

GRE Mathematics Practice Test 78

Upper-Intermediate Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

Intermediate reinforcement, combining standard applications with selected more demanding arguments. This is cycle 15 of 19, position 3 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. If $y' = 32xy$ and $y(0) = 1$, find the coefficient of x^8 in the Maclaurin series of y .

- (A) $-\frac{8192}{3}$
- (B) $\frac{8192}{3}$
- (C) $\frac{16384}{3}$
- (D) $\frac{4096}{3}$
- (E) $\frac{8195}{3}$

2. Let $u = (1, 2, \dots, 80)^T$, $v = (1, 1, \dots, 1)^T$, and $B = I + uv^T$. Find $\det B$.

- (A) 3241
- (B) 3242
- (C) 3239
- (D) 3240
- (E) 3243

3. If $y'' + 16y = 16$ and $y(0) = y'(0) = 0$, find $y(\pi/4)$.

- (A) 3
- (B) 2
- (C) 4
- (D) -2
- (E) 1

Questions 4–6

4. A coin success probability p has uniform prior on $[0, 1]$. After observing 30 successes and 2 failures in independent tosses conditional on p , what is the predictive probability of success on the next toss?

- (A) $\frac{65}{68}$
- (B) $\frac{31}{102}$
- (C) $\frac{3}{34}$
- (D) $\frac{31}{34}$
- (E) $\frac{31}{68}$

5. Evaluate $\iiint_B 16z^2(x^2 + y^2 + z^2)^4 dV$, where B is the unit ball.

- (A) $\frac{32\pi}{39}$
- (B) $-\frac{64\pi}{39}$
- (C) $1 + \frac{64\pi}{39}$
- (D) $\frac{64\pi}{39}$
- (E) $\frac{128\pi}{39}$

6. Let $D = \text{diag}(\underbrace{1, \dots, 1}_{14}, \underbrace{2, \dots, 2}_{66})$. Find the dimension of the space of real matrices X with $XD = DX$.

- (A) 4551
- (B) 4550
- (C) 4554
- (D) 4553
- (E) 4552

Questions 7–9

7. Let C be $x^2/16 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C \left(-\frac{16y^3}{3}\right) dx + \left(\frac{x^3}{3}\right) dy$.
- (A) -32π
(B) 16π
(C) 32π
(D) $1 + 32\pi$
(E) 64π
8. A random variable has mean 0 and variance 30. What upper bound does the Chebyshev inequality give for $P(|X| \geq 60)$?
- (A) $\frac{121}{240}$
(B) $\frac{1}{360}$
(C) $\frac{1}{120}$
(D) $\frac{1}{240}$
(E) $\frac{119}{120}$
9. For $f(x, y) = x^2 + 16xy + 4y^2$, what is its maximum directional derivative at $(1, -1)$?
- (A) 260
(B) -14
(C) 22
(D) 8
(E) $2\sqrt{65}$

Questions 10–12

10. When $x^3 + 1$ is divided by $(x - 75)(x - 76)$, the remainder is $ux + v$. Find u .

- (A) 17100
- (B) 17101
- (C) 17103
- (D) 17102
- (E) 17099

11. Let $\mathbf{F} = (16y^4, z^4, x^4)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (1, 1, 4)/\sqrt{18}$.

- (A) $1 - 44\sqrt{2}$
- (B) $-44\sqrt{2}$
- (C) $44\sqrt{2}$
- (D) $-88\sqrt{2}$
- (E) $-22\sqrt{2}$

12. A box is chosen uniformly from two boxes. Box A contains 35 red and 37 blue balls; box B contains 37 red and 35 blue balls. Two balls are drawn without replacement and both are red. What is the conditional probability that A was chosen?

- (A) $\frac{928}{1261}$
- (B) $\frac{666}{1261}$
- (C) $\frac{595}{3783}$
- (D) $\frac{1261}{595}$
- (E) $\frac{1261}{2522}$

Questions 13–15

13. If $y'' - 32y' + 256y = 4e^{16x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) e^{16}
- (B) $-2e^{16}$
- (C) $4e^{16}$
- (D) $2e^{16}$
- (E) $1 + 2e^{16}$

14. Let $P = \text{diag}(\underbrace{1, \dots, 1}_1, \underbrace{0, \dots, 0}_{80})$. What is the dimension of the space of 81×81 real matrices X satisfying $PX = X$ and $XP = 0$?

- (A) 78
- (B) 81
- (C) 82
- (D) 80
- (E) 79

15. Find the curvature of the helix $r(t) = (16 \cos t, 16 \sin t, 4t)$.

- (A) $\frac{1}{17}$
- (B) $\frac{2}{17}$
- (C) $-\frac{1}{17}$
- (D) $\frac{1}{34}$
- (E) $\frac{18}{17}$

Questions 16–18

16. Let $X_1, \dots, X_{35}$ be independent uniform random variables on $[0, 1]$. What is $E[(\max_i X_i)^2]$?

- (A) $\frac{35}{74}$
- (B) $\frac{2}{37}$
- (C) $\frac{35}{111}$
- (D) $\frac{35}{37}$
- (E) $\frac{36}{37}$

17. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(16j)!}{(j!)^{16}} x^{4j}$?

- (A) 16^{-4}
- (B) 16^{-16}
- (C) $4096 \left(16^{-4}\right)$
- (D) 16^4
- (E) $16^{-\frac{1}{4}}$

18. A 3×3 matrix A has characteristic polynomial $(t - 74)(t + 1)(t - 2)$. What is $\text{tr}(A^3)$?

- (A) 405233
- (B) 405229
- (C) 405231
- (D) 405230
- (E) 405232

Questions 19–21

19. Solve $y' + 16y = x^4e^{-16x}$, $y(0) = 0$, and find $y(1)$.

- (A) $\frac{16}{5e^{16}}$
- (B) $\frac{e^{16}}{5}$
- (C) $\frac{1}{4e^{16}}$
- (D) e^{-16}
- (E) $\frac{1}{5e^{16}}$

20. Independent trials have success probability $1/35$. Let T count trials up to and including the first success. Find $E[T]$.

- (A) 46
- (B) 13
- (C) 57
- (D) 24
- (E) 35

21. For $\mathbf{F} = (16x^4y, y^4z, xz^4)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) 82
- (B) -164
- (C) 328
- (D) 165
- (E) 164

Questions 22–24

22. Let N be the 80×80 matrix with ones immediately above the diagonal and zeros elsewhere. Find $\dim(\ker N^{26} / \ker N^{13})$.

- (A) 11
- (B) 12
- (C) 13
- (D) 14
- (E) 15

23. Let $F(x) = \int_0^x (x-t)^3 e^{16t} dt$. Find $F^{(4)}(0)$.

- (A) 96
- (B) 24
- (C) 6
- (D) 0
- (E) $\frac{1}{6}$

24. A continuous $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(q) = q^{25}$ for every rational q . Find $f(2)$.

- (A) 2^{26}
- (B) 2^{27}
- (C) 2^{28}
- (D) 2^{24}
- (E) 2^{25}

Questions 25–27

25. Evaluate $\int_0^1 x^{16}(1-x)^3 dx$.

- (A) $17 \left(\frac{16! 3!}{20!} \right)$
- (B) $\frac{16! 3!}{20!}$
- (C) $285 \left(\frac{16! 3!}{20!} \right)$
- (D) $20 \left(\frac{16! 3!}{20!} \right)$
- (E) $\frac{1}{4} \left(\frac{16! 3!}{20!} \right)$

26. Let A be the 80×80 matrix whose diagonal entries are 77 and whose off-diagonal entries are 1. What is its largest eigenvalue?

- (A) 158
- (B) 154
- (C) 155
- (D) 157
- (E) 156

27. Let $f(x) = \log(1 + 16x)$. Find $f^{(4)}(0)$.

- (A) $4 \left((-1)^3 3! 16^4 \right)$
- (B) $-(-1)^3 3! 16^4$
- (C) $\frac{1}{16} \left((-1)^3 3! 16^4 \right)$
- (D) $\frac{1}{4} \left((-1)^3 3! 16^4 \right)$
- (E) $(-1)^3 3! 16^4$

Questions 28–30

28. Find the radius of convergence of $\sum_{k=1}^{\infty} k^{25} x^k / 25^k$.

- (A) 41
- (B) 25
- (C) 17
- (D) 9
- (E) 33

29. Evaluate $\int_C (4x^3 + 16y) dx + (16x + 4y^3) dy$, where C is $r(t) = (t, 4t^2)$, $0 \leq t \leq 1$.

- (A) 322
- (B) -321
- (C) 321
- (D) $\frac{321}{2}$
- (E) 642

30. Positive real numbers x, y satisfy $\log_2 x + \log_2 y = \log_2 962$ and $x + y = 87$. Find $x^2 + y^2$.

- (A) 5645
- (B) 5647
- (C) 5643
- (D) 5646
- (E) 5644

Questions 31–33

31. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(16+4j)(20+4j)}$.

- (A) $\frac{1}{64}$
- (B) $\frac{1}{32}$
- (C) $\frac{1}{20}$
- (D) $\frac{1}{4}$
- (E) $\frac{1}{16}$

32. On $[0, 1]$, f is 25 on rationals and $1/25$ on irrationals. Find its upper Darboux integral minus its lower Darboux integral.

- (A) $\frac{674}{25}$
- (B) $\frac{1248}{25}$
- (C) $\frac{624}{25}$
- (D) $\frac{312}{25}$
- (E) $\frac{649}{25}$

33. Let $f(x, y) = e^{16x+4y}$. Find $\partial^4 f / \partial x^3 \partial y$ at $(0, 0)$.

- (A) $4(4 \cdot 16^3)$
- (B) $\frac{1}{4}(4 \cdot 16^3)$
- (C) $\frac{1}{16}(4 \cdot 16^3)$
- (D) $\frac{1}{64}(4 \cdot 16^3)$
- (E) $4 \cdot 16^3$

Questions 34–36

34. A sequence satisfies $u_0 = 74$ and $u_{k+1} = 2u_k + 13$ for all $k \geq 0$. Find u_7 .

- (A) 11123
- (B) 11125
- (C) 11122
- (D) 11124
- (E) 11121

35. Evaluate $\lim_{x \rightarrow 0} \frac{(1 + 16x)^{1/(3x)} - e^{16/3}}{x}$.

- (A) $-\frac{8e^{\frac{16}{3}}}{3}$
- (B) $e^{\frac{16}{3}}$
- (C) $-\frac{128e^{\frac{16}{3}}}{3}$
- (D) $\frac{128e^{\frac{16}{3}}}{3}$
- (E) $-\frac{256e^{\frac{16}{3}}}{3}$

36. A set with 25 elements has the discrete topology. How many of its subsets are simultaneously open and closed?

- (A) 2^{28}
- (B) 2^{25}
- (C) 2^{26}
- (D) 2^{27}
- (E) 2^{24}

Questions 37–39

37. Evaluate $\int_0^{\log 2} e^x (e^x + 16)^4 dx$.

- (A) $-\frac{18^5 - 17^5}{5}$
- (B) $\frac{18^5 - 17^5}{5}$
- (C) $18^4 - 17^4$
- (D) $\frac{18^5}{5}$
- (E) $5 \left(\frac{18^5 - 17^5}{5} \right)$

38. Linear maps $T : \mathbb{R}^{81} \rightarrow \mathbb{R}^{81}$ and $U : \mathbb{R}^{81} \rightarrow \mathbb{R}^{81}$ have ranks 4 and 2. What is the largest possible rank of UT ?

- (A) 2
- (B) 1
- (C) 3
- (D) 4
- (E) 0

39. For $D = \{(x, y) : x^2 + y^2 \leq 16\}$, evaluate $\iint_D (x^2 + y^2)^3 dA$.

- (A) 65536π
- (B) 16384π
- (C) 1024π
- (D) 32768π
- (E) $\frac{65536\pi}{7}$

Questions 40–42

40. Let $\zeta = e^{2\pi i/22}$. Find $\sum_{k=0}^{21} \zeta^k$.

- (A) 1
- (B) 2
- (C) 4
- (D) 3
- (E) 0

41. Evaluate $\iiint_{x^2+y^2+z^2 \leq 16} (x^2 + y^2 + z^2)^3 dV$.

- (A) $\frac{9}{112} \left(\frac{4\pi 16^{\frac{9}{2}}}{9} \right)$
- (B) $\frac{1}{3} \left(\frac{4\pi 16^{\frac{9}{2}}}{9} \right)$
- (C) $\frac{4\pi 16^{\frac{9}{2}}}{9}$
- (D) $\frac{1}{2} \left(\frac{4\pi 16^{\frac{9}{2}}}{9} \right)$
- (E) $9 \left(\frac{4\pi 16^{\frac{9}{2}}}{9} \right)$

42. In $R = \mathbb{F}_5[x]/(x^{16})$, how many elements are units?

- (A) $5^{17} - 5^{16}$
- (B) $5^{18} - 5^{17}$
- (C) $5^{16} - 5^0$
- (D) $5^{16} - 5^{15}$
- (E) $5^{17} - 5^{15}$

Questions 43–45

43. Find the total outward flux of $\mathbf{F} = (16x^4y, y^4z, xz^4)$ through the boundary of $[0, 1]^3$.
- (A) $\frac{9}{2}$
(B) 18
(C) 10
(D) 9
(E) -9
44. An analytic function has real part $u(x, y) = x^2 - y^2 + 20x$ and satisfies $f(0) = 0$. Find $\text{Im } f(1 + i)$.
- (A) 22
(B) 15
(C) 8
(D) 36
(E) 29
45. Evaluate $\int_0^{\pi/2} \sin^9 t \cos^{33} t \, dt$.
- (A) $21 \left(\frac{4! 16!}{2 \cdot 21!} \right)$
(B) $\frac{1}{2} \left(\frac{4! 16!}{2 \cdot 21!} \right)$
(C) $2 \left(\frac{4! 16!}{2 \cdot 21!} \right)$
(D) $4845 \left(\frac{4! 16!}{2 \cdot 21!} \right)$
(E) $\frac{4! 16!}{2 \cdot 21!}$

Questions 46–48

46. Integers x satisfy $x \equiv 2 \pmod{5}$ and $x \equiv 4 \pmod{8}$. Find the least nonnegative solution.

- (A) 13
- (B) 12
- (C) 14
- (D) 10
- (E) 11

47. Evaluate $\int_0^1 \int_0^x x^{16} y^4 dy dx$.

- (A) $\frac{1}{105}$
- (B) $\frac{1}{110}$
- (C) $\frac{1}{22}$
- (D) $\frac{1}{85}$
- (E) $\frac{1}{132}$

48. What is the coefficient of x^2 in $(1 + 2x)^{30}$?

- (A) 2900
- (B) 2320
- (C) 580
- (D) 1740
- (E) 1160

Questions 49–51

49. Evaluate $\lim_{x \rightarrow 0} \frac{e^{16x} - \sum_{j=0}^3 (16x)^j / j!}{x^4}$.

- (A) $4 \left(\frac{16^4}{4!} \right)$
- (B) $\frac{16^4}{4!}$
- (C) $24 \left(\frac{16^4}{4!} \right)$
- (D) $\frac{1}{16} \left(\frac{16^4}{4!} \right)$
- (E) $-\frac{16^4}{4!}$

50. The symmetric group S_{80} acts on the two-element subsets of $\{1, \dots, 80\}$. What is the index of the setwise stabilizer of $\{1, 2\}$?

- (A) 3160
- (B) 3162
- (C) 3161
- (D) 3159
- (E) 3158

51. The solution of $y' = 16x^3y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\sqrt{2}$
- (B) $\frac{\sqrt[4]{2}}{2}$
- (C) $\frac{\sqrt{2}}{2}$
- (D) $\frac{1}{2}$
- (E) $\frac{1}{4}$

Questions 52–54

52. How many edges does the complete bipartite graph $K_{30,32}$ have?

- (A) 320
- (B) 640
- (C) 960
- (D) 1280
- (E) 1600

53. What is the maximum of x^4y^{16} over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\frac{4}{25}$
- (B) $\left(\frac{4}{20}\right)^{16} \left(\frac{16}{20}\right)^4$
- (C) $\frac{1}{2^{20}}$
- (D) $20 \left(\left(\frac{4}{20}\right)^4 \left(\frac{16}{20}\right)^{16} \right)$
- (E) $\left(\frac{4}{20}\right)^4 \left(\frac{16}{20}\right)^{16}$

54. In $\mathbb{Z}/2406\mathbb{Z}$, how many residues are not idempotent ($x^2 \neq x$)?

- (A) 2398
- (B) 2400
- (C) 2399
- (D) 2396
- (E) 2397

Questions 55–57

55. Find the coefficient of x^4 in $e^{3x} \log(1 + 16x)$.

- (A) 12792
- (B) -16384
- (C) -12776
- (D) -12792
- (E) -307008

56. How many functions from a set of 35 elements onto a set of 3 elements exist?

- (A) 3^{35}
- (B) $3^{35} - 3(2^{35}) - 3$
- (C) $3^{35} - 2^{35}$
- (D) $3^{35} - 3(2^{35}) + 3$
- (E) $3^{35} - 3(2^{35})$

57. Let $f(x, y) = x^2 + 32xy + (260)y^2$. What is the product of the eigenvalues of the Hessian of f ?

- (A) 16
- (B) 8
- (C) 260
- (D) 1040
- (E) 4

Questions 58–60

58. In the additive group $\mathbb{Z}/108\mathbb{Z}$, find the index of the subgroup generated by $[6]$.

- (A) 5
- (B) 4
- (C) 6
- (D) 7
- (E) 8

59. The region $0 \leq x \leq 1$, $0 \leq y \leq 16x^4$ is revolved about the line $x = -4$. Find its volume.

- (A) $-\frac{464\pi}{15}$
- (B) $\frac{232\pi}{15}$
- (C) $\frac{464\pi}{15}$
- (D) $\frac{928\pi}{15}$
- (E) $1 + \frac{464\pi}{15}$

60. How many nonnegative integer triples sum to 70, if each coordinate is at most 35?

- (A) 222
- (B) 888
- (C) 1110
- (D) 666
- (E) 444

Questions 61–63

61. Let $F(x) = \int_0^{x^4} e^{16t} dt$. Find $F'(1)$.

- (A) $\frac{e^{16}}{4}$
- (B) $4e^{16}$
- (C) e^{16}
- (D) $64e^{16}$
- (E) e^{64}

62. How many invertible 3×3 matrices over $\mathbb{F}_{47}$ are there whose first column is $(1, 0, 0)^T$?

- (A) $\frac{(47^3 - 47^0)(47^3 - 47^0)(47^3 - 47^0)}{47^3 - 1}$
- (B) $\frac{47^3 47^3 47^3}{47^3 - 1}$
- (C) $\frac{(47^3 - 47^1)(47^3 - 47^2)(47^3 - 47^3)}{47^3 - 1}$
- (D) $\frac{(47^3 - 47^0)(47^3 - 47^1)(47^3 - 47^2)}{47^3 - 1}$
- (E) $\frac{(47^3 - 47^0)(47^3 - 47^1)}{47^3 - 1}$

63. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 16z^2 = x + 4y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.

- (A) -32
- (B) -64
- (C) 128
- (D) -4
- (E) -128

Questions 64–66

64. A tree has 39 vertices. What is the sum of its vertex degrees?

- (A) 76
- (B) 126
- (C) 26
- (D) 51
- (E) 101

65. For $x > 0$, $x^2y'' + (1 - 20)xy' + 64y = 0$, $y(1) = 0$, $y'(1) = 16$. Find $y(e)$.

- (A) $-\frac{4(1 - e^{12})e^4}{3}$
- (B) $-e^4 + e^{16}$
- (C) $e^4 + e^{16}$
- (D) $\frac{4(1 - e^{12})e^4}{3}$
- (E) $\frac{e^{16}}{20}$

66. How many permutations in S_{80} consist of one 6-cycle and 74 fixed points?

- (A) $2^5 \cdot 3 \cdot 5^3 \cdot 7 \cdot 11 \cdot 13 \cdot 19 \cdot 79$
- (B) $2^6 \cdot 3^2 \cdot 5^3 \cdot 7 \cdot 11 \cdot 13 \cdot 19 \cdot 79$
- (C) $2^6 \cdot 3 \cdot 5^3 \cdot 7 \cdot 11 \cdot 13 \cdot 19 \cdot 79$
- (D) $2^6 \cdot 3 \cdot 5^4 \cdot 7 \cdot 11 \cdot 13 \cdot 19 \cdot 79$
- (E) $2^7 \cdot 3 \cdot 5^3 \cdot 7 \cdot 11 \cdot 13 \cdot 19 \cdot 79$

Answer Key

Practice Test 78 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	B	23	C	45	E
2	A	24	E	46	B
3	B	25	B	47	B
4	D	26	E	48	D
5	D	27	E	49	B
6	E	28	B	50	A
7	C	29	C	51	C
8	C	30	A	52	C
9	E	31	A	53	E
10	B	32	C	54	A
11	B	33	E	55	D
12	D	34	A	56	D
13	D	35	C	57	A
14	D	36	B	58	C
15	A	37	B	59	C
16	D	38	A	60	D
17	A	39	B	61	B
18	C	40	E	62	D
19	E	41	C	63	E
20	E	42	D	64	A
21	E	43	D	65	A
22	C	44	A	66	C

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