

GRE Mathematics Practice Test 84

Bridge to Standard

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A bridge toward standard practice, with routine multi-step questions and a smaller number of challenging concepts. This is cycle 16 of 19, position 4 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. For the curve $x = t^3$, $y = t^4 + 17t$, find d^2y/dx^2 at $t = 1$.

- (A) $-\frac{17}{3}$
- (B) 7
- (C) $\frac{38}{9}$
- (D) -10
- (E) $-\frac{10}{3}$

2. The roots r, s of $x^2 - 91x + 880 = 0$ are counted with multiplicity. Find $r^2 + s^2$.

- (A) 6520
- (B) 6522
- (C) 6519
- (D) 6523
- (E) 6521

3. Evaluate $\int_0^1 x^{17}(1-x)^2 dx$.

- (A) $20 \left(\frac{17! 2!}{20!} \right)$
- (B) $\frac{190}{3} \left(\frac{17! 2!}{20!} \right)$
- (C) $\frac{1}{3} \left(\frac{17! 2!}{20!} \right)$
- (D) $\frac{17! 2!}{20!}$
- (E) $18 \left(\frac{17! 2!}{20!} \right)$

Questions 4–6

4. A uniformly random subset of 36 objects is selected from a collection with 37 red and 38 blue objects. If X is the number of red objects selected, find $E[X]$.

- (A) $\frac{494}{25}$
- (B) $\frac{444}{25}$
- (C) $\frac{469}{25}$
- (D) $\frac{222}{25}$
- (E) $\frac{888}{25}$

5. Evaluate $\int_0^1 \frac{x^2}{1 + 17x^3} dx$.

- (A) $\frac{\log(18)}{17}$
- (B) $\frac{\log(18)}{3}$
- (C) $\log(18)$
- (D) $-\frac{\log(18)}{51}$
- (E) $\frac{\log(18)}{51}$

6. For real t , let $M(t) = \begin{pmatrix} t - 80 & 11 & 0 \\ 0 & t - 80 & 1 \\ 0 & 0 & t - 80 \end{pmatrix}$. At $t = 80$, what is $\text{rank } M(t)$?

- (A) 2
- (B) 1
- (C) 0
- (D) 4
- (E) 3

Questions 7–9

7. For $f(x, y) = x^2 + 17xy + 3y^2$, what is its maximum directional derivative at $(1, -1)$?
- (A) $\sqrt{346}$
(B) 26
(C) 11
(D) -15
(E) 346
8. X, Y are independent Poisson variables with means 36 and 37. Given $X + Y = 38$, find $\text{Var}(X \mid X + Y)$.
- (A) $\frac{25308}{5329}$
(B) $\frac{101232}{5329}$
(C) $\frac{55945}{5329}$
(D) $\frac{61274}{5329}$
(E) $\frac{50616}{5329}$
9. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 17z^2 = x + 3y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.
- (A) -34
(B) -51
(C) 102
(D) -3
(E) -102

Questions 10–12

10. Let $u = (16, 3)^T$ and $v = (1, 0)^T$. Find the squared norm of the orthogonal projection of v onto $\text{span}(u)$.

- (A) $\frac{521}{265}$
- (B) $\frac{1307}{530}$
- (C) $\frac{256}{265}$
- (D) $\frac{777}{530}$
- (E) $\frac{247}{530}$

11. For $y' = 17y(1 - y/3)$, what is the derivative of the right-hand side with respect to y at the positive equilibrium?

- (A) -16
- (B) -17
- (C) 17
- (D) $-\frac{17}{2}$
- (E) -34

12. X, Y are independent, with variances 36 and 37. Find $\text{Cov}(X + 2Y, 3X - Y)$.

- (A) 34
- (B) 56
- (C) 23
- (D) 45
- (E) 12

Questions 13–15

13. Let C be the unit circle $x^2 + y^2 = 1$, $z = 3$, counterclockwise viewed from above. Evaluate $\int_C -17y(x^2 + y^2)^2 dx + (17x(x^2 + y^2)^2) dy$.

- (A) 34π
- (B) 17π
- (C) $1 + 34\pi$
- (D) -34π
- (E) 68π

14. An idempotent 86×86 real matrix P has rank 5. What is $\text{tr } P$?

- (A) 4
- (B) 6
- (C) 5
- (D) 3
- (E) 7

15. Evaluate $\int_0^{\log 2} e^x(e^x + 17)^3 dx$.

- (A) $19^3 - 18^3$
- (B) $\frac{19^4 - 18^4}{4}$
- (C) $\frac{19^4}{4}$
- (D) $-\frac{19^4 - 18^4}{4}$
- (E) $4 \left(\frac{19^4 - 18^4}{4} \right)$

Questions 16–18

16. X has density cx^{36} on $0 < x < 1$ and zero elsewhere. Find the normalizing constant c .

- (A) 61
- (B) 13
- (C) 37
- (D) 49
- (E) 25

17. Evaluate $\iiint_B 17(x^2 + y^2 + z^2)^{3/2} dV$, where B is the unit ball.

- (A) $\frac{34\pi}{3}$
- (B) $-\frac{34\pi}{3}$
- (C) $1 + \frac{34\pi}{3}$
- (D) $\frac{17\pi}{3}$
- (E) $\frac{68\pi}{3}$

18. For $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $b = \begin{bmatrix} 80 \\ 11 \\ 91 \end{bmatrix}$, let x minimize $\|Ax - b\|^2$. Find x_2 .

- (A) 13
- (B) 12
- (C) 10
- (D) 11
- (E) 9

Questions 19–21

19. If $y' = 17y$, $y(0) = 1$, what is the coefficient of x^3 in the Maclaurin series of y ?
- (A) $\frac{4913}{6}$
(B) $-\frac{4913}{6}$
(C) $\frac{4913}{12}$
(D) $\frac{4919}{6}$
(E) $\frac{4913}{3}$
20. Independent exponential waiting times X, Y have rates 36 and 38. Find $P(X < Y)$.
- (A) $\frac{9}{37}$
(B) $\frac{18}{37}$
(C) $\frac{6}{37}$
(D) $\frac{19}{37}$
(E) $\frac{55}{74}$
21. The region $0 \leq x \leq 1$, $0 \leq y \leq 17x^3$ is revolved about the x -axis. Find its volume.
- (A) $\frac{289\pi}{7}$
(B) $\frac{578\pi}{7}$
(C) $-\frac{289\pi}{7}$
(D) $\frac{289\pi}{14}$
(E) $1 + \frac{289\pi}{7}$

Questions 22–24

22. What is the coefficient of x^3 in $(x + 16)^5$?

- (A) 2559
- (B) 2561
- (C) 2562
- (D) 2558
- (E) 2560

23. Evaluate $\int_0^1 \int_0^x x^{17} y^3 dy dx$.

- (A) $\frac{1}{110}$
- (B) $\frac{1}{72}$
- (C) $\frac{1}{84}$
- (D) $\frac{1}{22}$
- (E) $\frac{1}{88}$

24. For which real a does $f_m(x) = x^{31m}/m^a$ converge uniformly to zero on $[0, 1]$?

- (A) $a > 0$
- (B) $a < 0$
- (C) $a > 1$
- (D) All real a
- (E) $a = 0$

Questions 25–27

25. Find the coefficient of x^6 in $\cos(17x)/(1 - 2x^2)$.

- (A) $-\frac{19536709}{720}$
- (B) -19536709
- (C) $\frac{19536709}{720}$
- (D) $-\frac{19537429}{720}$
- (E) $-\frac{24137569}{720}$

26. A matrix A satisfies $(A - 81I)(A - 83I) = 0$, and its minimal polynomial has degree two. If $A^2 = uA + vI$, find u .

- (A) 166
- (B) 165
- (C) 162
- (D) 163
- (E) 164

27. Find the arc length of $r(t) = (17t, 3t)$ for $0 \leq t \leq 1$.

- (A) $\sqrt{298}$
- (B) $-\sqrt{298}$
- (C) $1 + \sqrt{298}$
- (D) $2\sqrt{298}$
- (E) $\frac{\sqrt{298}}{2}$

Questions 28–30

28. For which real p does $\sum_{k=2}^{\infty} 1/[k(\log k)^{31p}]$ converge?

- (A) $p > 0$
- (B) All real p
- (C) $p \geq 1/31$
- (D) $p > 1/31$
- (E) $p < 1/31$

29. Let $F(x) = \int_0^x (x-t)^2 e^{17t} dt$. Find $F^{(3)}(0)$.

- (A) 34
- (B) 2
- (C) $\frac{1}{2}$
- (D) 0
- (E) 6

30. Let $V = \begin{bmatrix} 1 & 80 & 6400 \\ 1 & 82 & 6724 \\ 1 & 85 & 7225 \end{bmatrix}$. Find $\det V$.

- (A) 29
- (B) 31
- (C) 32
- (D) 30
- (E) 28

Questions 31–33

31. The solution of $y' = 17x^3y^2$ has $y(0) = 1/18$. Find $y(1)$.

- (A) $\frac{8}{55}$
- (B) $\frac{59}{55}$
- (C) $-\frac{4}{55}$
- (D) $\frac{4}{55}$
- (E) $\frac{2}{55}$

32. How many accumulation points in $\mathbb{R}$ does $S = \{1/k : k \geq 1\} \cup \{31 + 1/k : k \geq 1\}$ have?

- (A) 0
- (B) 2
- (C) 3
- (D) 4
- (E) 1

33. Evaluate $\lim_{x \rightarrow 0} \frac{e^{17x} - \sum_{j=0}^2 (17x)^j / j!}{x^3}$.

- (A) $-\frac{17^3}{3!}$
- (B) $3 \left(\frac{17^3}{3!} \right)$
- (C) $\frac{17^3}{3!}$
- (D) $\frac{1}{17} \left(\frac{17^3}{3!} \right)$
- (E) $6 \left(\frac{17^3}{3!} \right)$

Questions 34–36

34. Let $u = (1, 2, \dots, 86)^T$, $v = (1, 1, \dots, 1)^T$, and $B = I + uv^T$. Find $\det B$.

- (A) 3740
- (B) 3743
- (C) 3741
- (D) 3744
- (E) 3742

35. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^{17}y^2 dV$.

- (A) $22 \left(\frac{17! 2!}{22!} \right)$
- (B) $6 \left(\frac{17! 2!}{22!} \right)$
- (C) $\frac{17! 2!}{22!}$
- (D) $\frac{1}{54}$
- (E) $\frac{1}{6} \left(\frac{17! 2!}{22!} \right)$

36. How many connected components does $\mathbb{R} \setminus \{1, 2, \dots, 31\}$ have?

- (A) 12
- (B) 22
- (C) 32
- (D) 52
- (E) 42

Questions 37–39

37. Find the directional derivative of $f(x, y, z) = x^2 + 17y^2 + 3z^2$ at $(1, 1, 1)$ in the direction $(1, 5, 0)$.

- (A) $\frac{43\sqrt{26}}{13}$
- (B) $\frac{86\sqrt{26}}{13}$
- (C) $1 + \frac{86\sqrt{26}}{13}$
- (D) $\frac{172\sqrt{26}}{13}$
- (E) $-\frac{86\sqrt{26}}{13}$

38. Let D be a diagonal 86×86 real matrix with distinct diagonal entries. What is the dimension of the space of matrices X satisfying $XD = DX$?

- (A) 87
- (B) 84
- (C) 88
- (D) 86
- (E) 85

39. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{17t} \cos(3t)$, and $y = e^{17t} \sin(3t)$, find d^2f/dt^2 at $t = 1$.

- (A) 34
- (B) 0
- (C) 1156
- (D) 596
- (E) 6

Questions 40–42

40. What is the coefficient of z^{-1} in the Laurent expansion of e^{21z}/z^3 about zero?

- (A) $\frac{441}{4}$
- (B) $\frac{441}{2}$
- (C) $\frac{443}{2}$
- (D) 441
- (E) $\frac{445}{2}$

41. Two solutions of $y'' + 17x^1y' + 3y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $-\frac{1}{e^{\frac{17}{2}}}$
- (B) $e^{-\frac{17}{2}}$
- (C) $e^{-\frac{17}{2}} + 1$
- (D) $\frac{2}{e^{\frac{17}{2}}}$
- (E) $\frac{1}{2e^{\frac{17}{2}}}$

42. What is the remainder when $x^2 + 1$ is divided by $x - 81$?

- (A) 6561
- (B) 6562
- (C) 6560
- (D) 6563
- (E) 6564

Questions 43–45

43. Evaluate $\int_C (3x^2 + 17y) dx + (17x + 3y^2) dy$, where C is the line segment from $(0, 0)$ to $(5, 1)$.

- (A) -211
- (B) 422
- (C) 211
- (D) $\frac{211}{2}$
- (E) 212

44. An entire function satisfies $|f(z)| \leq C(1 + |z|^{21})$ for some $C > 0$. What is the largest possible degree of f ?

- (A) 35
- (B) 21
- (C) 7
- (D) 14
- (E) 28

45. If $y' + 17y = 3e^{-17x}$ and $y(0) = 5$, find $y(1)$.

- (A) $\frac{16}{e^{17}}$
- (B) $\frac{8}{e^{17}} + 1$
- (C) $\frac{8}{e^{17}}$
- (D) $\frac{4}{e^{17}}$
- (E) $-\frac{8}{e^{17}}$

Questions 46–48

46. Let $G = \mathbb{Z}/510\mathbb{Z} \times \mathbb{Z}/344\mathbb{Z}$. Find the order of $(1, 1)$.

- (A) 87719
- (B) 87718
- (C) 87720
- (D) 87722
- (E) 87721

47. For $x > 0$, $xy' = 17y$ and $y(1) = 1$. Find $y(e^3)$.

- (A) $1 + e^{51}$
- (B) $\frac{e^{51}}{2}$
- (C) e^{51}
- (D) $2e^{51}$
- (E) $-e^{51}$

48. A connected planar graph has 39 vertices and 73 edges. How many faces, including the unbounded face, are in a planar embedding?

- (A) 12
- (B) 36
- (C) 48
- (D) 24
- (E) 60

Questions 49–51

49. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(17j)!}{(j!)^{17}} x^{3j}$?

- (A) $\frac{1}{17}$
- (B) $17^{-\frac{3}{17}}$
- (C) 17^{-17}
- (D) $17^{-\frac{17}{3}}$
- (E) $17^{\frac{17}{3}}$

50. How many group homomorphisms $\mathbb{Z}/510\mathbb{Z} \rightarrow \mathbb{Z}/344\mathbb{Z}$ are there?

- (A) 0
- (B) 3
- (C) 2
- (D) 4
- (E) 1

51. Evaluate $\lim_{x \rightarrow 0} (1 + 17x)^{1/(2x)}$.

- (A) 17
- (B) $e^{\frac{17}{2}}$
- (C) e^{289}
- (D) e^{18}
- (E) $e^{\frac{2}{17}}$

Questions 52–54

52. How many integers in $\{1, \dots, 216\}$ are divisible by 2 or 3?

- (A) 240
- (B) 192
- (C) 96
- (D) 144
- (E) 48

53. For $D = \{(x, y) : x^2 + y^2 \leq 17\}$, evaluate $\iint_D (x^2 + y^2)^2 dA$.

- (A) $\frac{9826\pi}{3}$
- (B) $\frac{4913\pi}{5}$
- (C) 4913π
- (D) $\frac{289\pi}{3}$
- (E) $\frac{4913\pi}{3}$

54. How many 1-dimensional subspaces does $\mathbb{F}_{53}^4$ have?

- (A) $\frac{53^6 - 1}{53 - 1}$
- (B) $\frac{53^3 - 1}{53 - 1}$
- (C) $\frac{53^5 - 1}{53 - 1}$
- (D) $\frac{53^7 - 1}{53 - 1}$
- (E) $\frac{53^4 - 1}{53 - 1}$

Questions 55–57

55. For $x > 0$, $x^2y'' + (1 - 20)xy' + 51y = 0$, $y(1) = 0$, $y'(1) = 17$. Find $y(e)$.

- (A) $\frac{e^{17}}{20}$
- (B) $-e^3 + e^{17}$
- (C) $\frac{17(1 - e^{14})e^3}{14}$
- (D) $-\frac{17(1 - e^{14})e^3}{14}$
- (E) $e^3 + e^{17}$

56. Let $F_0 = 0$, $F_1 = 1$, and $F_{k+2} = F_{k+1} + F_k$. How many binary strings of length 36 have no adjacent 1s?

- (A) F_{39}
- (B) F_{37}
- (C) F_{38}
- (D) F_{40}
- (E) F_{36}

57. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(17 + 3j)(20 + 3j)}$.

- (A) $\frac{3}{17}$
- (B) $\frac{1}{3}$
- (C) $\frac{1}{17}$
- (D) $\frac{1}{51}$
- (E) $\frac{1}{20}$

Questions 58–60

58. How many monic irreducible polynomials of degree 2 are there over $\mathbb{F}_{53}$?

- (A) 1380
- (B) 1379
- (C) 1378
- (D) 1376
- (E) 1377

59. For $f(x) = x^3 e^{17x}$, find $f^{(5)}(0)$.

- (A) $\frac{5! 17^2}{2}$
- (B) $\frac{1}{20} \left(\frac{5! 17^2}{2} \right)$
- (C) $2 \left(\frac{5! 17^2}{2} \right)$
- (D) $\frac{2}{17} \left(\frac{5! 17^2}{2} \right)$
- (E) $\frac{1}{60} \left(\frac{5! 17^2}{2} \right)$

60. Apply one Newton iteration to solve $x^2 - 36 = 0$, starting at $x_0 = 1$. What is x_1 ?

- (A) $\frac{37}{2}$
- (B) $\frac{37}{4}$
- (C) $\frac{39}{2}$
- (D) 37
- (E) $\frac{41}{2}$

Questions 61–63

61. Find the coefficient of x^3 in $e^{2x} \log(1 + 17x)$.

- (A) $\frac{4913}{3}$
- (B) $\frac{4199}{3}$
- (C) 8296
- (D) $-\frac{4148}{3}$
- (E) $\frac{4148}{3}$

62. How many elements z of $\mathbb{F}_{3^{17}}$ satisfy $z^3 = z$?

- (A) 1
- (B) 2
- (C) 4
- (D) 3
- (E) 5

63. Let $F(x) = \int_0^{x^3} e^{17t} dt$. Find $F'(1)$.

- (A) e^{51}
- (B) e^{17}
- (C) $51e^{17}$
- (D) $3e^{17}$
- (E) $\frac{e^{17}}{3}$

Questions 64–66

64. Bisection starts with an interval of length 2^{36} . After 38 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{1}{8}$
- (B) $\frac{1}{16}$
- (C) $\frac{7}{8}$
- (D) $\frac{9}{16}$
- (E) $\frac{1}{24}$

65. Let $f(x) = \log(1 + 17x)$. Find $f^{(3)}(0)$.

- (A) $\frac{1}{17} \left((-1)^2 2! 17^3 \right)$
- (B) $\frac{1}{3} \left((-1)^2 2! 17^3 \right)$
- (C) $-(-1)^2 2! 17^3$
- (D) $(-1)^2 2! 17^3$
- (E) $3 \left((-1)^2 2! 17^3 \right)$

66. How many generators does the cyclic group of order 53 have?

- (A) 50
- (B) 51
- (C) 53
- (D) 54
- (E) 52

Answer Key

Practice Test 84 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	E	23	E	45	C
2	E	24	A	46	C
3	D	25	A	47	C
4	B	26	E	48	B
5	E	27	A	49	D
6	A	28	D	50	C
7	A	29	B	51	B
8	E	30	D	52	D
9	E	31	D	53	E
10	C	32	B	54	E
11	B	33	C	55	D
12	A	34	E	56	C
13	A	35	C	57	D
14	C	36	C	58	C
15	B	37	B	59	A
16	C	38	D	60	A
17	A	39	B	61	E
18	D	40	B	62	D
19	A	41	B	63	D
20	B	42	B	64	A
21	A	43	C	65	D
22	E	44	B	66	E

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