

GRE Mathematics Practice Test 82

Advanced Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A demanding practice form with frequent multi-step arguments and a mix of computational and conceptual questions. This is cycle 16 of 19, position 2 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Find the upward flux of $\mathbf{F} = (x, y, z)$ through $z = 17x^2 + 5y^2$, $0 \leq x, y \leq 1$.
- (A) $\frac{22}{3}$
(B) $-\frac{22}{3}$
(C) $-\frac{19}{3}$
(D) $-\frac{44}{3}$
(E) $-\frac{11}{3}$
2. Let A be the 84×84 matrix whose diagonal entries are 81 and whose off-diagonal entries are 1. What is its largest eigenvalue?
- (A) 166
(B) 164
(C) 162
(D) 165
(E) 163
3. For $\mathbf{F} = (17x^5y, y^5z, xz^5)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.
- (A) 255
(B) $\frac{255}{2}$
(C) -255
(D) 510
(E) 256

Questions 4–6

4. A uniformly random subset of 34 objects is selected from a collection with 35 red and 36 blue objects. If X is the number of red objects selected, find $E[X]$.

- (A) $\frac{2380}{71}$
- (B) $\frac{1332}{71}$
- (C) $\frac{1261}{71}$
- (D) $\frac{1190}{71}$
- (E) $\frac{595}{71}$

5. For $D = \{(x, y) : x^2 + y^2 \leq 17\}$, evaluate $\iint_D (x^2 + y^2)^4 dA$.

- (A) $\frac{2839714\pi}{5}$
- (B) $\frac{1419857\pi}{5}$
- (C) $\frac{83521\pi}{5}$
- (D) $\frac{1419857\pi}{9}$
- (E) 1419857π

6. Positive real numbers x, y satisfy $\log_2 x + \log_2 y = \log_2 780$ and $x + y = 88$. Find $x^2 + y^2$.

- (A) 6185
- (B) 6186
- (C) 6182
- (D) 6184
- (E) 6183

Questions 7–9

7. Evaluate $\int_0^{\log 2} e^x(e^x + 17)^5 dx$.

- (A) $19^5 - 18^5$
- (B) $\frac{19^6}{6}$
- (C) $-\frac{19^6 - 18^6}{6}$
- (D) $\frac{19^6 - 18^6}{6}$
- (E) $6 \left(\frac{19^6 - 18^6}{6} \right)$

8. X, Y are independent Poisson variables with means 34 and 35. Given $X + Y = 36$, find $\text{Var}(X | X + Y)$.

- (A) $\frac{5289}{529}$
- (B) $\frac{2380}{529}$
- (C) $\frac{5818}{529}$
- (D) $\frac{9520}{529}$
- (E) $\frac{4760}{529}$

9. Under $x = u + 17v$, $y = u - 5v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) 484
- (B) 22
- (C) 85
- (D) 12
- (E) $\frac{1}{22}$

Questions 10–12

10. A sequence satisfies $u_0 = 78$ and $u_{k+1} = 2u_k + 10$ for all $k \geq 0$. Find u_6 .

- (A) 5621
- (B) 5624
- (C) 5620
- (D) 5623
- (E) 5622

11. Find the total outward flux of $\mathbf{F} = (17x^5y, y^5z, xz^5)$ through the boundary of $[0, 1]^3$.

- (A) $\frac{19}{2}$
- (B) $\frac{21}{2}$
- (C) $-\frac{19}{2}$
- (D) 19
- (E) $\frac{19}{4}$

12. X, Y are independent, with variances 34 and 35. Find $\text{Cov}(X + 2Y, 3X - Y)$.

- (A) 22
- (B) 12
- (C) 32
- (D) 52
- (E) 42

Questions 13–15

13. Evaluate $\lim_{x \rightarrow 0} \frac{e^{17x} - \sum_{j=0}^4 (17x)^j / j!}{x^5}$.

- (A) $\frac{17^5}{5!}$
- (B) $-\frac{17^5}{5!}$
- (C) $5 \left(\frac{17^5}{5!} \right)$
- (D) $120 \left(\frac{17^5}{5!} \right)$
- (E) $\frac{1}{17} \left(\frac{17^5}{5!} \right)$

14. Linear maps $T : \mathbb{R}^{85} \rightarrow \mathbb{R}^{85}$ and $U : \mathbb{R}^{85} \rightarrow \mathbb{R}^{85}$ have ranks 2 and 4. What is the largest possible rank of UT ?

- (A) 3
- (B) 0
- (C) 4
- (D) 2
- (E) 1

15. The solution of $y' = 17x^4y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\frac{5}{17}$
- (B) $\frac{17^{\frac{4}{5}}}{17}$
- (C) $\frac{17^{\frac{4}{5}} \sqrt[5]{5}}{17}$
- (D) $\frac{\sqrt[5]{17} \cdot 5^{\frac{4}{5}}}{5}$
- (E) $\frac{34^{\frac{4}{5}} \sqrt[5]{5}}{34}$

Questions 16–18

16. X has density cx^{34} on $0 < x < 1$ and zero elsewhere. Find $E[X^2]$.

- (A) $\frac{2}{37}$
- (B) $\frac{35}{111}$
- (C) $\frac{35}{74}$
- (D) $\frac{36}{37}$
- (E) $\frac{35}{37}$

17. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j(x-17)^j}{j^{\frac{4}{3}}(\log j)^m}$ converge absolutely at $x = 16$? Give the least such m , or 0 if none exists.

- (A) 2
- (B) 1
- (C) 4
- (D) 3
- (E) 0

18. Fix a nonzero vector $u \in \mathbb{R}^{84}$. What is the dimension of the space of skew-symmetric 84×84 matrices A satisfying $Au = 0$?

- (A) 3402
- (B) 3405
- (C) 3403
- (D) 3404
- (E) 3401

Questions 19–21

19. Find the coefficient of x^5 in the Taylor series of $(1 - 17x)^{-2}$ at zero.

- (A) $\frac{1}{6} (6 \cdot 17^5)$
- (B) $\frac{5}{6} (6 \cdot 17^5)$
- (C) $-6 \cdot 17^5$
- (D) $6 \cdot 17^5$
- (E) $\frac{1}{17} (6 \cdot 17^5)$

20. Independent exponential waiting times X, Y have rates 34 and 36. Find $P(X < Y)$.

- (A) $\frac{17}{35}$
- (B) $\frac{18}{35}$
- (C) $\frac{26}{35}$
- (D) $\frac{17}{105}$
- (E) $\frac{17}{70}$

21. What is the maximum of $x^5 y^{17}$ over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $22 \left(\left(\frac{5}{22} \right)^5 \left(\frac{17}{22} \right)^{17} \right)$
- (B) $\left(\frac{5}{22} \right)^{17} \left(\frac{17}{22} \right)^5$
- (C) $\left(\frac{5}{22} \right)^5 \left(\frac{17}{22} \right)^{17}$
- (D) $\frac{1}{2^{22}}$
- (E) $\frac{85}{484}$

Questions 22–24

22. Let $A = \begin{bmatrix} 1 & 78 & 0 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{bmatrix}$. What is the $(1, 3)$ entry of A^4 ?

- (A) 4679
- (B) 4681
- (C) 4680
- (D) 4682
- (E) 4678

23. Evaluate $\int_0^1 x^{16}(\log x)^4 dx$.

- (A) $-\frac{(-1)^4 4!}{17^5}$
- (B) $17 \left(\frac{(-1)^4 4!}{17^5} \right)$
- (C) $\frac{1}{5} \left(\frac{(-1)^4 4!}{17^5} \right)$
- (D) $\frac{83521}{24} \left(\frac{(-1)^4 4!}{17^5} \right)$
- (E) $\frac{(-1)^4 4!}{17^5}$

24. For which real a does $f_m(x) = m^a x^{29}(1-x)^m$ converge uniformly to zero on $[0, 1]$?

- (A) $a \leq 29$
- (B) $a < 0$
- (C) $a > 29$
- (D) $a < 29$
- (E) All real a

Questions 25–27

25. Let $f(x) = \log(1 + 17x)$. Find $f^{(5)}(0)$.

- (A) $\frac{1}{5} \left((-1)^4 4! 17^5 \right)$
- (B) $(-1)^4 4! 17^5$
- (C) $5 \left((-1)^4 4! 17^5 \right)$
- (D) $-(-1)^4 4! 17^5$
- (E) $\frac{1}{17} \left((-1)^4 4! 17^5 \right)$

26. An invertible 84×84 matrix A has determinant 3. Find $\det(\text{adj } A)$.

- (A) 3^{84}
- (B) 3^{83}
- (C) $2 \cdot 3^{83}$
- (D) $3^{83} \cdot 5$
- (E) 3^{82}

27. For $f(x, y) = x^2 + 17xy + 5y^2$, what is its maximum directional derivative at $(1, -1)$?

- (A) 274
- (B) -15
- (C) $\sqrt{274}$
- (D) 22
- (E) 7

Questions 28–30

28. For which real p does $\sum_{k=2}^{\infty} 1/[k(\log k)^{29p}]$ converge?

- (A) All real p
- (B) $p > 1/29$
- (C) $p < 1/29$
- (D) $p > 0$
- (E) $p \geq 1/29$

29. Evaluate $\int_C (5x^4 + 17y) dx + (17x + 5y^4) dy$, where C is $r(t) = (t, 3t^2)$, $0 \leq t \leq 1$.

- (A) 295
- (B) 296
- (C) $\frac{295}{2}$
- (D) -295
- (E) 590

30. On P_{85} , the real polynomials of degree at most 85, define $T(p) = x^2 p'' - 28xp' + 210p$. Find its rank.

- (A) 85
- (B) 82
- (C) 86
- (D) 83
- (E) 84

Questions 31–33

31. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 17x$ and has density $\rho(x, y) = x^3y$. Find its centroid coordinate $\bar{x}$.

- (A) $-\frac{6}{7}$
- (B) $\frac{3}{7}$
- (C) $\frac{6}{7}$
- (D) $\frac{12}{7}$
- (E) $\frac{13}{7}$

32. How many accumulation points in $\mathbb{R}$ does $S = \{1/k : k \geq 1\} \cup \{29 + 1/k : k \geq 1\}$ have?

- (A) 1
- (B) 4
- (C) 0
- (D) 2
- (E) 3

33. If $y'' - 34y' + 289y = 5e^{17x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) $5e^{17}$
- (B) $-\frac{5e^{17}}{2}$
- (C) $\frac{5e^{17}}{4}$
- (D) $1 + \frac{5e^{17}}{2}$
- (E) $\frac{5e^{17}}{2}$

Questions 34–36

34. Subspaces U, V of $\mathbb{R}^{86}$ have dimensions 4 and 5. What is the smallest possible value of $\dim(U \cap V)$?

- (A) 3
- (B) 0
- (C) 4
- (D) 1
- (E) 2

35. Two solutions of $y'' + 17x^5y' + 5y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $e^{-\frac{17}{6}}$
- (B) $\frac{2}{e^{\frac{17}{6}}}$
- (C) $-\frac{1}{e^{\frac{17}{6}}}$
- (D) $e^{-\frac{17}{6}} + 1$
- (E) $\frac{1}{2e^{\frac{17}{6}}}$

36. How many connected components does $\mathbb{R} \setminus \{1, 2, \dots, 29\}$ have?

- (A) 50
- (B) 30
- (C) 40
- (D) 20
- (E) 10

Questions 37–39

37. Let C be $x^2/25 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C \left(-\frac{17y^3}{3}\right) dx + \left(\frac{x^3}{3}\right) dy$.

- (A) $-\frac{105\pi}{2}$
- (B) $1 + \frac{105\pi}{2}$
- (C) $\frac{105\pi}{4}$
- (D) 105π
- (E) $\frac{105\pi}{2}$

38. Let V consist of real polynomials of degree at most 11 that have both value and first derivative zero at 78, 79, and 81. Find $\dim V$.

- (A) 6
- (B) 5
- (C) 8
- (D) 7
- (E) 4

39. For the curve $x = t^5$, $y = t^6 + 17t$, find d^2y/dx^2 at $t = 1$.

- (A) $-\frac{62}{5}$
- (B) $-\frac{17}{5}$
- (C) $\frac{23}{5}$
- (D) $-\frac{62}{25}$
- (E) $\frac{74}{25}$

Questions 40–42

40. What is the coefficient of z^{-1} in the Laurent expansion of e^{19z}/z^3 about zero?

- (A) $\frac{361}{2}$
- (B) $\frac{365}{2}$
- (C) 361
- (D) $\frac{363}{2}$
- (E) $\frac{361}{4}$

41. For $f(x) = x^5 e^{17x}$, find $f^{(7)}(0)$.

- (A) $\frac{2}{17} \left(\frac{7! 17^2}{2} \right)$
- (B) $2 \left(\frac{7! 17^2}{2} \right)$
- (C) $\frac{1}{42} \left(\frac{7! 17^2}{2} \right)$
- (D) $\frac{1}{2520} \left(\frac{7! 17^2}{2} \right)$
- (E) $\frac{7! 17^2}{2}$

42. Let $R = (\mathbb{Z}/152\mathbb{Z})/([19])$, where $([19])$ is the ideal generated by $[19]$. How many units does R have?

- (A) 19
- (B) 20
- (C) 17
- (D) 16
- (E) 18

Questions 43–45

43. Solve $y' + 17y = x^5e^{-17x}$, $y(0) = 0$, and find $y(1)$.

- (A) $\frac{1}{6e^{17}}$
- (B) $\frac{17}{6e^{17}}$
- (C) $\frac{1}{5e^{17}}$
- (D) $\frac{e^{17}}{6}$
- (E) e^{-17}

44. An entire function satisfies $|f(z)| \leq C(1 + |z|^{19})$ for some $C > 0$. What is the largest possible degree of f ?

- (A) 31
- (B) 13
- (C) 7
- (D) 19
- (E) 25

45. Let $\mathbf{F} = (17y^5, z^5, x^5)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (1, 1, 3)/\sqrt{11}$.

- (A) $-\frac{530\sqrt{11}}{11}$
- (B) $\frac{265\sqrt{11}}{11}$
- (C) $-\frac{265\sqrt{11}}{11}$
- (D) $1 - \frac{265\sqrt{11}}{11}$
- (E) $-\frac{265\sqrt{11}}{22}$

Questions 46–48

46. What is the largest integer k such that 3^k divides $87!$?

- (A) 43
- (B) 40
- (C) 41
- (D) 42
- (E) 44

47. Evaluate $\int_0^{\pi/2} \sin^{11} t \cos^{35} t \, dt$.

- (A) $2 \left(\frac{5! 17!}{2 \cdot 23!} \right)$
- (B) $\frac{1}{2} \left(\frac{5! 17!}{2 \cdot 23!} \right)$
- (C) $23 \left(\frac{5! 17!}{2 \cdot 23!} \right)$
- (D) $26334 \left(\frac{5! 17!}{2 \cdot 23!} \right)$
- (E) $\frac{5! 17!}{2 \cdot 23!}$

48. A connected planar graph has 37 vertices and 69 edges. How many faces, including the unbounded face, are in a planar embedding?

- (A) 45
- (B) 56
- (C) 34
- (D) 12
- (E) 23

Questions 49–51

49. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(17+5j)(22+5j)}$.

- (A) $\frac{1}{5}$
- (B) $\frac{1}{85}$
- (C) $\frac{5}{17}$
- (D) $\frac{1}{17}$
- (E) $\frac{1}{22}$

50. In the ring $\mathbb{Z}/492\mathbb{Z}$, how many elements satisfy $x^2 = 1$?

- (A) 6
- (B) 10
- (C) 7
- (D) 8
- (E) 9

51. The solution of $y' = 17x^5y^3$ has $y(0) = 1/18$. Find $y(1)$.

- (A) $\frac{2\sqrt{2865}}{955}$
- (B) $-\frac{\sqrt{2865}}{955}$
- (C) $\frac{\sqrt{2865}}{955} + 1$
- (D) $\frac{\sqrt{2865}}{1910}$
- (E) $\frac{\sqrt{2865}}{955}$

Questions 52–54

- 52.** How many integers in $\{1, \dots, 204\}$ are divisible by 2 or 3?
- (A) 226
 - (B) 136
 - (C) 91
 - (D) 181
 - (E) 46
- 53.** Let $F(x) = \int_0^x (x - t)^4 e^{17t} dt$. Find $F^{(5)}(0)$.
- (A) 24
 - (B) $\frac{1}{24}$
 - (C) 120
 - (D) 408
 - (E) 0
- 54.** A permutation consists of disjoint cycles of lengths 8 and 7, together with a disjoint transposition when the sum of these two lengths is odd. It fixes all other points. What is its order?
- (A) 55
 - (B) 56
 - (C) 54
 - (D) 58
 - (E) 57

Questions 55–57

55. For $x > 0$, $x^2y'' + (-38)xy' + 374y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

- (A) $\frac{(1 - e^5)e^{17}}{5}$
- (B) $-\frac{e^{17}}{5} + 1 + \frac{e^{22}}{5}$
- (C) $-\frac{(1 - e^5)e^{17}}{5}$
- (D) $-\frac{(1 - e^5)e^{17}}{10}$
- (E) $-\frac{2(1 - e^5)e^{17}}{5}$

56. Let $F_0 = 0$, $F_1 = 1$, and $F_{k+2} = F_{k+1} + F_k$. How many binary strings of length 34 have no adjacent 1s?

- (A) F_{37}
- (B) F_{38}
- (C) F_{36}
- (D) F_{34}
- (E) F_{35}

57. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{17t} \cos(5t)$, and $y = e^{17t} \sin(5t)$, find d^2f/dt^2 at $t = 1$.

- (A) 628
- (B) 34
- (C) 10
- (D) 1156
- (E) 0

Questions 58–60

58. In $R = \mathbb{F}_7[x]/(x^{17})$, how many elements are units?

- (A) $7^{19} - 7^{18}$
- (B) $7^{18} - 7^{17}$
- (C) $7^{18} - 7^{16}$
- (D) $7^{17} - 7^0$
- (E) $7^{17} - 7^{16}$

59. Evaluate $\int_0^1 \int_0^x x^{17} y^5 dy dx$.

- (A) $\frac{1}{108}$
- (B) $\frac{1}{138}$
- (C) $\frac{1}{24}$
- (D) $\frac{1}{168}$
- (E) $\frac{1}{144}$

60. For Newton iteration on $f(x) = x^2 - 1156$ near the root $r = 34$, find $\lim_{k \rightarrow \infty} (x_{k+1} - r)/(x_k - r)^2$ for nonterminating convergent iterates.

- (A) $\frac{1}{68}$
- (B) $\frac{1}{136}$
- (C) $\frac{1}{204}$
- (D) $\frac{67}{68}$
- (E) $\frac{69}{136}$

Questions 61–63

61. Let $F(x) = \int_0^{x^5} e^{17t} dt$. Find $F'(1)$.

- (A) $\frac{e^{17}}{5}$
- (B) e^{85}
- (C) e^{17}
- (D) $5e^{17}$
- (E) $85e^{17}$

62. Integers x satisfy $x \equiv 6 \pmod{7}$ and $x \equiv 3 \pmod{9}$. Find the sum of all such x with $0 \leq x < 126$.

- (A) 161
- (B) 157
- (C) 160
- (D) 159
- (E) 158

63. Evaluate $\int_0^\infty x^5 e^{-17x} dx$.

- (A) $\frac{5!}{17^6}$
- (B) $\frac{1}{17} \left(\frac{5!}{17^6} \right)$
- (C) $\frac{1}{5} \left(\frac{5!}{17^6} \right)$
- (D) 120
- (E) $17 \left(\frac{5!}{17^6} \right)$

Questions 64–66

64. Bisection starts with an interval of length 2^{34} . After 36 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{9}{16}$
- (B) $\frac{1}{24}$
- (C) $\frac{1}{8}$
- (D) $\frac{1}{16}$
- (E) $\frac{7}{8}$

65. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(17x) - \sum_{j=0}^3 (-1)^j (17x)^{2j+1} / (2j+1)!}{x^9}$.

- (A) $\frac{(-1)^4 17^9}{9!}$
- (B) $-\frac{(-1)^4 17^9}{9!}$
- (C) $17 \left(\frac{(-1)^4 17^9}{9!} \right)$
- (D) $\frac{1}{17} \left(\frac{(-1)^4 17^9}{9!} \right)$
- (E) $9 \left(\frac{(-1)^4 17^9}{9!} \right)$

66. The symmetric group S_{84} acts on the two-element subsets of $\{1, \dots, 84\}$. What is the index of the setwise stabilizer of $\{1, 2\}$?

- (A) 3484
- (B) 3488
- (C) 3486
- (D) 3485
- (E) 3487

Answer Key

Practice Test 82 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	B	23	E	45	C
2	B	24	D	46	D
3	A	25	B	47	E
4	D	26	B	48	C
5	B	27	C	49	B
6	D	28	B	50	D
7	D	29	A	51	E
8	E	30	E	52	B
9	B	31	C	53	A
10	E	32	D	54	B
11	A	33	E	55	C
12	C	34	B	56	C
13	A	35	A	57	E
14	D	36	B	58	E
15	C	37	E	59	E
16	E	38	A	60	A
17	B	39	D	61	D
18	C	40	A	62	D
19	D	41	E	63	A
20	A	42	E	64	C
21	C	43	A	65	A
22	C	44	D	66	C

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