

GRE Mathematics Practice Test 81

Advanced Challenge

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

The most demanding form in this five-test cycle. Emphasis on connected ideas, non-routine reasoning, and careful conceptual distinctions. This is cycle 16 of 19, position 1 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Evaluate $\int_0^\infty x^6 e^{-17x} dx$.

- (A) 720
- (B) $17 \left(\frac{6!}{17^7} \right)$
- (C) $\frac{1}{6} \left(\frac{6!}{17^7} \right)$
- (D) $\frac{6!}{17^7}$
- (E) $\frac{1}{17} \left(\frac{6!}{17^7} \right)$

2. Let $D = \text{diag}(\underbrace{1, \dots, 1}_{15}, \underbrace{2, \dots, 2}_{68})$. Find the dimension of the space of real matrices X with $XD = DX$ and $\text{tr } X = 0$.

- (A) 4846
- (B) 4847
- (C) 4848
- (D) 4850
- (E) 4849

3. The region $0 \leq x \leq 1$, $0 \leq y \leq 17x^6$ is revolved about the line $x = -2$. Find its volume.

- (A) $\frac{391\pi}{28}$
- (B) $1 + \frac{391\pi}{28}$
- (C) $-\frac{391\pi}{28}$
- (D) $\frac{391\pi}{56}$
- (E) $\frac{391\pi}{14}$

Questions 4–6

4. A uniformly random subset of 33 objects is selected from a collection with 34 red and 35 blue objects. If X is the number of red objects selected, find $\text{Var}(X)$.

- (A) $\frac{2839}{529}$
- (B) $\frac{4620}{529}$
- (C) $\frac{1155}{529}$
- (D) $\frac{3368}{529}$
- (E) $\frac{2310}{529}$

5. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-17)^j}{j^{\frac{5}{3}} (\log j)^m}$ converge absolutely at $x = 16$? Give the least such m , or 0 if none exists.

- (A) 0
- (B) 2
- (C) 4
- (D) 3
- (E) 1

6. When $x^3 + 1$ is divided by $(x - 78)(x - 79)$, the remainder is $ux + v$. Find u .

- (A) 18488
- (B) 18486
- (C) 18489
- (D) 18487
- (E) 18485

Questions 7–9

7. For $f(x) = x^6 e^{17x}$, find $f^{(8)}(0)$.

- (A) $2 \left(\frac{8! 17^2}{2} \right)$
- (B) $\frac{2}{17} \left(\frac{8! 17^2}{2} \right)$
- (C) $\frac{1}{20160} \left(\frac{8! 17^2}{2} \right)$
- (D) $\frac{8! 17^2}{2}$
- (E) $\frac{1}{56} \left(\frac{8! 17^2}{2} \right)$

8. X, Y are independent Poisson variables with means 33 and 34. Given $X + Y = 35$, find $\text{Var}(X | X + Y)$.

- (A) $\frac{48248}{4489}$
- (B) $\frac{43759}{4489}$
- (C) $\frac{39270}{4489}$
- (D) $\frac{19635}{4489}$
- (E) $\frac{78540}{4489}$

9. Let $f(x, y) = e^{17x+6y}$. Find $\partial^6 f / \partial x^5 \partial y$ at $(0, 0)$.

- (A) $\frac{1}{6} (6 \cdot 17^5)$
- (B) $\frac{1296}{83521} (6 \cdot 17^5)$
- (C) $\frac{17}{6} (6 \cdot 17^5)$
- (D) $6 \cdot 17^5$
- (E) 46656

Questions 10–12

10. Let $P = \text{diag}(\underbrace{1, \dots, 1}_1, \underbrace{0, \dots, 0}_{83})$. What is the dimension of the space of 84×84 real matrices X satisfying $PX = X$ and $XP = 0$?

- (A) 85
- (B) 83
- (C) 82
- (D) 84
- (E) 81

11. The solution of $y' = 17x^5y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\frac{17^{\frac{5}{6}}}{17}$
- (B) $\frac{\sqrt[6]{17} \cdot 6^{\frac{5}{6}}}{6}$
- (C) $\frac{17^{\frac{5}{6}} \sqrt[6]{3}}{17}$
- (D) $\frac{6}{17}$
- (E) $\frac{17^{\frac{5}{6}} \sqrt[6]{6}}{17}$

12. X, Y are independent, with variances 33 and 34. Find $\text{Cov}(X + 2Y, 3X - Y)$.

- (A) 41
- (B) 21
- (C) 31
- (D) 51
- (E) 11

Questions 13–15

13. Solve $y' + 17y = x^6 e^{-17x}$, $y(0) = 0$, and find $y(1)$.

- (A) $\frac{1}{7e^{17}}$
- (B) $\frac{1}{6e^{17}}$
- (C) $\frac{e^{17}}{7}$
- (D) $\frac{17}{7e^{17}}$
- (E) e^{-17}

14. A 3×3 matrix A has characteristic polynomial $(t - 77)(t + 1)(t - 2)$. What is $\text{tr}(A^3 - 2A + I)$?

- (A) 456389
- (B) 456385
- (C) 456388
- (D) 456386
- (E) 456387

15. Let $f(x) = \log(1 + 17x)$. Find $f^{(6)}(0)$.

- (A) $-(-1)^5 5! 17^6$
- (B) $(-1)^5 5! 17^6$
- (C) $6 \left((-1)^5 5! 17^6 \right)$
- (D) $\frac{1}{17} \left((-1)^5 5! 17^6 \right)$
- (E) $\frac{1}{6} \left((-1)^5 5! 17^6 \right)$

Questions 16–18

16. X has density cx^{33} on $0 < x < 1$ and zero elsewhere. Find $E[X^2]$.

- (A) $\frac{17}{18}$
- (B) $\frac{35}{36}$
- (C) $\frac{17}{54}$
- (D) $\frac{17}{36}$
- (E) $\frac{1}{18}$

17. For $\mathbf{F} = (17x^6y, y^6z, xz^6)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) -402
- (B) 402
- (C) 804
- (D) 403
- (E) 201

18. Let N be the 83×83 matrix with ones immediately above the diagonal and zeros elsewhere. Find $\dim(\ker N^{28} / \ker N^{14})$.

- (A) 15
- (B) 13
- (C) 14
- (D) 16
- (E) 12

Questions 19–21

19. Evaluate $\iiint_{x^2+y^2+z^2 \leq 17} (x^2 + y^2 + z^2)^5 dV$.

- (A) $\frac{4\pi 17^{\frac{13}{2}}}{13}$
- (B) $\frac{13}{187} \left(\frac{4\pi 17^{\frac{13}{2}}}{13} \right)$
- (C) $\frac{1}{3} \left(\frac{4\pi 17^{\frac{13}{2}}}{13} \right)$
- (D) $\frac{1}{2} \left(\frac{4\pi 17^{\frac{13}{2}}}{13} \right)$
- (E) $13 \left(\frac{4\pi 17^{\frac{13}{2}}}{13} \right)$

20. Independent exponential waiting times X, Y have rates 33 and 35. Find $E[\min(X, Y)]$.

- (A) $\frac{1}{136}$
- (B) $\frac{69}{136}$
- (C) $\frac{67}{68}$
- (D) $\frac{1}{68}$
- (E) $\frac{1}{204}$

21. Evaluate $\int_0^1 \frac{x^5}{1 + 17x^6} dx$.

- (A) $\frac{\log(18)}{102}$
- (B) $\log(18)$
- (C) $\frac{\log(18)}{17}$
- (D) $-\frac{\log(18)}{102}$
- (E) $\frac{\log(18)}{6}$

Questions 22–24

22. Let A be the 83×83 matrix whose diagonal entries are 80 and whose off-diagonal entries are 1. What is its largest eigenvalue?

- (A) 162
- (B) 163
- (C) 161
- (D) 160
- (E) 164

23. Two solutions of $y'' + 17x^6y' + 6y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $-\frac{1}{e^{\frac{17}{7}}}$
- (B) $e^{-\frac{17}{7}}$
- (C) $\frac{2}{e^{\frac{17}{7}}}$
- (D) $\frac{1}{2e^{\frac{17}{7}}}$
- (E) $e^{-\frac{17}{7}} + 1$

24. For which real a does $f_m(x) = m^a x^{28}(1-x)^m$ converge uniformly to zero on $[0, 1]$?

- (A) $a < 0$
- (B) $a \leq 28$
- (C) $a < 28$
- (D) All real a
- (E) $a > 28$

Questions 25–27

25. What is the maximum of x^6y^{17} over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\left(\frac{6}{23}\right)^{17} \left(\frac{17}{23}\right)^6$
- (B) $\frac{102}{529}$
- (C) $\frac{1}{2^{23}}$
- (D) $\left(\frac{6}{23}\right)^6 \left(\frac{17}{23}\right)^{17}$
- (E) $23 \left(\left(\frac{6}{23}\right)^6 \left(\frac{17}{23}\right)^{17} \right)$

26. Positive real numbers x, y satisfy $\log_2 x + \log_2 y = \log_2 924$ and $x + y = 89$. Find $x^2 + y^2$.

- (A) 6075
- (B) 6071
- (C) 6072
- (D) 6073
- (E) 6074

27. Find the directional derivative of $f(x, y, z) = x^2y + 6xz^2 + 17y^2z$ at $(1, 1, 1)$ in the direction $(1, 2, 0)$.

- (A) $1 + \frac{78\sqrt{5}}{5}$
- (B) $\frac{39\sqrt{5}}{5}$
- (C) $-\frac{78\sqrt{5}}{5}$
- (D) $\frac{156\sqrt{5}}{5}$
- (E) $\frac{78\sqrt{5}}{5}$

Questions 28–30

28. For which real p does $\sum_{k=2}^{\infty} 1/[k(\log k)^{28p}]$ converge?

- (A) $p \geq 1/28$
- (B) All real p
- (C) $p > 1/28$
- (D) $p > 0$
- (E) $p < 1/28$

29. For $x > 0$, $x^2y'' + (-39)xy' + 391y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

- (A) $-\frac{(1 - e^6)e^{17}}{6}$
- (B) $\frac{(1 - e^6)e^{17}}{6}$
- (C) $-\frac{e^{17}}{6} + 1 + \frac{e^{23}}{6}$
- (D) $-\frac{(1 - e^6)e^{17}}{12}$
- (E) $-\frac{(1 - e^6)e^{17}}{3}$

30. A sequence satisfies $u_0 = 77$ and $u_{k+1} = 2u_k + 12$ for all $k \geq 0$. Find u_7 .

- (A) 11378
- (B) 11381
- (C) 11379
- (D) 11380
- (E) 11382

Questions 31–33

31. Find the curvature of the helix $r(t) = (17 \cos t, 17 \sin t, 6t)$.

- (A) $-\frac{17}{325}$
- (B) $\frac{17}{650}$
- (C) $\frac{342}{325}$
- (D) $\frac{34}{325}$
- (E) $\frac{17}{325}$

32. How many accumulation points in $\mathbb{R}$ does $S = \{1/k : k \geq 1\} \cup \{28 + 1/k : k \geq 1\}$ have?

- (A) 3
- (B) 0
- (C) 4
- (D) 1
- (E) 2

33. If $y'' + 36y = 17$ and $y(0) = y'(0) = 0$, find $y(\pi/6)$.

- (A) $-\frac{17}{18}$
- (B) $\frac{17}{18}$
- (C) $\frac{35}{18}$
- (D) $\frac{17}{36}$
- (E) $\frac{17}{9}$

Questions 34–36

34. Linear maps $T : \mathbb{R}^{84} \rightarrow \mathbb{R}^{84}$ and $U : \mathbb{R}^{84} \rightarrow \mathbb{R}^{84}$ have ranks 3 and 4. What is the largest possible rank of UT ?

- (A) 3
- (B) 1
- (C) 4
- (D) 5
- (E) 2

35. Let $f(x, y) = x^2 + 34xy + (295)y^2$. What is the product of the eigenvalues of the Hessian of f ?

- (A) 1180
- (B) 295
- (C) 12
- (D) 6
- (E) 24

36. How many connected components does $\mathbb{R} \setminus \{1, 2, \dots, 28\}$ have?

- (A) 38
- (B) 47
- (C) 29
- (D) 20
- (E) 11

Questions 37–39

37. For the curve $x = t^6$, $y = t^7 + 17t$, find d^2y/dx^2 at $t = 1$.

- (A) 4
- (B) $\frac{23}{9}$
- (C) $-\frac{17}{6}$
- (D) $-\frac{13}{6}$
- (E) -13

38. Fix a nonzero vector $u \in \mathbb{R}^{83}$. What is the dimension of the space of skew-symmetric 83×83 matrices A satisfying $Au = 0$?

- (A) 3319
- (B) 3323
- (C) 3320
- (D) 3321
- (E) 3322

39. Find the upward flux of $\mathbf{F} = (x, y, z)$ through $z = 17x^2 + 6y^2$, $0 \leq x, y \leq 1$.

- (A) $-\frac{20}{3}$
- (B) $\frac{23}{3}$
- (C) $-\frac{23}{6}$
- (D) $-\frac{46}{3}$
- (E) $-\frac{23}{3}$

Questions 40–42

40. Find the coefficient of z^{-1} in $e^{18z}/[z^2(1-z)]$ for $0 < |z| < 1$.

- (A) 7
- (B) 25
- (C) 13
- (D) 19
- (E) 31

41. Evaluate $\int_C (6x^5 + 17y) dx + (17x + 6y^5) dy$, where C is $r(t) = (t, 2t^2)$, $0 \leq t \leq 1$.

- (A) -99
- (B) 198
- (C) 100
- (D) $\frac{99}{2}$
- (E) 99

42. The field K has 11^{17} elements. Find the largest possible multiplicative order of a nonzero element of K .

- (A) 11^{17}
- (B) $11^{16} - 1$
- (C) $11^{17} - 1$
- (D) $11^{18} - 1$
- (E) $11^{17} + 1$

Questions 43–45

43. Find the coefficient of x^6 in $\cos(17x)/(1 - 5x^2)$.

- (A) -14120419
- (B) $-\frac{24137569}{720}$
- (C) $-\frac{14121139}{720}$
- (D) $\frac{14120419}{720}$
- (E) $-\frac{14120419}{720}$

44. An entire function satisfies $|f(z)| \leq C(1 + |z|^{18})$ for some $C > 0$. What is the largest possible degree of f ?

- (A) 18
- (B) 24
- (C) 6
- (D) 12
- (E) 30

45. Let $\mathbf{F} = (17y^6, z^6, x^6)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (1, 1, 2)/\sqrt{6}$.

- (A) $-18\sqrt{6}$
- (B) $-36\sqrt{6}$
- (C) $1 - 36\sqrt{6}$
- (D) $-72\sqrt{6}$
- (E) $36\sqrt{6}$

Questions 46–48

46. Let $G = \mathbb{Z}/492\mathbb{Z} \times \mathbb{Z}/332\mathbb{Z}$. Find the order of G/H , where $H = \{g \in G : 4g = 0\}$.

- (A) 10210
- (B) 10209
- (C) 10211
- (D) 10208
- (E) 10207

47. If $y'' - 34y' + 289y = 6e^{17x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) $1 + 3e^{17}$
- (B) $3e^{17}$
- (C) $-3e^{17}$
- (D) $6e^{17}$
- (E) $\frac{3e^{17}}{2}$

48. A connected planar graph has 36 vertices and 67 edges. How many faces, including the unbounded face, are in a planar embedding?

- (A) 11
- (B) 33
- (C) 22
- (D) 44
- (E) 55

Questions 49–51

49. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(17j)!}{(j!)^{17}} x^{6j}$?
- (A) 17^{-17}
(B) $17^{\frac{17}{6}}$
(C) $\frac{1}{17}$
(D) $17^{-\frac{6}{17}}$
(E) $17^{-\frac{17}{6}}$
50. How many group homomorphisms $\mathbb{Z}/492\mathbb{Z} \rightarrow \mathbb{Z}/332\mathbb{Z}$ have image of order 4?
- (A) 0
(B) 2
(C) 1
(D) 3
(E) 4
51. Evaluate $\int_0^{\pi/2} \sin^{13} t \cos^{35} t \, dt$.
- (A) $\frac{1}{48}$
(B) $2 \left(\frac{6! 17!}{2 \cdot 24!} \right)$
(C) $\frac{6! 17!}{2 \cdot 24!}$
(D) $24 \left(\frac{6! 17!}{2 \cdot 24!} \right)$
(E) $\frac{1}{2} \left(\frac{6! 17!}{2 \cdot 24!} \right)$

Questions 52–54

52. How many integers in $\{1, \dots, 198\}$ are divisible by 2 or 3?

- (A) 176
- (B) 88
- (C) 132
- (D) 44
- (E) 220

53. Evaluate $\int_0^1 x^{16}(\log x)^5 dx$.

- (A) $-\frac{1}{17}$
- (B) $\frac{(-1)^5 5!}{17^6}$
- (C) $-\frac{(-1)^5 5!}{17^6}$
- (D) $\frac{1}{6} \left(\frac{(-1)^5 5!}{17^6} \right)$
- (E) $17 \left(\frac{(-1)^5 5!}{17^6} \right)$

54. How many 2-dimensional subspaces does $\mathbb{F}_{53}^7$ have?

- (A) $\frac{(53^7 - 1)(53^7 - 53)}{(53^2 - 1)(53^2 - 53)}$
- (B) $\frac{(53^6 - 1)(53^6 - 53)}{(53^2 - 1)(53^2 - 53)}$
- (C) $\frac{(53^{10} - 1)(53^{10} - 53)}{(53^2 - 1)(53^2 - 53)}$
- (D) $\frac{(53^8 - 1)(53^8 - 53)}{(53^2 - 1)(53^2 - 53)}$
- (E) $\frac{(53^9 - 1)(53^9 - 53)}{(53^2 - 1)(53^2 - 53)}$

Questions 55–57

55. Evaluate $\iiint_B 17z^2(x^2 + y^2 + z^2)^6 dV$, where B is the unit ball.

- (A) $\frac{8\pi}{3}$
- (B) $\frac{4\pi}{3}$
- (C) $-\frac{4\pi}{3}$
- (D) $1 + \frac{4\pi}{3}$
- (E) $\frac{2\pi}{3}$

56. Let $F_0 = 0$, $F_1 = 1$, and $F_{k+2} = F_{k+1} + F_k$. How many binary strings of length 33 have no adjacent 1s?

- (A) F_{33}
- (B) F_{34}
- (C) F_{36}
- (D) F_{35}
- (E) F_{37}

57. If $y' + 17y = 6x^6e^{-17x}$ and $y(0) = 2$, find $y(1)$.

- (A) $\frac{20}{7e^{17}} + 1$
- (B) $\frac{40}{7e^{17}}$
- (C) $\frac{20}{7e^{17}}$
- (D) $-\frac{20}{7e^{17}}$
- (E) $\frac{10}{7e^{17}}$

Questions 58–60

58. How many monic irreducible polynomials of degree 5 are there over $\mathbb{F}_{53}$?

- (A) $\frac{2(53^5 - 53)}{5}$
- (B) $53^5 - 53$
- (C) $\frac{3(53^5 - 53)}{5}$
- (D) $\frac{53^5 - 53}{5}$
- (E) $2(53^5 - 53)$

59. Evaluate $\lim_{x \rightarrow 0} \frac{(1 + 17x)^{1/(5x)} - e^{17/5}}{x}$.

- (A) $-\frac{289e^{\frac{17}{5}}}{5}$
- (B) $e^{\frac{17}{5}}$
- (C) $-\frac{289e^{\frac{17}{5}}}{10}$
- (D) $-\frac{17e^{\frac{17}{5}}}{10}$
- (E) $\frac{289e^{\frac{17}{5}}}{10}$

60. For Newton iteration on $f(x) = x^2 - 1089$ near the root $r = 33$, find $\lim_{k \rightarrow \infty} (x_{k+1} - r)/(x_k - r)^2$ for nonterminating convergent iterates.

- (A) $\frac{1}{66}$
- (B) $\frac{65}{66}$
- (C) $\frac{67}{132}$
- (D) $\frac{1}{198}$
- (E) $\frac{1}{132}$

Questions 61–63

61. For $D = \{(x, y) : x^2 + y^2 \leq 17\}$, evaluate $\iint_D (x^2 + y^2)^5 dA$.

- (A) 24137569π
- (B) $\frac{24137569\pi}{6}$
- (C) $\frac{24137569\pi}{3}$
- (D) $\frac{24137569\pi}{11}$
- (E) $\frac{1419857\pi}{6}$

62. How many elements z of $\mathbb{F}_{11^{17}}$ satisfy $z^{121} = z$?

- (A) 9
- (B) 12
- (C) 13
- (D) 10
- (E) 11

63. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 17z^2 = x + 6y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.

- (A) -6
- (B) -102
- (C) 204
- (D) -34
- (E) -204

Questions 64–66

64. Bisection starts with an interval of length 2^{33} . After 35 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{1}{8}$
- (B) $\frac{9}{16}$
- (C) $\frac{1}{16}$
- (D) $\frac{7}{8}$
- (E) $\frac{1}{24}$

65. Let $F(x) = \int_0^{x^6} e^{17t} dt$. Find $F'(1)$.

- (A) $6e^{17}$
- (B) e^{17}
- (C) $\frac{e^{17}}{6}$
- (D) e^{102}
- (E) $102e^{17}$

66. How many generators does the cyclic group of order 22164361129 have?

- (A) $53^6(53 - 1)$
- (B) $53^7(53 - 1)$
- (C) $53^5(53 - 1)$
- (D) $53^8(53 - 1)$
- (E) $53^4(53 - 1)$

Answer Key

Practice Test 81 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	D	23	B	45	B
2	C	24	C	46	B
3	A	25	D	47	B
4	E	26	D	48	B
5	E	27	E	49	E
6	D	28	C	50	B
7	D	29	A	51	C
8	C	30	D	52	C
9	D	31	E	53	B
10	B	32	E	54	A
11	E	33	B	55	B
12	C	34	A	56	D
13	A	35	E	57	C
14	E	36	C	58	D
15	B	37	D	59	C
16	A	38	D	60	A
17	B	39	E	61	B
18	C	40	D	62	E
19	A	41	E	63	E
20	D	42	C	64	A
21	A	43	E	65	A
22	A	44	A	66	C

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