

GRE Mathematics Practice Test 83

Upper-Intermediate Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

Intermediate reinforcement, combining standard applications with selected more demanding arguments. This is cycle 16 of 19, position 3 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Solve $y' + 17y = x^4e^{-17x}$, $y(0) = 0$, and find $y(1)$.

- (A) $\frac{1}{5e^{17}}$
- (B) $\frac{1}{4e^{17}}$
- (C) $\frac{17}{5e^{17}}$
- (D) $\frac{e^{17}}{5}$
- (E) e^{-17}

2. Let $A = \begin{bmatrix} 1 & 79 & 0 \\ 0 & 1 & 13 \\ 0 & 0 & 1 \end{bmatrix}$. What is the $(1, 3)$ entry of A^5 ?

- (A) 10271
- (B) 10270
- (C) 10269
- (D) 10272
- (E) 10268

3. The solution of $y' = 17x^4y^3$ has $y(0) = 1/18$. Find $y(1)$.

- (A) $\frac{\sqrt{7930}}{1586}$
- (B) $-\frac{\sqrt{7930}}{1586}$
- (C) $\frac{\sqrt{7930}}{1586} + 1$
- (D) $\frac{\sqrt{7930}}{793}$
- (E) $\frac{\sqrt{7930}}{3172}$

Questions 4–6

4. A uniformly random subset of 35 objects is selected from a collection with 36 red and 37 blue objects. If X is the number of red objects selected, find $E[X]$.

- (A) $\frac{2520}{73}$
- (B) $\frac{630}{73}$
- (C) $\frac{1333}{73}$
- (D) $\frac{1260}{73}$
- (E) $\frac{1406}{73}$

5. For $x > 0$, $x^2 y'' + (-37)xy' + 357y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

- (A) $-\frac{(1 - e^4) e^{17}}{8}$
- (B) $-\frac{(1 - e^4) e^{17}}{2}$
- (C) $\frac{(1 - e^4) e^{17}}{4}$
- (D) $-\frac{e^{17}}{4} + 1 + \frac{e^{21}}{4}$
- (E) $-\frac{(1 - e^4) e^{17}}{4}$

6. An invertible 85×85 matrix A has determinant 4. Find $\det(\text{adj } A)$.

- (A) $2^{168} \cdot 5$
- (B) 2^{167}
- (C) $2^{168} \cdot 3$
- (D) 2^{169}
- (E) 2^{168}

Questions 7–9

7. If $y' = 34xy$ and $y(0) = 1$, find the coefficient of x^8 in the Maclaurin series of y .

- (A) $\frac{83545}{24}$
- (B) $\frac{83521}{24}$
- (C) $\frac{83521}{48}$
- (D) $-\frac{83521}{24}$
- (E) $\frac{83521}{12}$

8. X, Y are independent Poisson variables with means 35 and 36. Given $X + Y = 37$, find $\text{Var}(X | X + Y)$.

- (A) $\frac{93240}{5041}$
- (B) $\frac{51661}{5041}$
- (C) $\frac{56702}{5041}$
- (D) $\frac{46620}{5041}$
- (E) $\frac{23310}{5041}$

9. Evaluate $\int_C (4x^3 + 17y) dx + (17x + 4y^3) dy$, where C is $r(t) = (t, 4t^2)$, $0 \leq t \leq 1$.

- (A) $\frac{325}{2}$
- (B) -325
- (C) 650
- (D) 326
- (E) 325

Questions 10–12

10. On P_{86} , the real polynomials of degree at most 86, define $T(p) = x^2 p'' - 182p$. Find its rank.

- (A) 87
- (B) 86
- (C) 84
- (D) 85
- (E) 88

11. Evaluate $\int_0^1 x^{16}(\log x)^3 dx$.

- (A) $17 \left(\frac{(-1)^3 3!}{17^4} \right)$
- (B) $\frac{(-1)^3 3!}{17^4}$
- (C) $-\frac{(-1)^3 3!}{17^4}$
- (D) $\frac{4913}{6} \left(\frac{(-1)^3 3!}{17^4} \right)$
- (E) $\frac{1}{4} \left(\frac{(-1)^3 3!}{17^4} \right)$

12. X, Y are independent, with variances 35 and 36. Find $\text{Cov}(X + 2Y, 3X - Y)$.

- (A) 22
- (B) 44
- (C) 11
- (D) 55
- (E) 33

Questions 13–15

13. Find the coefficient of x^6 in $\cos(17x)/(1 - 3x^2)$.

- (A) $-\frac{17537599}{720}$
- (B) $-\frac{24137569}{720}$
- (C) $\frac{17537599}{720}$
- (D) $-\frac{17538319}{720}$
- (E) -17537599

14. Subspaces U, V of $\mathbb{R}^{87}$ have dimensions 3 and 4. What is the smallest possible value of $\dim(U \cap V)$?

- (A) 1
- (B) 3
- (C) 0
- (D) 4
- (E) 2

15. Evaluate $\iiint_{x^2+y^2+z^2 \leq 17} (x^2 + y^2 + z^2)^3 dV$.

- (A) $\frac{9}{119} \left(\frac{4\pi 17^{\frac{9}{2}}}{9} \right)$
- (B) $9 \left(\frac{4\pi 17^{\frac{9}{2}}}{9} \right)$
- (C) $\frac{1}{3} \left(\frac{4\pi 17^{\frac{9}{2}}}{9} \right)$
- (D) $\frac{1}{2} \left(\frac{4\pi 17^{\frac{9}{2}}}{9} \right)$
- (E) $\frac{4\pi 17^{\frac{9}{2}}}{9}$

Questions 16–18

16. X has density cx^{35} on $0 < x < 1$ and zero elsewhere. Find the normalizing constant c .

- (A) 48
- (B) 12
- (C) 36
- (D) 24
- (E) 60

17. For $f(x) = x^4 e^{17x}$, find $f^{(6)}(0)$.

- (A) $\frac{6! 17^2}{2}$
- (B) $\frac{1}{360} \left(\frac{6! 17^2}{2} \right)$
- (C) $\frac{1}{30} \left(\frac{6! 17^2}{2} \right)$
- (D) $\frac{2}{17} \left(\frac{6! 17^2}{2} \right)$
- (E) $2 \left(\frac{6! 17^2}{2} \right)$

18. Let V consist of real polynomials of degree at most 10 that vanish at 79, 80, and 82. Find $\dim V$.

- (A) 8
- (B) 7
- (C) 10
- (D) 9
- (E) 6

Questions 19–21

19. For $\mathbf{F} = (17x^4y, y^4z, xz^4)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) -172
- (B) 173
- (C) 344
- (D) 172
- (E) 86

20. Independent exponential waiting times X, Y have rates 35 and 37. Find $P(X < Y)$.

- (A) $\frac{35}{72}$
- (B) $\frac{37}{72}$
- (C) $\frac{107}{144}$
- (D) $\frac{35}{216}$
- (E) $\frac{35}{144}$

21. Evaluate $\int_0^1 \int_0^x x^{17} y^4 dy dx$.

- (A) $\frac{1}{23}$
- (B) $\frac{1}{110}$
- (C) $\frac{1}{90}$
- (D) $\frac{1}{115}$
- (E) $\frac{1}{138}$

Questions 22–24

22. The roots r, s of $x^2 - 92x + 1027 = 0$ are counted with multiplicity. Find $r^3 + s^3$.

- (A) 495237
- (B) 495234
- (C) 495238
- (D) 495235
- (E) 495236

23. Evaluate $\lim_{x \rightarrow 0} \frac{e^{17x} - \sum_{j=0}^3 (17x)^j / j!}{x^4}$.

- (A) $24 \left(\frac{17^4}{4!} \right)$
- (B) $\frac{17^4}{4!}$
- (C) $-\frac{17^4}{4!}$
- (D) $4 \left(\frac{17^4}{4!} \right)$
- (E) $\frac{1}{17} \left(\frac{17^4}{4!} \right)$

24. For which real a does $f_m(x) = m^a x^{30} (1-x)^m$ converge uniformly to zero on $[0, 1]$?

- (A) $a < 0$
- (B) $a \leq 30$
- (C) All real a
- (D) $a < 30$
- (E) $a > 30$

Questions 25–27

25. Find the coefficient of x^4 in the Taylor series of $(1 - 17x)^{-2}$ at zero.

- (A) $2(5 \cdot 17^4)$
- (B) $\frac{1}{17}(5 \cdot 17^4)$
- (C) $5 \cdot 17^4$
- (D) $\frac{4}{5}(5 \cdot 17^4)$
- (E) $\frac{1}{5}(5 \cdot 17^4)$

26. For real t , let $M(t) = \begin{pmatrix} t-79 & 13 & 0 \\ 0 & t-79 & 1 \\ 0 & 0 & t-79 \end{pmatrix}$. At $t = 79$, what is $\dim \ker(M(t)^2) - \dim \ker M(t)$?

- (A) 3
- (B) 1
- (C) 0
- (D) 4
- (E) 2

27. Find the curvature of the helix $r(t) = (17 \cos t, 17 \sin t, 4t)$.

- (A) $-\frac{17}{305}$
- (B) $\frac{17}{305}$
- (C) $\frac{34}{305}$
- (D) $\frac{17}{610}$
- (E) $\frac{322}{305}$

Questions 28–30

28. For which real p does $\sum_{k=2}^{\infty} 1/[k(\log k)^{30p}]$ converge?

- (A) $p > 0$
- (B) $p \geq 1/30$
- (C) All real p
- (D) $p < 1/30$
- (E) $p > 1/30$

29. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 17x$ and has density $\rho(x, y) = x^2y$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{5}{12}$
- (B) $\frac{5}{3}$
- (C) $\frac{11}{6}$
- (D) $-\frac{5}{6}$
- (E) $\frac{5}{6}$

30. Let $u = (16, 4)^T$ and $v = (1, 0)^T$. Find the squared distance from $(16, 0)^T$ to $\text{span}(u)$.

- (A) $\frac{273}{17}$
- (B) $\frac{239}{17}$
- (C) $\frac{529}{34}$
- (D) $\frac{495}{34}$
- (E) $\frac{256}{17}$

Questions 31–33

31. Find the coefficient of x^4 in $e^{3x} \log(1 + 17x)$.

- (A) -396984
- (B) -16541
- (C) $-\frac{83521}{4}$
- (D) 16541
- (E) -16524

32. How many accumulation points in $\mathbb{R}$ does $S = \{1/k : k \geq 1\} \cup \{30 + 1/k : k \geq 1\}$ have?

- (A) 4
- (B) 2
- (C) 1
- (D) 3
- (E) 0

33. Evaluate $\int_0^{\log 2} e^x (e^x + 17)^4 dx$.

- (A) $\frac{19^5 - 18^5}{5}$
- (B) $-\frac{19^5 - 18^5}{5}$
- (C) $\frac{19^5}{5}$
- (D) $19^4 - 18^4$
- (E) $5 \left(\frac{19^5 - 18^5}{5} \right)$

Questions 34–36

34. An idempotent 85×85 real matrix P has rank 4. What is $\det(I + P)$?

- (A) 16
- (B) 15
- (C) 18
- (D) 17
- (E) 14

35. Find the upward flux of $\mathbf{F} = (x, y, z)$ through $z = 17x^2 + 4y^2$, $0 \leq x, y \leq 1$.

- (A) -6
- (B) 7
- (C) $-\frac{7}{2}$
- (D) -7
- (E) -14

36. How many connected components does $\mathbb{R} \setminus \{1, 2, \dots, 30\}$ have?

- (A) 41
- (B) 51
- (C) 21
- (D) 31
- (E) 11

Questions 37–39

37. Evaluate $\int_0^1 \frac{x^3}{1+17x^4} dx$.

- (A) $-\frac{\log(18)}{68}$
- (B) $\frac{\log(18)}{4}$
- (C) $\frac{\log(18)}{68}$
- (D) $\log(18)$
- (E) $\frac{\log(18)}{17}$

38. For $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} 79 \\ 13 \\ 92 \end{bmatrix}$, let x minimize $\|Ax - b\|^2$. Find x_2 .

- (A) 7
- (B) $\frac{15}{2}$
- (C) $\frac{13}{2}$
- (D) $\frac{11}{2}$
- (E) 6

39. Let C be the unit circle $x^2 + y^2 = 1$, $z = 4$, counterclockwise viewed from above. Evaluate $\int_C -17y(x^2 + y^2)^3 dx + (17x(x^2 + y^2)^3) dy + 4z dz$.

- (A) 17π
- (B) -34π
- (C) 68π
- (D) 34π
- (E) $1 + 34\pi$

Questions 40–42

40. What is the coefficient of z^{-1} in the Laurent expansion of e^{20z}/z^3 about zero?
- (A) 68
(B) 134
(C) 266
(D) 200
(E) 332
41. Let C be $x^2/16 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C \left(-\frac{17y^3}{3}\right) dx + \left(\frac{x^3}{3}\right) dy$.
- (A) $1 + 33\pi$
(B) $\frac{33\pi}{2}$
(C) 33π
(D) -33π
(E) 66π
42. In $\mathbb{Z}/2598\mathbb{Z}$, how many residues are not idempotent ($x^2 \neq x$)?
- (A) 2588
(B) 2591
(C) 2589
(D) 2590
(E) 2592

Questions 43–45

43. If $y' = y(1 - y/4)$ and $y(0) = 2$, find $y(-\log 17)$.

- (A) $\frac{2}{9}$
- (B) $\frac{11}{9}$
- (C) $\frac{1}{9}$
- (D) $-\frac{2}{9}$
- (E) $\frac{4}{9}$

44. An entire function satisfies $|f(z)| \leq C(1 + |z|^{20})$ for some $C > 0$. What is the largest possible degree of f ?

- (A) 20
- (B) 32
- (C) 14
- (D) 8
- (E) 26

45. Let $f(x, y) = x^2 + 34xy + (293)y^2$. What is the product of the eigenvalues of the Hessian of f ?

- (A) 8
- (B) 1172
- (C) 16
- (D) 4
- (E) 293

Questions 46–48

46. In the additive group $\mathbb{Z}/120\mathbb{Z}$, find the index of the subgroup generated by $[6]$.

- (A) 5
- (B) 7
- (C) 4
- (D) 6
- (E) 8

47. Two solutions of $y'' + 17x^4y' + 4y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $e^{-\frac{17}{5}}$
- (B) $\frac{2}{e^{\frac{17}{5}}}$
- (C) $e^{-\frac{17}{5}} + 1$
- (D) $\frac{1}{2e^{\frac{17}{5}}}$
- (E) $-\frac{1}{e^{\frac{17}{5}}}$

48. A connected planar graph has 38 vertices and 71 edges. How many faces, including the unbounded face, are in a planar embedding?

- (A) 24
- (B) 46
- (C) 13
- (D) 35
- (E) 57

Questions 49–51

49. Evaluate $\lim_{x \rightarrow 0} \frac{(1 + 17x)^{1/(3x)} - e^{17/3}}{x}$.

- (A) $-\frac{17e^{\frac{17}{3}}}{6}$
- (B) $\frac{289e^{\frac{17}{3}}}{6}$
- (C) $e^{\frac{17}{3}}$
- (D) $-\frac{289e^{\frac{17}{3}}}{3}$
- (E) $-\frac{289e^{\frac{17}{3}}}{6}$

50. How many invertible 3×3 matrices over $\mathbb{F}_{53}$ are there whose first column is $(1, 0, 0)^T$?

- (A) $\frac{(53^3 - 53^0)(53^3 - 53^0)(53^3 - 53^0)}{53^3 - 1}$
- (B) $\frac{(53^3 - 53^0)(53^3 - 53^1)}{53^3 - 1}$
- (C) $\frac{(53^3 - 53^1)(53^3 - 53^2)(53^3 - 53^3)}{53^3 - 1}$
- (D) $\frac{(53^3 - 53^0)(53^3 - 53^1)(53^3 - 53^2)}{53^3 - 1}$
- (E) $\frac{53^3 53^3 53^3}{53^3 - 1}$

51. Let $F(x) = \int_0^x (x - t)^3 e^{17t} dt$. Find $F^{(4)}(0)$.

- (A) 24
- (B) 6
- (C) $\frac{1}{6}$
- (D) 102
- (E) 0

Questions 52–54

52. How many integers in $\{1, \dots, 210\}$ are divisible by 2 or 3?

- (A) 48
- (B) 140
- (C) 232
- (D) 94
- (E) 186

53. Evaluate $\int_0^\infty x^4 e^{-17x} dx$.

- (A) $\frac{4!}{17^5}$
- (B) 24
- (C) $17 \left(\frac{4!}{17^5} \right)$
- (D) $\frac{1}{17} \left(\frac{4!}{17^5} \right)$
- (E) $\frac{1}{4} \left(\frac{4!}{17^5} \right)$

54. How many permutations in S_{85} consist of one 4-cycle and 81 fixed points?

- (A) $2 \cdot 3 \cdot 5^2 \cdot 7 \cdot 17 \cdot 41 \cdot 83$
- (B) $2 \cdot 3 \cdot 5 \cdot 7 \cdot 17 \cdot 41 \cdot 83$
- (C) $2 \cdot 3^2 \cdot 5 \cdot 7 \cdot 17 \cdot 41 \cdot 83$
- (D) $3 \cdot 5 \cdot 7 \cdot 17 \cdot 41 \cdot 83$
- (E) $2^2 \cdot 3 \cdot 5 \cdot 7 \cdot 17 \cdot 41 \cdot 83$

Questions 55–57

55. Find the directional derivative of $f(x, y, z) = x^2y + 4xz^2 + 17y^2z$ at $(1, 1, 1)$ in the direction $(1, 4, 0)$.

- (A) $\frac{73\sqrt{17}}{17}$
- (B) $\frac{146\sqrt{17}}{17}$
- (C) $1 + \frac{146\sqrt{17}}{17}$
- (D) $\frac{292\sqrt{17}}{17}$
- (E) $-\frac{146\sqrt{17}}{17}$

56. Let $F_0 = 0$, $F_1 = 1$, and $F_{k+2} = F_{k+1} + F_k$. How many binary strings of length 35 have no adjacent 1s?

- (A) F_{38}
- (B) F_{36}
- (C) F_{35}
- (D) F_{39}
- (E) F_{37}

57. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{17t} \cos(4t)$, and $y = e^{17t} \sin(4t)$, find d^2f/dt^2 at $t = 1$.

- (A) 0
- (B) 34
- (C) 610
- (D) 1156
- (E) 8

Questions 58–60

58. How many maximal proper subgroups does the cyclic group of order 996 have?

- (A) 1
- (B) 4
- (C) 3
- (D) 2
- (E) 5

59. Find the arc length of $r(t) = (17(t - t^3/3), 17t^2)$ for $0 \leq t \leq 1/4$.

- (A) $-\frac{833}{192}$
- (B) $\frac{1025}{192}$
- (C) $\frac{833}{192}$
- (D) $\frac{384}{833}$
- (E) $\frac{833}{96}$

60. Apply one Newton iteration to solve $x^2 - 35 = 0$, starting at $x_0 = 1$. What is x_1 ?

- (A) 6
- (B) 24
- (C) 30
- (D) 12
- (E) 18

Questions 61–63

61. What is the maximum of x^4y^{17} over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $21 \left(\left(\frac{4}{21} \right)^4 \left(\frac{17}{21} \right)^{17} \right)$
- (B) $\left(\frac{4}{21} \right)^{17} \left(\frac{17}{21} \right)^4$
- (C) $\left(\frac{4}{21} \right)^4 \left(\frac{17}{21} \right)^{17}$
- (D) $\frac{1}{2^{21}}$
- (E) $\frac{68}{441}$

62. How many residues x modulo 120 satisfy $4x \equiv 4 \pmod{120}$ and $2x \equiv 0 \pmod{120}$?

- (A) 2
- (B) 1
- (C) 0
- (D) 4
- (E) 3

63. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j(x-17)^j}{j^1(\log j)^m}$ converge absolutely at $x = 16$? Give the least such m , or 0 if none exists.

- (A) 4
- (B) 1
- (C) 2
- (D) 0
- (E) 3

Questions 64–66

64. Bisection starts with an interval of length 2^{35} . After 37 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{1}{24}$
- (B) $\frac{7}{8}$
- (C) $\frac{9}{16}$
- (D) $\frac{1}{16}$
- (E) $\frac{1}{8}$

65. For $D = \{(x, y) : x^2 + y^2 \leq 17\}$, evaluate $\iint_D (x^2 + y^2)^3 dA$.

- (A) $\frac{83521\pi}{2}$
- (B) 83521π
- (C) $\frac{83521\pi}{4}$
- (D) $\frac{83521\pi}{7}$
- (E) $\frac{4913\pi}{4}$

66. The field K has 5^{17} elements. Find the largest possible multiplicative order of a nonzero element of K .

- (A) $5^{18} - 1$
- (B) $5^{17} - 1$
- (C) $5^{16} - 1$
- (D) $5^{17} + 1$
- (E) 5^{17}

Answer Key

Practice Test 83 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	A	23	B	45	C
2	B	24	D	46	D
3	A	25	C	47	A
4	D	26	B	48	D
5	E	27	B	49	E
6	E	28	E	50	D
7	B	29	E	51	B
8	D	30	E	52	B
9	E	31	B	53	A
10	B	32	B	54	B
11	B	33	A	55	B
12	E	34	A	56	E
13	A	35	D	57	A
14	C	36	D	58	C
15	E	37	C	59	C
16	C	38	C	60	E
17	A	39	D	61	C
18	A	40	D	62	C
19	D	41	C	63	C
20	A	42	D	64	E
21	D	43	A	65	C
22	E	44	A	66	B

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