

GRE Mathematics Practice Test 85

Standard-Level Target

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A mixed standard-level target, intended to approximate public GRE Mathematics practice in scope and overall demand. This is cycle 16 of 19, position 5 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Two solutions of $y'' + 17x^1y' + 2y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $\frac{2}{e^{\frac{17}{2}}}$
- (B) $e^{-\frac{17}{2}} + 1$
- (C) $e^{-\frac{17}{2}}$
- (D) $\frac{1}{2e^{\frac{17}{2}}}$
- (E) $-\frac{1}{e^{\frac{17}{2}}}$

2. What is the coefficient of x^2 in $(x + 16)^4$?

- (A) 1534
- (B) 1536
- (C) 1538
- (D) 1535
- (E) 1537

3. Evaluate $\lim_{x \rightarrow 0} (1 + 17x)^{1/(1x)}$.

- (A) e^{17}
- (B) $e^{\frac{1}{17}}$
- (C) e^{289}
- (D) e^{18}
- (E) 17

Questions 4–6

4. A uniformly random subset of 37 objects is selected from a collection with 38 red and 39 blue objects. If X is the number of red objects selected, find $E[X]$.

- (A) $\frac{1406}{77}$
- (B) $\frac{1560}{77}$
- (C) $\frac{2812}{77}$
- (D) $\frac{1483}{77}$
- (E) $\frac{703}{77}$

5. Find the coefficient of x^2 in the Taylor series of $(1 - 17x)^{-2}$ at zero.

- (A) $\frac{2}{3}(3 \cdot 17^2)$
- (B) $3 \cdot 17^2$
- (C) $\frac{1}{17}(3 \cdot 17^2)$
- (D) $2(3 \cdot 17^2)$
- (E) $\frac{1}{3}(3 \cdot 17^2)$

6. A matrix A satisfies $(A - 82I)(A - 84I) = 0$, and its minimal polynomial has degree two. If $A^2 = uA + vI$, find u .

- (A) 165
- (B) 166
- (C) 164
- (D) 167
- (E) 168

Questions 7–9

7. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 17x$ and has density $\rho(x, y) = 1$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{4}{3}$
- (B) $\frac{5}{3}$
- (C) $\frac{2}{3}$
- (D) $\frac{1}{3}$
- (E) $-\frac{2}{3}$

8. X, Y are independent Poisson variables with means 37 and 38. Given $X+Y = 39$, find $E[X \mid X+Y]$.

- (A) $\frac{506}{25}$
- (B) $\frac{531}{25}$
- (C) $\frac{962}{25}$
- (D) $\frac{481}{25}$
- (E) $\frac{481}{50}$

9. For $f(x, y) = x^2 + 17xy + 2y^2$, what is its maximum directional derivative at $(1, -1)$?

- (A) -15
- (B) 394
- (C) $\sqrt{394}$
- (D) 28
- (E) 13

Questions 10–12

10. Let $V = \begin{bmatrix} 1 & 81 & 6561 \\ 1 & 83 & 6889 \\ 1 & 86 & 7396 \end{bmatrix}$. Find $\det V$.

- (A) 29
- (B) 30
- (C) 31
- (D) 28
- (E) 32

11. For $x > 0$, $xy' = 17y$ and $y(1) = 1$. Find $y(e^2)$.

- (A) $2e^{34}$
- (B) $-e^{34}$
- (C) $1 + e^{34}$
- (D) e^{34}
- (E) $\frac{e^{34}}{2}$

12. X, Y are independent, with variances 37 and 38. Find $\text{Cov}(X + 2Y, 3X - Y)$.

- (A) 35
- (B) 46
- (C) 24
- (D) 13
- (E) 57

Questions 13–15

13. If $y' = 17y$, $y(0) = 1$, what is the coefficient of x^2 in the Maclaurin series of y ?

- (A) $-\frac{289}{2}$
- (B) $\frac{291}{2}$
- (C) $\frac{289}{2}$
- (D) 289
- (E) $\frac{289}{4}$

14. Let $u = (1, 2, \dots, 87)^T$, $v = (1, 1, \dots, 1)^T$, and $B = I + uv^T$. Find $\det B$.

- (A) 3831
- (B) 3828
- (C) 3827
- (D) 3830
- (E) 3829

15. Evaluate $\int_0^1 x^{17}(1-x)^1 dx$.

- (A) $\frac{17! 1!}{19!}$
- (B) $18 \left(\frac{17! 1!}{19!} \right)$
- (C) $19 \left(\frac{17! 1!}{19!} \right)$
- (D) $\frac{1}{2} \left(\frac{17! 1!}{19!} \right)$
- (E) $\frac{19}{2} \left(\frac{17! 1!}{19!} \right)$

Questions 16–18

16. X has density cx^{37} on $0 < x < 1$ and zero elsewhere. Find the normalizing constant c .

- (A) 50
- (B) 26
- (C) 14
- (D) 38
- (E) 62

17. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{17t} \cos(2t)$, and $y = e^{17t} \sin(2t)$, find d^2f/dt^2 at $t = 1$.

- (A) 0
- (B) 586
- (C) 34
- (D) 4
- (E) 1156

18. Let D be a diagonal 87×87 real matrix with distinct diagonal entries. What is the dimension of the space of matrices X satisfying $XD = DX$?

- (A) 85
- (B) 88
- (C) 89
- (D) 86
- (E) 87

Questions 19–21

19. The function $y(x)$ satisfies $y^3 + 17y = 1x$ and $y(0) = 0$. Find $y'(0)$.

- (A) $\frac{1}{20}$
- (B) $\frac{1}{17}$
- (C) 0
- (D) $-\frac{1}{17}$
- (E) 17

20. Independent exponential waiting times X, Y have rates 37 and 39. Find $P(X < Y)$.

- (A) $\frac{37}{228}$
- (B) $\frac{37}{76}$
- (C) $\frac{113}{152}$
- (D) $\frac{39}{76}$
- (E) $\frac{37}{152}$

21. For $x > 0$, $x^2y'' + (1 - 19)xy' + 34y = 0$, $y(1) = 0$, $y'(1) = 17$. Find $y(e)$.

- (A) $-\frac{17(1 - e^{15})e^2}{15}$
- (B) $e^2 + e^{17}$
- (C) $\frac{17(1 - e^{15})e^2}{15}$
- (D) $-e^2 + e^{17}$
- (E) $\frac{e^{17}}{19}$

Questions 22–24

22. What is the remainder when $x^2 + 1$ is divided by $x - 82$?
- (A) 6726
(B) 6723
(C) 6724
(D) 6725
(E) 6727
23. What is the maximum of x^2y^{17} over $x, y \geq 0$ subject to $x + y = 1$?
- (A) $\left(\frac{2}{19}\right)^2 \left(\frac{17}{19}\right)^{17}$
(B) $\frac{34}{361}$
(C) $19 \left(\left(\frac{2}{19}\right)^2 \left(\frac{17}{19}\right)^{17} \right)$
(D) $\frac{1}{2^{19}}$
(E) $\left(\frac{2}{19}\right)^{17} \left(\frac{17}{19}\right)^2$
24. For which real a does $f_m(x) = x^{32m}/m^a$ converge uniformly to zero on $[0, 1]$?
- (A) $a > 0$
(B) $a > 1$
(C) $a = 0$
(D) $a < 0$
(E) All real a

Questions 25–27

25. Under $x = u + 17v$, $y = u - 2v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) $\frac{1}{19}$
- (B) 19
- (C) 361
- (D) 34
- (E) 15

26. Let $P = \text{diag}(\underbrace{1, \dots, 1}_1, \underbrace{0, \dots, 0}_{87})$. What is the dimension of the space of 88×88 real matrices X satisfying $PX = X$?

- (A) 90
- (B) 88
- (C) 89
- (D) 86
- (E) 87

27. Let $\mathbf{F} = (17y^2, z^2, x^2)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (0, 0, 1)$.

- (A) -34
- (B) 34
- (C) -17
- (D) -68
- (E) -33

Questions 28–30

28. For which real p does $\sum_{k=2}^{\infty} 1/[k(\log k)^{32p}]$ converge?

- (A) All real p
- (B) $p > 0$
- (C) $p \geq 1/32$
- (D) $p < 1/32$
- (E) $p > 1/32$

29. Evaluate $\int_0^1 \int_0^x x^{17} y^2 dy dx$.

- (A) $\frac{1}{84}$
- (B) $\frac{1}{60}$
- (C) $\frac{1}{21}$
- (D) $\frac{1}{63}$
- (E) $\frac{1}{54}$

30. A 3×3 matrix A has characteristic polynomial $(t - 81)(t + 1)(t - 2)$. What is $\text{tr}(A^2)$?

- (A) 6568
- (B) 6565
- (C) 6564
- (D) 6567
- (E) 6566

Questions 31–33

31. Evaluate $\int_C (2x + 17y) dx + (17x + 2y) dy$, where C is the line segment from $(0, 0)$ to $(6, 1)$.

- (A) 278
- (B) 140
- (C) $\frac{139}{2}$
- (D) -139
- (E) 139

32. How many accumulation points in $\mathbb{R}$ does $S = \{1/k : k \geq 1\} \cup \{32 + 1/k : k \geq 1\}$ have?

- (A) 1
- (B) 2
- (C) 3
- (D) 0
- (E) 4

33. Let $f(x, y) = x^2 + 34xy + (291)y^2$. What is the product of the eigenvalues of the Hessian of f ?

- (A) 1164
- (B) 2
- (C) 291
- (D) 8
- (E) 4

Questions 34–36

34. Let N be the 87×87 matrix with ones immediately above the diagonal and zeros elsewhere. Find $\text{rank } N^{14}$.

- (A) 72
- (B) 75
- (C) 74
- (D) 73
- (E) 71

35. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(17j)!}{(j!)^{17}} x^{2j}$?

- (A) $17^{\frac{17}{2}}$
- (B) 17^{-17}
- (C) $17^{-\frac{2}{17}}$
- (D) $\frac{1}{17}$
- (E) $17^{-\frac{17}{2}}$

36. How many connected components does $\mathbb{R} \setminus \{1, 2, \dots, 32\}$ have?

- (A) 55
- (B) 11
- (C) 33
- (D) 44
- (E) 22

Questions 37–39

37. Let $F(x) = \int_0^x (x-t)^1 e^{17t} dt$. Find $F^{(2)}(0)$.

- (A) 0
- (B) 17
- (C) 1
- (D) 2
- (E) $\frac{1}{2}$

38. Let A be the 87×87 matrix whose diagonal entries are 84 and whose off-diagonal entries are 1. What is its smallest eigenvalue?

- (A) 81
- (B) 85
- (C) 82
- (D) 84
- (E) 83

39. If $y' + 17y = 2e^{-17x}$ and $y(0) = 6$, find $y(1)$.

- (A) $-\frac{8}{e^{17}}$
- (B) $\frac{4}{e^{17}}$
- (C) $\frac{8}{e^{17}}$
- (D) $\frac{16}{e^{17}}$
- (E) $\frac{8}{e^{17}} + 1$

Questions 40–42

40. What is the coefficient of z^{-1} in the Laurent expansion of e^{22z}/z^3 about zero?

- (A) 82
- (B) 322
- (C) 402
- (D) 162
- (E) 242

41. Evaluate $\int_0^1 \int_x^1 17y^2 \, dy \, dx$.

- (A) $\frac{17}{3}$
- (B) $\frac{17}{2}$
- (C) $\frac{17}{4}$
- (D) $\frac{17}{8}$
- (E) $\frac{17}{12}$

42. Positive real numbers x, y satisfy $\log_2 x + \log_2 y = \log_2 729$ and $x + y = 90$. Find xy .

- (A) 731
- (B) 728
- (C) 730
- (D) 729
- (E) 727

Questions 43–45

43. If $y'' + 4y = 0$, $y(0) = 17$, and $y'(0) = 6$, find $y(\pi/4)$.

- (A) 6
- (B) -3
- (C) $\frac{3}{2}$
- (D) 4
- (E) 3

44. An entire function satisfies $|f(z)| \leq 22$ for all complex z . What is $f'(1)$?

- (A) 0
- (B) 2
- (C) 4
- (D) 3
- (E) 1

45. Evaluate $\int_0^\infty x^2 e^{-17x} dx$.

- (A) $\frac{1}{2} \left(\frac{2!}{17^3} \right)$
- (B) $4913 \left(\frac{2!}{17^3} \right)$
- (C) $\frac{1}{17} \left(\frac{2!}{17^3} \right)$
- (D) $\frac{2!}{17^3}$
- (E) $17 \left(\frac{2!}{17^3} \right)$

Questions 46–48

46. A sequence satisfies $u_0 = 81$ and $u_{k+1} = 1u_k + 9$ for all $k \geq 0$. Find u_3 .

- (A) 106
- (B) 109
- (C) 107
- (D) 110
- (E) 108

47. Solve $y'' - 36y' + 323y = 0$ with $y(0) = 1$, $y'(0) = 0$. Find $y(1)$.

- (A) $\frac{(19 - 17e^2)e^{17}}{2} + 1$
- (B) $\frac{(19 - 17e^2)e^{17}}{4}$
- (C) $\frac{(-19 + 17e^2)e^{17}}{2}$
- (D) $\frac{(19 - 17e^2)e^{17}}{2}$
- (E) $(19 - 17e^2)e^{17}$

48. A connected planar graph has 40 vertices and 75 edges. How many faces, including the unbounded face, are in a planar embedding?

- (A) 13
- (B) 25
- (C) 37
- (D) 61
- (E) 49

Questions 49–51

49. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 17z^2 = x + 2y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.

- (A) 68
- (B) -2
- (C) -136
- (D) -68
- (E) -34

50. What is the largest integer k such that 2^k divides $90!$?

- (A) 84
- (B) 87
- (C) 85
- (D) 86
- (E) 88

51. Evaluate $\int_0^{\log 2} e^x (e^x + 17)^2 dx$.

- (A) $3 \left(\frac{19^3 - 18^3}{3} \right)$
- (B) $\frac{19^3 - 18^3}{3}$
- (C) $\frac{19^3}{3}$
- (D) $-\frac{19^3 - 18^3}{3}$
- (E) $19^2 - 18^2$

Questions 52–54

52. How many integers in $\{1, \dots, 222\}$ are divisible by 2 or 3?

- (A) 148
- (B) 197
- (C) 50
- (D) 99
- (E) 246

53. Evaluate $\iiint_B 17(x^2 + y^2 + z^2)^1 dV$, where B is the unit ball.

- (A) $1 + \frac{68\pi}{5}$
- (B) $\frac{68\pi}{5}$
- (C) $-\frac{68\pi}{5}$
- (D) $\frac{34\pi}{5}$
- (E) $\frac{136\pi}{5}$

54. In the ring $\mathbb{Z}/510\mathbb{Z}$, how many elements are units?

- (A) 127
- (B) 126
- (C) 129
- (D) 128
- (E) 130

Questions 55–57

55. Evaluate $\lim_{x \rightarrow 0} \frac{e^{17x} - \sum_{j=0}^1 (17x)^j / j!}{x^2}$.

- (A) $\frac{1}{17} \left(\frac{17^2}{2!} \right)$
- (B) $\frac{1}{2} \left(\frac{17^2}{2!} \right)$
- (C) $2 \left(\frac{17^2}{2!} \right)$
- (D) $-\frac{17^2}{2!}$
- (E) $\frac{17^2}{2!}$

56. Let $F_0 = 0$, $F_1 = 1$, and $F_{k+2} = F_{k+1} + F_k$. How many binary strings of length 37 have no adjacent 1s?

- (A) F_{39}
- (B) F_{37}
- (C) F_{41}
- (D) F_{40}
- (E) F_{38}

57. Let $f(x, y) = e^{17x+2y}$. Find $\partial^2 f / \partial x^1 \partial y$ at $(0, 0)$.

- (A) $2 \left(2 \cdot 17^1 \right)$
- (B) $\frac{17}{2} \left(2 \cdot 17^1 \right)$
- (C) $2 \cdot 17^1$
- (D) $\frac{1}{2} \left(2 \cdot 17^1 \right)$
- (E) $\frac{2}{17} \left(2 \cdot 17^1 \right)$

Questions 58–60

58. A permutation consists of disjoint cycles of lengths 4 and 10 and fixes all other points. What is its order?

- (A) 18
- (B) 22
- (C) 20
- (D) 21
- (E) 19

59. Find the curvature at $t = 0$ of $r(t) = (2t, 17t^2)$.

- (A) $\frac{19}{2}$
- (B) $\frac{17}{4}$
- (C) $\frac{17}{2}$
- (D) $-\frac{17}{2}$
- (E) 17

60. Apply one Newton iteration to solve $x^2 - 37 = 0$, starting at $x_0 = 1$. What is x_1 ?

- (A) 13
- (B) 7
- (C) 19
- (D) 25
- (E) 31

Questions 61–63

61. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(17x) - \sum_{j=0}^0 (-1)^j (17x)^{2j+1} / (2j+1)!}{x^3}$.

(A) $3 \left(\frac{(-1)^1 17^3}{3!} \right)$

(B) $\frac{(-1)^1 17^3}{3!}$

(C) $-\frac{(-1)^1 17^3}{3!}$

(D) $17 \left(\frac{(-1)^1 17^3}{3!} \right)$

(E) $\frac{1}{17} \left(\frac{(-1)^1 17^3}{3!} \right)$

62. In $R = \mathbb{F}_2[x]/(x^{17})$, how many elements are there?

(A) 2^{20}

(B) 2^{16}

(C) 2^{19}

(D) 2^{18}

(E) 2^{17}

63. For $\mathbf{F} = (17x^2, y^2, z^2)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

(A) 40

(B) 41

(C) -40

(D) 20

(E) 80

Questions 64–66

64. Bisection starts with an interval of length 2^{37} . After 39 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{9}{16}$
- (B) $\frac{7}{8}$
- (C) $\frac{1}{24}$
- (D) $\frac{1}{16}$
- (E) $\frac{1}{8}$

65. For the curve $x = t^2$, $y = t^3 + 17t$, find d^2y/dx^2 at $t = 1$.

- (A) 10
- (B) $-\frac{7}{2}$
- (C) 5
- (D) $-\frac{17}{2}$
- (E) -7

66. Integers x satisfy $x \equiv 2 \pmod{7}$ and $x \equiv 6 \pmod{9}$. Find the least nonnegative solution.

- (A) 50
- (B) 52
- (C) 49
- (D) 53
- (E) 51

Answer Key

Practice Test 85 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	C	23	A	45	D
2	B	24	A	46	E
3	A	25	B	47	D
4	A	26	B	48	C
5	B	27	A	49	D
6	B	28	E	50	D
7	C	29	D	51	B
8	D	30	E	52	A
9	C	31	E	53	B
10	B	32	B	54	D
11	D	33	D	55	E
12	A	34	D	56	A
13	C	35	E	57	C
14	E	36	C	58	C
15	A	37	C	59	C
16	D	38	E	60	C
17	A	39	C	61	B
18	E	40	E	62	E
19	B	41	C	63	A
20	B	42	D	64	E
21	A	43	E	65	B
22	D	44	A	66	E

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