}{575}$
- (D) $\frac{12}{575}$
- (E) $\frac{623}{24}$

33. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 16z^2 = x + 5y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.

- (A) 160
- (B) -80
- (C) -5
- (D) -160
- (E) -32

Questions 34–36

34. Let $P = \text{diag}(\underbrace{1, \dots, 1}_4, \underbrace{0, \dots, 0}_{76})$. What is the dimension of the space of 80×80 real matrices X satisfying $PX = X$ and $XP = 0$?

- (A) 303
- (B) 302
- (C) 304
- (D) 306
- (E) 305

35. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-16)^j}{j^{\frac{4}{3}} (\log j)^m}$ converge absolutely at $x = 15$? Give the least such m , or 0 if none exists.

- (A) 0
- (B) 3
- (C) 4
- (D) 1
- (E) 2

36. A set with 24 elements has the discrete topology. How many of its subsets are simultaneously open and closed?

- (A) 2^{25}
- (B) 2^{24}
- (C) 2^{23}
- (D) 2^{26}
- (E) 2^{27}

Questions 37–39

37. Let C be $x^2/25 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C \left(-\frac{16y^3}{3}\right) dx + \left(\frac{x^3}{3}\right) dy$.
- (A) $\frac{205\pi}{2}$
(B) $-\frac{205\pi}{4}$
(C) $\frac{205\pi}{8}$
(D) $1 + \frac{205\pi}{4}$
(E) $\frac{205\pi}{4}$
38. A 3×3 matrix A has characteristic polynomial $(t - 73)(t + 1)(t - 2)$. What is $\text{tr}(A^3 - 2A + I)$?
- (A) 388879
(B) 388877
(C) 388880
(D) 388881
(E) 388878
39. For $x > 0$, $x^2y'' + (1 - 21)xy' + 80y = 0$, $y(1) = 0$, $y'(1) = 16$. Find $y(e)$.
- (A) $\frac{e^{16}}{21}$
(B) $e^5 + e^{16}$
(C) $\frac{16(1 - e^{11})e^5}{11}$
(D) $-e^5 + e^{16}$
(E) $-\frac{16(1 - e^{11})e^5}{11}$

Questions 40–42

40. Let $\zeta = e^{2\pi i/21}$. Find $\sum_{k=0}^{20} \zeta^k$.

- (A) 2
- (B) 3
- (C) 4
- (D) 0
- (E) 1

41. If $y' = 32xy$ and $y(0) = 1$, find the coefficient of x^{10} in the Maclaurin series of y .

- (A) $\frac{262144}{15}$
- (B) $\frac{65536}{15}$
- (C) $\frac{131087}{15}$
- (D) $-\frac{131072}{15}$
- (E) $\frac{131072}{15}$

42. How many monic irreducible polynomials of degree 4 are there over $\mathbb{F}_{47}$?

- (A) $\frac{2(47^4 - 47^2)}{4}$
- (B) $\frac{3(47^4 - 47^2)}{4}$
- (C) $47^4 - 47^2$
- (D) $2(47^4 - 47^2)$
- (E) $\frac{47^4 - 47^2}{4}$

Questions 43–45

43. Let $f(x) = \log(1 + 16x)$. Find $f^{(5)}(0)$.

- (A) $-(-1)^4 4! 16^5$
- (B) $\frac{1}{16} \left((-1)^4 4! 16^5 \right)$
- (C) $(-1)^4 4! 16^5$
- (D) $\frac{1}{5} \left((-1)^4 4! 16^5 \right)$
- (E) $5 \left((-1)^4 4! 16^5 \right)$

44. An analytic function has real part $u(x, y) = x^2 - y^2 + 19x$ and satisfies $f(0) = 0$. Find $\text{Im } f(1 + i)$.

- (A) 7
- (B) 21
- (C) 35
- (D) 14
- (E) 28

45. For $x > 0$, $x^2 y'' + (-36)xy' + 336y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

- (A) $-\frac{2(1 - e^5)e^{16}}{5}$
- (B) $-\frac{(1 - e^5)e^{16}}{10}$
- (C) $\frac{(1 - e^5)e^{16}}{5}$
- (D) $-\frac{e^{16}}{5} + 1 + \frac{e^{21}}{5}$
- (E) $-\frac{(1 - e^5)e^{16}}{5}$

Questions 46–48

46. How many elements z of $\mathbb{F}_{7^{16}}$ satisfy $z^{49} = z$?

- (A) 50
- (B) 51
- (C) 49
- (D) 47
- (E) 48

47. Two solutions of $y'' + 16x^5y' + 5y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $\frac{1}{2e^{\frac{8}{3}}}$
- (B) $e^{-\frac{8}{3}}$
- (C) $e^{-\frac{8}{3}} + 1$
- (D) $\frac{2}{e^{\frac{8}{3}}}$
- (E) $-\frac{1}{e^{\frac{8}{3}}}$

48. Find the coefficient of x^3 in $(1+x)^{29}(1-x)^{29}(1+2x)^2$.

- (A) -192
- (B) -154
- (C) -78
- (D) -40
- (E) -116

Questions 49–51

49. For $\mathbf{F} = (16x^5y, y^5z, xz^5)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) $\frac{245}{2}$
- (B) 245
- (C) 490
- (D) -245
- (E) 246

50. How many generators does the cyclic group of order 229345007 have?

- (A) $47^3(47 - 1)$
- (B) $47^6(47 - 1)$
- (C) $47^4(47 - 1)$
- (D) $47^5(47 - 1)$
- (E) $47^7(47 - 1)$

51. For $D = \{(x, y) : x^2 + y^2 \leq 16\}$, evaluate $\iint_D (x^2 + y^2)^4 dA$.

- (A) $\frac{65536\pi}{5}$
- (B) 1048576π
- (C) $\frac{1048576\pi}{9}$
- (D) $\frac{1048576\pi}{5}$
- (E) $\frac{2097152\pi}{5}$

Questions 52–54

52. How many edges does the complete bipartite graph $K_{29,31}$ have?

- (A) 1198
- (B) 1497
- (C) 301
- (D) 600
- (E) 899

53. Let $f(x, y) = x^2 + 32xy + (261)y^2$. What is the product of the eigenvalues of the Hessian of f ?

- (A) 20
- (B) 5
- (C) 1044
- (D) 10
- (E) 261

54. Let $R = (\mathbb{Z}/36\mathbb{Z})/([18])$, where $([18])$ is the ideal generated by $[18]$. How many units does R have?

- (A) 8
- (B) 6
- (C) 7
- (D) 5
- (E) 4

Questions 55–57

55. Evaluate $\int_0^1 x^{15}(\log x)^4 dx$.

- (A) $-\frac{(-1)^4 4!}{16^5}$
- (B) $\frac{(-1)^4 4!}{16^5}$
- (C) $\frac{1}{5} \left(\frac{(-1)^4 4!}{16^5} \right)$
- (D) $\frac{8192}{3} \left(\frac{(-1)^4 4!}{16^5} \right)$
- (E) $16 \left(\frac{(-1)^4 4!}{16^5} \right)$

56. How many functions from a set of 34 elements onto a set of 3 elements exist?

- (A) 3^{34}
- (B) $3^{34} - 3(2^{34})$
- (C) $3^{34} - 2^{34}$
- (D) $3^{34} - 3(2^{34}) + 3$
- (E) $3^{34} - 3(2^{34}) - 3$

57. Under $x = u + 16v$, $y = u - 5v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) 21
- (B) 80
- (C) $\frac{1}{21}$
- (D) 441
- (E) 11

Questions 58–60

58. What is the largest integer k such that 3^k divides $82!$?

- (A) 38
- (B) 39
- (C) 40
- (D) 41
- (E) 42

59. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{16t} \cos(5t)$, and $y = e^{16t} \sin(5t)$, find d^2f/dt^2 at $t = 1$.

- (A) 0
- (B) 10
- (C) 562
- (D) 1024
- (E) 32

60. How many nonnegative integer triples sum to 68, if each coordinate is at most 34?

- (A) 1050
- (B) 840
- (C) 420
- (D) 630
- (E) 210

Questions 61–63

61. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 16x$ and has density $\rho(x, y) = x^3y$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{6}{7}$
- (B) $\frac{3}{7}$
- (C) $\frac{12}{7}$
- (D) $\frac{13}{7}$
- (E) $-\frac{6}{7}$

62. In the ring $\mathbb{Z}/462\mathbb{Z}$, how many elements satisfy $x^2 = 1$?

- (A) 8
- (B) 7
- (C) 6
- (D) 10
- (E) 9

63. The solution of $y' = 16x^4y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\frac{10^{\frac{4}{5}}}{5}$
- (B) $\frac{\sqrt[5]{2}}{2}$
- (C) $\frac{\sqrt[5]{5}}{2}$
- (D) $\frac{5}{16}$
- (E) $\frac{\sqrt[5]{10}}{2}$

Questions 64–66

64. A tree has 34 vertices of degree 3; every other vertex has degree 1. How many leaves does it have?

- (A) 24
- (B) 60
- (C) 12
- (D) 48
- (E) 36

65. Find the coefficient of x^4 in $e^{4x} \log(1 + 16x)$.

- (A) -16384
- (B) -11760
- (C) -282624
- (D) -11776
- (E) 11776

66. A permutation consists of disjoint cycles of lengths 2 and 9, together with a disjoint transposition when the sum of these two lengths is odd. It fixes all other points. What is its order?

- (A) 18
- (B) 19
- (C) 17
- (D) 16
- (E) 20

Answer Key

Practice Test 77 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	B	23	C	45	E
2	A	24	E	46	C
3	A	25	B	47	B
4	A	26	D	48	E
5	B	27	A	49	B
6	B	28	C	50	C
7	E	29	D	51	D
8	E	30	B	52	E
9	D	31	B	53	A
10	C	32	C	54	B
11	C	33	D	55	B
12	C	34	C	56	D
13	D	35	D	57	A
14	D	36	B	58	C
15	D	37	E	59	A
16	C	38	A	60	D
17	C	39	E	61	A
18	A	40	D	62	A
19	B	41	E	63	E
20	B	42	E	64	E
21	E	43	C	65	D
22	A	44	B	66	A

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