

GRE Mathematics Practice Test 88

Upper-Intermediate Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

Intermediate reinforcement, combining standard applications with selected more demanding arguments. This is cycle 17 of 19, position 3 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. If $y' = y(1 - y/4)$ and $y(0) = 2$, find $y(-\log 18)$.

- (A) $-\frac{4}{19}$
- (B) $\frac{2}{19}$
- (C) $\frac{8}{19}$
- (D) $\frac{4}{19}$
- (E) $\frac{23}{19}$

2. A matrix A satisfies $(A - 85I)(A - 87I) = 0$, and its minimal polynomial has degree two. If $A^4 = uA + vI$, find u .

- (A) $2^3 \cdot 5 \cdot 13 \cdot 43 \cdot 569$
- (B) $2^2 \cdot 13 \cdot 43 \cdot 569$
- (C) $2^3 \cdot 13 \cdot 43 \cdot 569$
- (D) $2^3 \cdot 3 \cdot 13 \cdot 43 \cdot 569$
- (E) $2^4 \cdot 13 \cdot 43 \cdot 569$

3. Evaluate $\int_C (4x^3 + 18y) dx + (18x + 4y^3) dy$, where C is $r(t) = (t, 4t^2)$, $0 \leq t \leq 1$.

- (A) -329
- (B) $\frac{329}{2}$
- (C) 330
- (D) 658
- (E) 329

Questions 4–6

4. $X_1, \dots, X_{35}$ are independent with common mean and variance 37. With $\bar{X}$ their sample mean, find $E[\sum_i (X_i - \bar{X})^2]$.

- (A) 1258
- (B) 839
- (C) 2096
- (D) 1677
- (E) 420

5. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-18)^j}{j^1 (\log j)^m}$ converge absolutely at $x = 17$? Give the least such m , or 0 if none exists.

- (A) 0
- (B) 2
- (C) 4
- (D) 3
- (E) 1

6. Let $V = \begin{bmatrix} 1 & 84 & 7056 \\ 1 & 86 & 7396 \\ 1 & 89 & 7921 \end{bmatrix}$. Find $\det(V^T V)$.

- (A) 899
- (B) 902
- (C) 898
- (D) 901
- (E) 900

Questions 7–9

7. Let $f(x) = \log(1 + 18x)$. Find $f^{(4)}(0)$.

- (A) $\frac{1}{4}((-1)^3 3! 18^4)$
- (B) $\frac{1}{18}((-1)^3 3! 18^4)$
- (C) $4((-1)^3 3! 18^4)$
- (D) $-(-1)^3 3! 18^4$
- (E) $(-1)^3 3! 18^4$

8. A permutation of 37 objects is chosen uniformly. Let Z be the number of fixed points. Find $E[Z^2]$.

- (A) 1
- (B) 2
- (C) 3
- (D) 4
- (E) 0

9. Find the curvature of the helix $r(t) = (18 \cos t, 18 \sin t, 4t)$.

- (A) $\frac{9}{85}$
- (B) $\frac{9}{340}$
- (C) $\frac{179}{170}$
- (D) $\frac{9}{170}$
- (E) $-\frac{9}{170}$

Questions 10–12

10. Let $u = (1, 2, \dots, 90)^T$, $v = (1, 1, \dots, 1)^T$, and $B = I + uv^T$. Find $\det B$.

- (A) 4096
- (B) 4095
- (C) 4097
- (D) 4094
- (E) 4098

11. Let $F(x) = \int_0^{x^4} e^{18t} dt$. Find $F'(1)$.

- (A) e^{18}
- (B) $\frac{e^{18}}{4}$
- (C) $4e^{18}$
- (D) $72e^{18}$
- (E) e^{72}

12. A coin success probability p has uniform prior on $[0, 1]$. After observing 35 successes and 2 failures in independent tosses conditional on p , what is the predictive probability of success on the next toss?

- (A) $\frac{1}{13}$
- (B) $\frac{4}{13}$
- (C) $\frac{25}{26}$
- (D) $\frac{6}{13}$
- (E) $\frac{12}{13}$

Questions 13–15

13. For $x > 0$, $x^2 y'' + (-39)xy' + 396y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

- (A) $-\frac{(1 - e^4)e^{18}}{4}$
(B) $-\frac{e^{18}}{4} + 1 + \frac{e^{22}}{4}$
(C) $-\frac{(1 - e^4)e^{18}}{2}$
(D) $\frac{(1 - e^4)e^{18}}{4}$
(E) $-\frac{(1 - e^4)e^{18}}{8}$

14. Let $D = \text{diag}(\underbrace{1, \dots, 1}_{16}, \underbrace{2, \dots, 2}_{74})$. Find the dimension of the space of real matrices X with $XD = DX$.

- (A) 5731
(B) 5730
(C) 5732
(D) 5734
(E) 5733

15. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(18j)!}{(j!)^{18}} x^{4j}$?

- (A) $18^{\frac{9}{2}}$
(B) 18^{-18}
(C) $18^{-\frac{9}{2}}$
(D) $\frac{1}{18}$
(E) $18^{-\frac{2}{9}}$

Questions 16–18

16. A random variable has mean 0 and variance 35. What upper bound does the Chebyshev inequality give for $P(|X| \geq 70)$?

- (A) $\frac{141}{280}$
- (B) $\frac{1}{140}$
- (C) $\frac{1}{280}$
- (D) $\frac{1}{420}$
- (E) $\frac{139}{140}$

17. Let C be the unit circle $x^2 + y^2 = 1$, $z = 4$, counterclockwise viewed from above. Evaluate $\int_C -18y(x^2 + y^2)^3 dx + (18x(x^2 + y^2)^3) dy + 4z dz$.

- (A) -36π
- (B) $1 + 36\pi$
- (C) 36π
- (D) 72π
- (E) 18π

18. When $x^3 + 1$ is divided by $(x - 85)(x - 86)$, the remainder is $ux + v$. Find u .

- (A) 21933
- (B) 21931
- (C) 21930
- (D) 21929
- (E) 21932

Questions 19–21

19. Evaluate $\int_0^1 x^{18}(1-x)^3 dx$.

- (A) $19 \left(\frac{18! 3!}{22!} \right)$
- (B) $22 \left(\frac{18! 3!}{22!} \right)$
- (C) $\frac{1}{4} \left(\frac{18! 3!}{22!} \right)$
- (D) $\frac{18! 3!}{22!}$
- (E) $385 \left(\frac{18! 3!}{22!} \right)$

20. A box is chosen uniformly from two boxes. Box A contains 40 red and 42 blue balls; box B contains 42 red and 40 blue balls. Two balls are drawn without replacement and both are red. What is the conditional probability that A was chosen?

- (A) $\frac{130}{547}$
- (B) $\frac{287}{547}$
- (C) $\frac{260}{1641}$
- (D) $\frac{1641}{260}$
- (E) $\frac{547}{807}$

21. If $y' = 36xy$ and $y(0) = 1$, find the coefficient of x^8 in the Maclaurin series of y .

- (A) 2187
- (B) 4374
- (C) 4375
- (D) 8748
- (E) -4374

Questions 22–24

22. Let $P = \text{diag}(\underbrace{1, \dots, 1}_1, \underbrace{0, \dots, 0}_{90})$. What is the dimension of the space of 91×91 real matrices X satisfying $PX = X$ and $XP = 0$?

- (A) 92
- (B) 91
- (C) 90
- (D) 88
- (E) 89

23. Evaluate $\int_0^{\pi/2} \sin^9 t \cos^{37} t \, dt$.

- (A) $2 \left(\frac{4! 18!}{2 \cdot 23!} \right)$
- (B) $7315 \left(\frac{4! 18!}{2 \cdot 23!} \right)$
- (C) $\frac{1}{2} \left(\frac{4! 18!}{2 \cdot 23!} \right)$
- (D) $\frac{4! 18!}{2 \cdot 23!}$
- (E) $23 \left(\frac{4! 18!}{2 \cdot 23!} \right)$

24. Which subset of $\mathbb{R}$ is compact, with 30 fixed as displayed?

- (A) $\{0\} \cup \{30/k : k \geq 1\}$
- (B) $\{30/k : k \geq 1\}$
- (C) $(0, 30)$
- (D) $30\mathbb{Z}$
- (E) $\mathbb{Q} \cap [0, 30]$

Questions 25–27

25. Evaluate $\int_0^1 \frac{x^3}{1 + 18x^4} dx$.

- (A) $-\frac{\log(19)}{72}$
- (B) $\frac{\log(19)}{18}$
- (C) $\frac{\log(19)}{72}$
- (D) $\frac{\log(19)}{4}$
- (E) $\log(19)$

26. A 3×3 matrix A has characteristic polynomial $(t - 84)(t + 1)(t - 2)$. What is $\text{tr}(A^3)$?

- (A) 592709
- (B) 592710
- (C) 592712
- (D) 592713
- (E) 592711

27. Find the upward flux of $\mathbf{F} = (x, y, z)$ through $z = 18x^2 + 4y^2$, $0 \leq x, y \leq 1$.

- (A) $-\frac{11}{3}$
- (B) $-\frac{22}{3}$
- (C) $\frac{22}{3}$
- (D) $-\frac{19}{3}$
- (E) $-\frac{44}{3}$

Questions 28–30

28. For $a_k = (-1)^k + 30/k$, find $\limsup_{k \rightarrow \infty} a_k - \liminf_{k \rightarrow \infty} a_k$.

- (A) 0
- (B) 3
- (C) 2
- (D) 1
- (E) 4

29. Find the arc length of $r(t) = (18(t - t^3/3), 18t^2)$ for $0 \leq t \leq 1/4$.

- (A) $\frac{179}{32}$
- (B) $\frac{147}{32}$
- (C) $\frac{147}{64}$
- (D) $-\frac{147}{32}$
- (E) $\frac{147}{16}$

30. Let N be the 90×90 matrix with ones immediately above the diagonal and zeros elsewhere. Find $\dim(\ker N^{30} / \ker N^{15})$.

- (A) 14
- (B) 17
- (C) 13
- (D) 16
- (E) 15

Questions 31–33

31. The solution of $y' = 18x^3y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\frac{2^{\frac{3}{4}}\sqrt{3}}{2}$
- (B) $\frac{2}{9}$
- (C) $\frac{\sqrt[4]{2}\sqrt{3}}{3}$
- (D) $\frac{2^{\frac{3}{4}}\sqrt{3}}{6}$
- (E) $\frac{\sqrt{3}}{3}$

32. For which real a is $f(x) = x^a$ in $L^{30}(0, 1)$?

- (A) $a \geq -1/30$
- (B) $a > 0$
- (C) All real a
- (D) $a < 1/30$
- (E) $a > -1/30$

33. Under $x = u + 18v$, $y = u - 4v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) 22
- (B) $\frac{1}{22}$
- (C) 14
- (D) 484
- (E) 72

Questions 34–36

34. Let A be the 90×90 matrix whose diagonal entries are 87 and whose off-diagonal entries are 1. What is its largest eigenvalue?

- (A) 178
- (B) 174
- (C) 175
- (D) 176
- (E) 177

35. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^{18}y^3 dV$.

- (A) $\frac{1}{76}$
- (B) $\frac{18!3!}{24!}$
- (C) $6 \left(\frac{18!3!}{24!} \right)$
- (D) $24 \left(\frac{18!3!}{24!} \right)$
- (E) $\frac{1}{6} \left(\frac{18!3!}{24!} \right)$

36. A map $T : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $|T(x) - T(y)| \leq |x - y|/30$. If $T(x_*) = x_*$ and $|x_0 - x_*| \leq 1$, what is the smallest guaranteed upper bound for $|T^3(x_0) - x_*|$?

- (A) $\frac{26999}{27000}$
- (B) $\frac{27001}{54000}$
- (C) $\frac{1}{54000}$
- (D) $\frac{1}{27000}$
- (E) $\frac{1}{81000}$

Questions 37–39

37. Evaluate $\iiint_B 18z^2(x^2 + y^2 + z^2)^4 dV$, where B is the unit ball.

- (A) $\frac{12\pi}{13}$
- (B) $-\frac{24\pi}{13}$
- (C) $\frac{48\pi}{13}$
- (D) $1 + \frac{24\pi}{13}$
- (E) $\frac{24\pi}{13}$

38. Positive real numbers x, y satisfy $\log_2 x + \log_2 y = \log_2 1176$ and $x + y = 98$. Find $x^2 + y^2$.

- (A) 7253
- (B) 7254
- (C) 7251
- (D) 7252
- (E) 7250

39. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(18x) - \sum_{j=0}^2 (-1)^j (18x)^{2j+1} / (2j+1)!}{x^7}$.

- (A) $\frac{(-1)^3 18^7}{7!}$
- (B) $7 \left(\frac{(-1)^3 18^7}{7!} \right)$
- (C) $\frac{1}{18} \left(\frac{(-1)^3 18^7}{7!} \right)$
- (D) $-\frac{(-1)^3 18^7}{7!}$
- (E) $18 \left(\frac{(-1)^3 18^7}{7!} \right)$

Questions 40–42

40. The circle $|z| = 2$ is traversed counterclockwise. What is $\frac{1}{2\pi i} \int_{|z|=2} \frac{e^z}{z^{26}} dz$?

- (A) $25!$
- (B) $1/26!$
- (C) $1/24!$
- (D) $1/50!$
- (E) $1/25!$

41. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 18z^2 = x + 4y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.

- (A) -72
- (B) -36
- (C) -144
- (D) 144
- (E) -4

42. How many permutations in S_{90} consist of one 5-cycle and 85 fixed points?

- (A) $2^5 \cdot 3^4 \cdot 11 \cdot 29 \cdot 43 \cdot 89$
- (B) $2^5 \cdot 3^3 \cdot 11 \cdot 29 \cdot 43 \cdot 89$
- (C) $2^5 \cdot 3^3 \cdot 5 \cdot 11 \cdot 29 \cdot 43 \cdot 89$
- (D) $2^4 \cdot 3^3 \cdot 11 \cdot 29 \cdot 43 \cdot 89$
- (E) $2^6 \cdot 3^3 \cdot 11 \cdot 29 \cdot 43 \cdot 89$

Questions 43–45

43. Find the coefficient of x^6 in $\cos(18x)/(1 - 3x^2)$.

- (A) $\frac{177741}{5}$
- (B) -25594704
- (C) $-\frac{177741}{5}$
- (D) $-\frac{236196}{5}$
- (E) $-\frac{177746}{5}$

44. Find the residue of $z^{25}/(z - 1)^3$ at $z = 1$.

- (A) 200
- (B) 100
- (C) 500
- (D) 400
- (E) 300

45. Solve $y' + 18y = x^4e^{-18x}$, $y(0) = 0$, and find $y(1)$.

- (A) e^{-18}
- (B) $\frac{1}{5e^{18}}$
- (C) $\frac{1}{4e^{18}}$
- (D) $\frac{18}{5e^{18}}$
- (E) $\frac{e^{18}}{5}$

Questions 46–48

46. How many maximal proper subgroups does the cyclic group of order 1056 have?

- (A) 3
- (B) 4
- (C) 1
- (D) 2
- (E) 5

47. Let $f(x, y) = e^{18x+4y}$. Find $\partial^4 f / \partial x^3 \partial y$ at $(0, 0)$.

- (A) $\frac{1}{4} (4 \cdot 18^3)$
- (B) $\frac{8}{729} (4 \cdot 18^3)$
- (C) $\frac{9}{2} (4 \cdot 18^3)$
- (D) $4 \cdot 18^3$
- (E) $\frac{4}{81} (4 \cdot 18^3)$

48. What is the least positive multiplicative inverse of 2 modulo 71?

- (A) 48
- (B) 60
- (C) 24
- (D) 12
- (E) 36

Questions 49–51

49. Evaluate $\int_0^\infty x^4 e^{-18x} dx$.

- (A) 24
- (B) $18 \left(\frac{4!}{18^5} \right)$
- (C) $\frac{1}{18} \left(\frac{4!}{18^5} \right)$
- (D) $\frac{1}{4} \left(\frac{4!}{18^5} \right)$
- (E) $\frac{4!}{18^5}$

50. How many residues x modulo 108 satisfy $2x \equiv 14 \pmod{108}$ and $2x \equiv 0 \pmod{108}$?

- (A) 4
- (B) 0
- (C) 2
- (D) 3
- (E) 1

51. Evaluate $\lim_{x \rightarrow 0} \frac{(1 + 18x)^{1/(3x)} - e^{18/3}}{x}$.

- (A) $54e^6$
- (B) e^6
- (C) $-108e^6$
- (D) $-54e^6$
- (E) $-3e^6$

Questions 52–54

52. How many truth assignments to $p, q, r_1, \dots, r_{33}$ make $p \Rightarrow q$ false?

- (A) 2^{33}
- (B) 2^{35}
- (C) 2^{36}
- (D) 2^{32}
- (E) 2^{34}

53. Evaluate $\int_0^1 \int_x^1 18y^4 \, dy \, dx$.

- (A) 3
- (B) $\frac{18}{5}$
- (C) $\frac{9}{2}$
- (D) 9
- (E) $\frac{3}{5}$

54. The field K has 5^{18} elements. Find the largest possible multiplicative order of a nonzero element of K .

- (A) $5^{19} - 1$
- (B) $5^{18} - 1$
- (C) $5^{18} + 1$
- (D) 5^{18}
- (E) $5^{17} - 1$

Questions 55–57

55. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(18+4j)(22+4j)}$.

- (A) $\frac{1}{72}$
- (B) $\frac{2}{9}$
- (C) $\frac{1}{18}$
- (D) $\frac{1}{4}$
- (E) $\frac{1}{22}$

56. What is the coefficient of x^2 in $(1+2x)^{35}$?

- (A) 2380
- (B) 3173
- (C) 3966
- (D) 1587
- (E) 794

57. If $y' + 18y = 4x^4e^{-18x}$ and $y(0) = 4$, find $y(1)$.

- (A) $-\frac{24}{5e^{18}}$
- (B) $\frac{48}{5e^{18}}$
- (C) $\frac{24}{5e^{18}}$
- (D) $\frac{12}{5e^{18}}$
- (E) $\frac{24}{5e^{18}} + 1$

Questions 58–60

58. Let $G = \mathbb{Z}/534\mathbb{Z} \times \mathbb{Z}/360\mathbb{Z}$. Find the order of G/H , where $H = \{g \in G : 6g = 0\}$.

- (A) 5341
- (B) 5342
- (C) 5338
- (D) 5339
- (E) 5340

59. If $y'' - 36y' + 324y = 4e^{18x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) $4e^{18}$
- (B) $-2e^{18}$
- (C) $1 + 2e^{18}$
- (D) $2e^{18}$
- (E) e^{18}

60. How many edges does the complete bipartite graph $K_{35,37}$ have?

- (A) 1726
- (B) 1295
- (C) 2157
- (D) 433
- (E) 864

Questions 61–63

61. Let $\mathbf{F} = (18y^4, z^4, x^4)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (1, 1, 4)/\sqrt{18}$.

- (A) $1 - \frac{148\sqrt{2}}{3}$
- (B) $-\frac{74\sqrt{2}}{3}$
- (C) $-\frac{148\sqrt{2}}{3}$
- (D) $\frac{148\sqrt{2}}{3}$
- (E) $-\frac{296\sqrt{2}}{3}$

62. How many group homomorphisms $\mathbb{Z}/534\mathbb{Z} \rightarrow \mathbb{Z}/360\mathbb{Z}$ are there?

- (A) 7
- (B) 4
- (C) 5
- (D) 6
- (E) 8

63. The local inverse $y(x)$ of $x = 18y + y^3$ satisfies $y(0) = 0$. Find $y^{(3)}(0)$.

- (A) $-\frac{6}{18^4}$
- (B) $\frac{1}{6} \left(-\frac{6}{18^4} \right)$
- (C) $\frac{1}{18} \left(-\frac{6}{18^4} \right)$
- (D) $18 \left(-\frac{6}{18^4} \right)$
- (E) $\frac{6}{18^4}$

Questions 64–66

64. How many functions from a set of 40 elements onto a set of 3 elements exist?

- (A) 3^{40}
- (B) $3^{40} - 3(2^{40})$
- (C) $3^{40} - 3(2^{40}) + 3$
- (D) $3^{40} - 3(2^{40}) - 3$
- (E) $3^{40} - 2^{40}$

65. For $D = \{(x, y) : x^2 + y^2 \leq 18\}$, evaluate $\iint_D (x^2 + y^2)^3 dA$.

- (A) 26244π
- (B) 52488π
- (C) 1458π
- (D) 104976π
- (E) $\frac{104976\pi}{7}$

66. How many 1-dimensional subspaces does $\mathbb{F}_{59}^5$ have?

- (A) $\frac{59^8 - 1}{59 - 1}$
- (B) $\frac{59^4 - 1}{59 - 1}$
- (C) $\frac{59^7 - 1}{59 - 1}$
- (D) $\frac{59^5 - 1}{59 - 1}$
- (E) $\frac{59^6 - 1}{59 - 1}$

Answer Key

Practice Test 88 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	D	23	D	45	B
2	C	24	A	46	A
3	E	25	C	47	D
4	A	26	E	48	E
5	B	27	B	49	E
6	E	28	C	50	B
7	E	29	B	51	D
8	B	30	E	52	A
9	D	31	C	53	A
10	A	32	E	54	B
11	C	33	A	55	A
12	E	34	D	56	A
13	A	35	B	57	C
14	C	36	D	58	E
15	C	37	E	59	D
16	B	38	D	60	B
17	C	39	A	61	C
18	B	40	E	62	D
19	D	41	C	63	A
20	D	42	B	64	C
21	B	43	C	65	A
22	C	44	E	66	D

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