

GRE Mathematics Practice Test 86

Advanced Challenge

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

The most demanding form in this five-test cycle. Emphasis on connected ideas, non-routine reasoning, and careful conceptual distinctions. This is cycle 17 of 19, position 1 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Find the coefficient of x^4 in $e^{5x} \log(1 + 18x)$.
- (A) -18174
(B) 18174
(C) -18156
(D) -26244
(E) -436176
2. An invertible 88×88 matrix A has determinant 4. Find $\det(\text{adj } A)$.
- (A) 2^{175}
(B) $2^{174} \cdot 3$
(C) $2^{174} \cdot 5$
(D) 2^{173}
(E) 2^{174}
3. Find the arc length of $r(t) = (18(t - t^3/3), 18t^2)$ for $0 \leq t \leq 1/6$.
- (A) $\frac{109}{18}$
(B) $\frac{109}{72}$
(C) $\frac{145}{36}$
(D) $\frac{109}{36}$
(E) $-\frac{109}{36}$

Questions 4–6

4. $X_1, \dots, X_{33}$ are independent with common mean and variance 35. With $\bar{X}$ their sample mean, find $E[\sum_i (X_i - \bar{X})^2]$.

- (A) 374
- (B) 1866
- (C) 747
- (D) 1493
- (E) 1120

5. Evaluate $\int_0^1 x^{18}(1-x)^5 dx$.

- (A) $24 \left(\frac{18! 5!}{24!} \right)$
- (B) $\frac{18! 5!}{24!}$
- (C) $\frac{1}{6} \left(\frac{18! 5!}{24!} \right)$
- (D) $19 \left(\frac{18! 5!}{24!} \right)$
- (E) $7084 \left(\frac{18! 5!}{24!} \right)$

6. On P_{89} , the real polynomials of degree at most 89, define $T(p) = x^2 p'' - 30xp' + 240p$. Find its rank.

- (A) 87
- (B) 90
- (C) 89
- (D) 86
- (E) 88

Questions 7–9

7. For the curve $x = t^6$, $y = t^7 + 18t$, find d^2y/dx^2 at $t = 1$.

- (A) $-\frac{83}{6}$
- (B) $\frac{25}{6}$
- (C) $\frac{97}{36}$
- (D) -3
- (E) $-\frac{83}{36}$

8. A permutation of 35 objects is chosen uniformly. Let Z be the number of fixed points. Find $E[Z^2]$.

- (A) 0
- (B) 2
- (C) 4
- (D) 3
- (E) 1

9. For $f(x) = x^6 e^{18x}$, find $f^{(8)}(0)$.

- (A) $2 \left(\frac{8! 18^2}{2} \right)$
- (B) $\frac{1}{9} \left(\frac{8! 18^2}{2} \right)$
- (C) $\frac{1}{56} \left(\frac{8! 18^2}{2} \right)$
- (D) $\frac{1}{20160} \left(\frac{8! 18^2}{2} \right)$
- (E) $\frac{8! 18^2}{2}$

Questions 10–12

10. Subspaces U, V of $\mathbb{R}^{90}$ have dimensions 5 and 6. What is the smallest possible value of $\dim(U \cap V)$?

- (A) 2
- (B) 3
- (C) 4
- (D) 0
- (E) 1

11. Find the coefficient of x^6 in $\cos(18x)/(1 - 5x^2)$.

- (A) $-\frac{146476}{5}$
- (B) $-\frac{236196}{5}$
- (C) $-\frac{146471}{5}$
- (D) $\frac{146471}{5}$
- (E) -21091824

12. A coin success probability p has uniform prior on $[0, 1]$. After observing 33 successes and 2 failures in independent tosses conditional on p , what is the predictive probability of success on the next toss?

- (A) $\frac{34}{37}$
- (B) $\frac{71}{74}$
- (C) $\frac{34}{111}$
- (D) $\frac{3}{37}$
- (E) $\frac{17}{37}$

Questions 13–15

13. Evaluate $\iiint_B 18z^2(x^2 + y^2 + z^2)^6 dV$, where B is the unit ball.

- (A) $-\frac{24\pi}{17}$
- (B) $\frac{48\pi}{17}$
- (C) $\frac{24\pi}{17}$
- (D) $\frac{12\pi}{17}$
- (E) $1 + \frac{24\pi}{17}$

14. Let V consist of real polynomials of degree at most 13 that have both value and first derivative zero at 82, 83, and 85. Find $\dim V$.

- (A) 6
- (B) 7
- (C) 9
- (D) 8
- (E) 10

15. The local inverse $y(x)$ of $x = 18y + y^3$ satisfies $y(0) = 0$. Find $y^{(7)}(0)$.

- (A) $18 \left(-\frac{60480}{18^{10}} \right)$
- (B) $\frac{60480}{18^{10}}$
- (C) $\frac{1}{5040} \left(-\frac{60480}{18^{10}} \right)$
- (D) $\frac{1}{18} \left(-\frac{60480}{18^{10}} \right)$
- (E) $-\frac{60480}{18^{10}}$

Questions 16–18

16. A random variable has mean 0 and variance 33. What upper bound does the Chebyshev inequality give for $P(|X| \geq 66)$?

- (A) $\frac{1}{396}$
- (B) $\frac{131}{132}$
- (C) $\frac{133}{264}$
- (D) $\frac{1}{264}$
- (E) $\frac{1}{132}$

17. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{18t} \cos(6t)$, and $y = e^{18t} \sin(6t)$, find d^2f/dt^2 at $t = 1$.

- (A) 36
- (B) 12
- (C) 1296
- (D) 720
- (E) 0

18. The roots r, s of $x^2 - 95x + 1066 = 0$ are counted with multiplicity. Find $r^4 + s^4$.

- (A) $95^4 + 2(1066^2)$
- (B) $95^4 - 2(1066)(95^2) + 2(1066^2)$
- (C) $95^4 - 4(1066)(95^2) + 2(1066^2)$
- (D) $95^4 - 4(1066)(95^2) + 1066^2$
- (E) $95^4 - 4(1066)(95^2)$

Questions 19–21

19. Evaluate $\int_0^1 x^{17}(\log x)^5 dx$.

- (A) $\frac{1}{6} \left(\frac{(-1)^5 5!}{18^6} \right)$
- (B) $\frac{(-1)^5 5!}{18^6}$
- (C) $\frac{78732}{5} \left(\frac{(-1)^5 5!}{18^6} \right)$
- (D) $18 \left(\frac{(-1)^5 5!}{18^6} \right)$
- (E) $-\frac{(-1)^5 5!}{18^6}$

20. A box is chosen uniformly from two boxes. Box A contains 38 red and 40 blue balls; box B contains 40 red and 38 blue balls. Two balls are drawn without replacement and both are red. What is the conditional probability that A was chosen?

- (A) $\frac{703}{4449}$
- (B) $\frac{703}{1483}$
- (C) $\frac{2966}{1093}$
- (D) $\frac{1483}{780}$
- (E) $\frac{780}{1483}$

21. For $D = \{(x, y) : x^2 + y^2 \leq 18\}$, evaluate $\iint_D (x^2 + y^2)^5 dA$.

- (A) 314928π
- (B) 5668704π
- (C) $\frac{34012224\pi}{11}$
- (D) 11337408π
- (E) 34012224π

Questions 22–24

22. For real t , let $M(t) = \begin{pmatrix} t-82 & 13 & 0 \\ 0 & t-82 & 1 \\ 0 & 0 & t-82 \end{pmatrix}$. At $t = 82$, what is $\dim \ker(M(t)^2) - \dim \ker M(t)$?

- (A) 3
- (B) 1
- (C) 0
- (D) 2
- (E) 4

23. Let $f(x, y) = e^{18x+6y}$. Find $\partial^6 f / \partial x^5 \partial y$ at $(0, 0)$.

- (A) $3(6 \cdot 18^5)$
- (B) $\frac{1}{6}(6 \cdot 18^5)$
- (C) $\frac{1}{243}(6 \cdot 18^5)$
- (D) $6 \cdot 18^5$
- (E) $\frac{1}{81}(6 \cdot 18^5)$

24. Which subset of $\mathbb{R}$ is compact, with 28 fixed as displayed?

- (A) $28\mathbb{Z}$
- (B) $\{0\} \cup \{28/k : k \geq 1\}$
- (C) $\mathbb{Q} \cap [0, 28]$
- (D) $(0, 28)$
- (E) $\{28/k : k \geq 1\}$

Questions 25–27

25. Two solutions of $y'' + 18x^6y' + 6y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $\frac{2}{e^{\frac{18}{7}}}$
- (B) $-\frac{1}{e^{\frac{18}{7}}}$
- (C) $e^{-\frac{18}{7}}$
- (D) $e^{-\frac{18}{7}} + 1$
- (E) $\frac{1}{2e^{\frac{18}{7}}}$

26. Let $u = (17, 6)^T$ and $v = (1, 0)^T$. Find the squared distance from $(17, 0)^T$ to $\text{span}(u)$.

- (A) $\frac{21133}{650}$
- (B) $\frac{10404}{325}$
- (C) $\frac{20483}{650}$
- (D) $\frac{10729}{325}$
- (E) $\frac{10079}{325}$

27. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^{18}y^5 dV$.

- (A) $\frac{18!5!}{26!}$
- (B) $\frac{1}{6} \left(\frac{18!5!}{26!} \right)$
- (C) $\frac{1}{114}$
- (D) $26 \left(\frac{18!5!}{26!} \right)$
- (E) $6 \left(\frac{18!5!}{26!} \right)$

Questions 28–30

28. For $a_k = (-1)^k + 28/k$, find $\limsup_{k \rightarrow \infty} a_k - \liminf_{k \rightarrow \infty} a_k$.

- (A) 1
- (B) 0
- (C) 4
- (D) 3
- (E) 2

29. Let $\mathbf{F} = (18y^6, z^6, x^6)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (1, 1, 2)/\sqrt{6}$.

- (A) $-76\sqrt{6}$
- (B) $1 - 38\sqrt{6}$
- (C) $38\sqrt{6}$
- (D) $-38\sqrt{6}$
- (E) $-19\sqrt{6}$

30. An idempotent 88×88 real matrix P has rank 3. What is $\det(I + P)$?

- (A) 6
- (B) 8
- (C) 9
- (D) 7
- (E) 10

Questions 31–33

31. If $y' = y(1 - y/6)$ and $y(0) = 3$, find $y(-\log 18)$.

- (A) $\frac{12}{19}$
- (B) $\frac{25}{19}$
- (C) $\frac{3}{19}$
- (D) $\frac{6}{19}$
- (E) $-\frac{6}{19}$

32. For which real a is $f(x) = x^a$ in $L^{28}(0, 1)$?

- (A) $a > -1/28$
- (B) $a > 0$
- (C) $a < 1/28$
- (D) $a \geq -1/28$
- (E) All real a

33. Evaluate $\int_0^{\pi/2} \sin^{13} t \cos^{37} t \, dt$.

- (A) $\frac{1}{50}$
- (B) $\frac{6! 18!}{2 \cdot 25!}$
- (C) $2 \left(\frac{6! 18!}{2 \cdot 25!} \right)$
- (D) $\frac{1}{2} \left(\frac{6! 18!}{2 \cdot 25!} \right)$
- (E) $25 \left(\frac{6! 18!}{2 \cdot 25!} \right)$

Questions 34–36

34. For $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} 82 \\ 13 \\ 95 \end{bmatrix}$, let x minimize $\|Ax - b\|^2$. Find the minimum squared residual.

- (A) $\frac{22801}{6}$
- (B) $\frac{11399}{3}$
- (C) $\frac{22795}{6}$
- (D) $\frac{22807}{6}$
- (E) $\frac{11402}{3}$

35. For $\mathbf{F} = (18x^6y, y^6z, xz^6)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) 414
- (B) -414
- (C) 828
- (D) 415
- (E) 207

36. $T : \mathbb{R} \rightarrow \mathbb{R}$ has Lipschitz constant at most $1/28$. If $x_1 = T(x_0)$ and $|x_1 - x_0| \leq 1$, what is the sharp upper bound on $|T^3(x_0) - x_*|$, where x_* is its fixed point?

- (A) $\frac{1}{42336}$
- (B) $\frac{21167}{21168}$
- (C) $\frac{21169}{42336}$
- (D) $\frac{1}{21168}$
- (E) $\frac{1}{63504}$

Questions 37–39

37. Evaluate $\int_0^1 \int_0^x x^{18} y^6 dy dx$.

- (A) $\frac{1}{182}$
- (B) $\frac{1}{26}$
- (C) $\frac{1}{133}$
- (D) $\frac{1}{175}$
- (E) $\frac{1}{208}$

38. What is the coefficient of x^8 in $(x + 17)^8(1 - x)^3$?

- (A) -251257
- (B) -251260
- (C) -251258
- (D) -251259
- (E) -251261

39. If $y'' + 36y = 18$ and $y(0) = y'(0) = 0$, find $y(\pi/6)$.

- (A) -1
- (B) $\frac{1}{2}$
- (C) 2
- (D) 1
- (E) 0

Questions 40–42

40. The circle $|z| = 2$ is traversed counterclockwise. What is $\frac{1}{2\pi i} \int_{|z|=2} \frac{e^z}{z^{24}} dz$?

- (A) $1/24!$
- (B) $1/46!$
- (C) $1/22!$
- (D) $1/23!$
- (E) $23!$

41. Let C be the unit circle $x^2 + y^2 = 1$, $z = 6$, counterclockwise viewed from above. Evaluate $\int_C -18y(x^2 + y^2)^5 dx + (18x(x^2 + y^2)^5) dy + 2z dz$.

- (A) 36π
- (B) 72π
- (C) $1 + 36\pi$
- (D) 18π
- (E) -36π

42. How many 2-dimensional subspaces does $\mathbb{F}_{59}^7$ have?

- (A) $\frac{(59^6 - 1)(59^6 - 59)}{(59^2 - 1)(59^2 - 59)}$
- (B) $\frac{(59^9 - 1)(59^9 - 59)}{(59^2 - 1)(59^2 - 59)}$
- (C) $\frac{(59^{10} - 1)(59^{10} - 59)}{(59^2 - 1)(59^2 - 59)}$
- (D) $\frac{(59^8 - 1)(59^8 - 59)}{(59^2 - 1)(59^2 - 59)}$
- (E) $\frac{(59^7 - 1)(59^7 - 59)}{(59^2 - 1)(59^2 - 59)}$

Questions 43–45

43. Find the directional derivative of $f(x, y, z) = x^2y + 6xz^2 + 18y^2z$ at $(1, 1, 1)$ in the direction $(1, 2, 0)$.

- (A) $1 + \frac{82\sqrt{5}}{5}$
- (B) $-\frac{82\sqrt{5}}{5}$
- (C) $\frac{41\sqrt{5}}{5}$
- (D) $\frac{164\sqrt{5}}{5}$
- (E) $\frac{82\sqrt{5}}{5}$

44. Find the residue of $z^{23}/(z - 1)^3$ at $z = 1$.

- (A) 253
- (B) 337
- (C) 169
- (D) 85
- (E) 421

45. The region $0 \leq x \leq 1$, $0 \leq y \leq 18x^6$ is revolved about the line $x = -2$. Find its volume.

- (A) $1 + \frac{207\pi}{14}$
- (B) $-\frac{207\pi}{14}$
- (C) $\frac{207\pi}{14}$
- (D) $\frac{207\pi}{7}$
- (E) $\frac{207\pi}{28}$

Questions 46–48

46. How many monic irreducible polynomials of degree 5 are there over $\mathbb{F}_{59}$?
- (A) $59^5 - 59$
(B) $2(59^5 - 59)$
(C) $\frac{59^5 - 59}{5}$
(D) $\frac{3(59^5 - 59)}{5}$
(E) $\frac{2(59^5 - 59)}{5}$
47. The solution of $y' = 18x^5y^2$, $y(0) = 1$, has its first positive singularity at which x ?
- (A) $\sqrt[6]{3}$
(B) $\frac{1}{3}$
(C) $\frac{3^{\frac{5}{6}}}{3}$
(D) $\frac{2^{\frac{5}{6}} \cdot 3^{\frac{2}{3}}}{6}$
(E) $\frac{6^{\frac{5}{6}}}{6}$
48. What is the least positive multiplicative inverse of 2 modulo 67?
- (A) 12
(B) 45
(C) 34
(D) 23
(E) 56

Questions 49–51

49. Find the coefficient of x^6 in the Taylor series of $(1 - 18x)^{-2}$ at zero.

- (A) $2(7 \cdot 18^6)$
- (B) $\frac{6}{7}(7 \cdot 18^6)$
- (C) $\frac{1}{7}(7 \cdot 18^6)$
- (D) $7 \cdot 18^6$
- (E) $\frac{1}{18}(7 \cdot 18^6)$

50. How many elements z of $\mathbb{F}_{11^{18}}$ satisfy $z^{121} = z$?

- (A) 123
- (B) 119
- (C) 121
- (D) 120
- (E) 122

51. Evaluate $\lim_{x \rightarrow 0} \frac{(1 + 18x)^{1/(5x)} - e^{18/5}}{x}$.

- (A) $-\frac{162e^{\frac{18}{5}}}{5}$
- (B) $-\frac{324e^{\frac{18}{5}}}{5}$
- (C) $-\frac{9e^{\frac{18}{5}}}{5}$
- (D) $\frac{162e^{\frac{18}{5}}}{5}$
- (E) $e^{\frac{18}{5}}$

Questions 52–54

52. How many truth assignments to $p, q, r_1, \dots, r_{31}$ make $p \Rightarrow q$ false?

- (A) 2^{31}
- (B) 2^{33}
- (C) 2^{30}
- (D) 2^{34}
- (E) 2^{32}

53. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-18)^j}{j^{\frac{5}{3}} (\log j)^m}$ converge absolutely at $x = 17$? Give the least such m , or 0 if none exists.

- (A) 1
- (B) 3
- (C) 4
- (D) 0
- (E) 2

54. How many generators does the cyclic group of order 42180533641 have?

- (A) $59^5(59-1)$
- (B) $59^6(59-1)$
- (C) $59^8(59-1)$
- (D) $59^4(59-1)$
- (E) $59^7(59-1)$

Questions 55–57

55. Let $f(x, y) = x^2 + 36xy + (330)y^2$. What is the product of the eigenvalues of the Hessian of f ?

- (A) 1320
- (B) 24
- (C) 12
- (D) 330
- (E) 6

56. Find the coefficient of x^3 in $(1 + x)^{33}(1 - x)^{33}(1 + 2x)^2$.

- (A) -176
- (B) -88
- (C) -44
- (D) -220
- (E) -132

57. Evaluate $\int_0^1 \int_x^1 18y^6 \, dy \, dx$.

- (A) 9
- (B) $\frac{9}{4}$
- (C) $\frac{18}{7}$
- (D) 3
- (E) $\frac{9}{28}$

Questions 58–60

58. Let $R = (\mathbb{Z}/114\mathbb{Z})/([19])$, where $([19])$ is the ideal generated by $[19]$. How many units does R have?

- (A) 19
- (B) 20
- (C) 18
- (D) 17
- (E) 16

59. If $y' + 18y = 6x^6e^{-18x}$ and $y(0) = 2$, find $y(1)$.

- (A) $\frac{40}{7e^{18}}$
- (B) $\frac{10}{7e^{18}}$
- (C) $\frac{20}{7e^{18}}$
- (D) $\frac{20}{7e^{18}} + 1$
- (E) $-\frac{20}{7e^{18}}$

60. How many distinct unoriented cycles of length 4 are in $K_{33,35}$?

- (A) 418880
- (B) 209440
- (C) 104720
- (D) 314160
- (E) 523600

Questions 61–63

- 61.** If $y'' - 36y' + 324y = 6e^{18x}$ and $y(0) = y'(0) = 0$, find $y(1)$.
- (A) $-3e^{18}$
 - (B) $6e^{18}$
 - (C) $\frac{3e^{18}}{2}$
 - (D) $1 + 3e^{18}$
 - (E) $3e^{18}$
- 62.** What is the largest integer k such that 3^k divides $91!$?
- (A) 46
 - (B) 45
 - (C) 44
 - (D) 42
 - (E) 43
- 63.** Under $x = u + 18v$, $y = u - 6v$, what is the area of the image of $0 \leq u, v \leq 1$?
- (A) $\frac{1}{24}$
 - (B) 24
 - (C) 108
 - (D) 12
 - (E) 576

Questions 64–66

64. How many functions from a set of 38 elements onto a set of 3 elements exist?

- (A) $3^{38} - 2^{38}$
- (B) 3^{38}
- (C) $3^{38} - 3(2^{38}) + 3$
- (D) $3^{38} - 3(2^{38})$
- (E) $3^{38} - 3(2^{38}) - 3$

65. Evaluate $\iiint_{x^2+y^2+z^2 \leq 18} (x^2 + y^2 + z^2)^5 dV$.

- (A) $\frac{1}{3} \left(\frac{4\pi 18^{\frac{13}{2}}}{13} \right)$
- (B) $\frac{13}{198} \left(\frac{4\pi 18^{\frac{13}{2}}}{13} \right)$
- (C) $\frac{1}{2} \left(\frac{4\pi 18^{\frac{13}{2}}}{13} \right)$
- (D) $\frac{4\pi 18^{\frac{13}{2}}}{13}$
- (E) $13 \left(\frac{4\pi 18^{\frac{13}{2}}}{13} \right)$

66. In the ring $\mathbb{Z}/516\mathbb{Z}$, how many elements satisfy $x^2 = 1$?

- (A) 9
- (B) 7
- (C) 6
- (D) 8
- (E) 10

Answer Key

Practice Test 86 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	A	23	D	45	C
2	E	24	B	46	C
3	D	25	C	47	C
4	E	26	B	48	C
5	B	27	A	49	D
6	E	28	E	50	C
7	E	29	D	51	A
8	B	30	B	52	A
9	E	31	D	53	A
10	D	32	A	54	A
11	C	33	B	55	B
12	A	34	A	56	E
13	C	35	A	57	B
14	D	36	D	58	C
15	E	37	A	59	C
16	E	38	D	60	D
17	E	39	D	61	E
18	C	40	D	62	C
19	B	41	A	63	B
20	B	42	E	64	C
21	B	43	E	65	D
22	B	44	A	66	D

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