

# GRE Mathematics Practice Test 89

Bridge to Standard

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

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## Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural,  $\mathbb{N} = \{1, 2, 3, \dots\}$ , and distances in  $\mathbb{R}^n$  are Euclidean.  $I_n$  denotes the  $n \times n$  identity matrix. Graphs are finite and simple unless stated otherwise.

## Difficulty target

A bridge toward standard practice, with routine multi-step questions and a smaller number of challenging concepts. This is cycle 17 of 19, position 4 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

## Coverage

|                 |                                                                                                                           |
|-----------------|---------------------------------------------------------------------------------------------------------------------------|
| Calculus / 33   | Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations. |
| Algebra / 17    | Elementary and linear algebra, groups, rings, fields, and number theory.                                                  |
| Additional / 16 | Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.              |

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

## Questions 1–3

1. For  $x > 0$ ,  $x^2 y'' + (1 - 21)xy' + 54y = 0$ ,  $y(1) = 0$ ,  $y'(1) = 18$ . Find  $y(e)$ .

- (A)  $e^3 + e^{18}$
- (B)  $\frac{e^{18}}{21}$
- (C)  $\frac{6(1 - e^{15})e^3}{5}$
- (D)  $-e^3 + e^{18}$
- (E)  $-\frac{6(1 - e^{15})e^3}{5}$

2. Let  $P = \text{diag}(\underbrace{1, \dots, 1}_2, \underbrace{0, \dots, 0}_{90})$ . What is the dimension of the space of  $92 \times 92$  real matrices  $X$  satisfying  $PX = X$ ?

- (A) 184
- (B) 182
- (C) 185
- (D) 186
- (E) 183

3. Evaluate  $\int_0^1 \int_0^x x^{18} y^3 dy dx$ .

- (A)  $\frac{1}{88}$
- (B)  $\frac{1}{23}$
- (C)  $\frac{1}{76}$
- (D)  $\frac{1}{115}$
- (E)  $\frac{1}{92}$

**Questions 4–6**

4.  $X_1, \dots, X_{36}$  are independent with common mean and variance 38. With  $\bar{X}$  their sample mean, find  $E[\sum_i (X_i - \bar{X})^2]$ .

- (A) 1330
- (B) 887
- (C) 444
- (D) 1773
- (E) 2216

5. Evaluate  $\int_C (3x^2 + 18y) dx + (18x + 3y^2) dy$ , where  $C$  is the line segment from  $(0, 0)$  to  $(5, 1)$ .

- (A) 432
- (B) 217
- (C)  $-216$
- (D) 216
- (E) 108

6. A  $3 \times 3$  matrix  $A$  has characteristic polynomial  $(t - 85)(t + 1)(t - 2)$ . What is  $\text{tr}(A^2)$ ?

- (A) 7228
- (B) 7231
- (C) 7230
- (D) 7229
- (E) 7232

## Questions 7–9

7. Solve  $y'' - 39y' + 378y = 0$  with  $y(0) = 1$ ,  $y'(0) = 0$ . Find  $y(1)$ .

- (A)  $(-7 + 6e^3)e^{18}$
- (B)  $(7 - 6e^3)e^{18}$
- (C)  $(14 - 12e^3)e^{18}$
- (D)  $(7 - 6e^3)e^{18} + 1$
- (E)  $\frac{(7 - 6e^3)e^{18}}{2}$

8. A permutation of 38 objects is chosen uniformly. Let  $Z$  be the number of fixed points. Find  $E[Z]$ .

- (A) 2
- (B) 0
- (C) 3
- (D) 4
- (E) 1

9. Find the upward flux of  $\mathbf{F} = (0, 0, z)$  through  $z = 18x^2 + 3y^2$ ,  $0 \leq x, y \leq 1$ .

- (A) 8
- (B) 14
- (C)  $\frac{7}{2}$
- (D) 7
- (E)  $-7$

**Questions 10–12**

**10.** Let  $N$  be the  $91 \times 91$  matrix with ones immediately above the diagonal and zeros elsewhere. Find  $\text{rank } N^{15}$ .

- (A) 76
- (B) 78
- (C) 74
- (D) 75
- (E) 77

**11.** Under  $x = u + 18v$ ,  $y = u - 3v$ , what is the area of the image of  $0 \leq u, v \leq 1$ ?

- (A)  $\frac{1}{21}$
- (B) 441
- (C) 15
- (D) 54
- (E) 21

**12.** A coin success probability  $p$  has uniform prior on  $[0, 1]$ . After observing 36 successes and 2 failures in independent tosses conditional on  $p$ , what is the predictive probability of success on the next toss?

- (A)  $\frac{37}{40}$
- (B)  $\frac{37}{80}$
- (C)  $\frac{3}{40}$
- (D)  $\frac{37}{120}$
- (E)  $\frac{77}{80}$

**Questions 13–15**

**13.** For  $y' = 18y(1 - y/3)$ , what is the derivative of the right-hand side with respect to  $y$  at the positive equilibrium?

- (A) 18
- (B)  $-36$
- (C)  $-9$
- (D)  $-18$
- (E)  $-17$

**14.** Let  $A$  be the  $91 \times 91$  matrix whose diagonal entries are 88 and whose off-diagonal entries are 1. What is its smallest eigenvalue?

- (A) 88
- (B) 85
- (C) 86
- (D) 87
- (E) 89

**15.** Find the directional derivative of  $f(x, y, z) = x^2 + 18y^2 + 3z^2$  at  $(1, 1, 1)$  in the direction  $(1, 5, 0)$ .

- (A)  $14\sqrt{26}$
- (B)  $1 + 7\sqrt{26}$
- (C)  $7\sqrt{26}$
- (D)  $-7\sqrt{26}$
- (E)  $\frac{7\sqrt{26}}{2}$

**Questions 16–18**

**16.** A random variable has mean 0 and variance 36. What upper bound does the Chebyshev inequality give for  $P(|X| \geq 72)$ ?

- (A)  $\frac{1}{432}$
- (B)  $\frac{1}{288}$
- (C)  $\frac{1}{144}$
- (D)  $\frac{143}{144}$
- (E)  $\frac{145}{288}$

**17.** If  $y' = 18y$ ,  $y(0) = 1$ , what is the coefficient of  $x^3$  in the Maclaurin series of  $y$ ?

- (A) 486
- (B)  $-972$
- (C) 972
- (D) 973
- (E) 1944

**18.** Positive real numbers  $x, y$  satisfy  $\log_2 x + \log_2 y = \log_2 1020$  and  $x + y = 97$ . Find  $xy$ .

- (A) 1021
- (B) 1020
- (C) 1022
- (D) 1019
- (E) 1018

**Questions 19–21**

**19.** If  $y' + 18y = 3e^{-18x}$  and  $y(0) = 5$ , find  $y(1)$ .

- (A)  $-\frac{8}{e^{18}}$
- (B)  $\frac{8}{e^{18}} + 1$
- (C)  $\frac{8}{e^{18}}$
- (D)  $\frac{16}{e^{18}}$
- (E)  $\frac{4}{e^{18}}$

**20.** A box is chosen uniformly from two boxes. Box A contains 41 red and 43 blue balls; box B contains 43 red and 41 blue balls. Two balls are drawn without replacement and both are red. What is the conditional probability that A was chosen?

- (A)  $\frac{903}{1723}$
- (B)  $\frac{410}{1723}$
- (C)  $\frac{820}{5169}$
- (D)  $\frac{2543}{3446}$
- (E)  $\frac{820}{1723}$

**21.** Find the coefficient of  $x^3$  in the Taylor series of  $(1 - 18x)^{-2}$  at zero.

- (A)  $-4 \cdot 18^3$
- (B)  $4 \cdot 18^3$
- (C)  $\frac{3}{4} (4 \cdot 18^3)$
- (D)  $\frac{1}{4} (4 \cdot 18^3)$
- (E)  $\frac{1}{18} (4 \cdot 18^3)$

## Questions 22–24

22. A sequence satisfies  $u_0 = 85$  and  $u_{k+1} = 1u_k + 12$  for all  $k \geq 0$ . Find  $u_5$ .
- (A) 146  
(B) 147  
(C) 143  
(D) 144  
(E) 145
23. If  $f(x, y) = \log(x^2 + y^2)$ ,  $x = e^{18t} \cos(3t)$ , and  $y = e^{18t} \sin(3t)$ , find  $d^2f/dt^2$  at  $t = 1$ .
- (A) 36  
(B) 6  
(C) 666  
(D) 1296  
(E) 0
24. Which subset of  $\mathbb{R}$  is compact, with 31 fixed as displayed?
- (A)  $\mathbb{Q} \cap [0, 31]$   
(B)  $(0, 31)$   
(C)  $\{0\} \cup \{31/k : k \geq 1\}$   
(D)  $31\mathbb{Z}$   
(E)  $\{31/k : k \geq 1\}$

## Questions 25–27

25. Evaluate  $\int_0^1 \frac{x^2}{1+18x^3} dx$ .

- (A)  $\frac{\log(19)}{18}$
- (B)  $\log(19)$
- (C)  $-\frac{\log(19)}{54}$
- (D)  $\frac{\log(19)}{54}$
- (E)  $\frac{\log(19)}{3}$

26. A linear map  $T : \mathbb{R}^{92} \rightarrow \mathbb{R}^{94}$  has rank 4. What is its nullity?

- (A) 87
- (B) 88
- (C) 90
- (D) 86
- (E) 89

27. For which positive integer  $m$  does  $\sum_{j=2}^{\infty} \frac{(-1)^j(x-18)^j}{j^{\frac{2}{3}}(\log j)^m}$  converge absolutely at  $x = 17$ ? Give the least such  $m$ , or 0 if none exists.

- (A) 2
- (B) 4
- (C) 3
- (D) 1
- (E) 0

**Questions 28–30**

**28.** For  $a_k = (-1)^k + 31/k$ , find  $\limsup_{k \rightarrow \infty} a_k - \liminf_{k \rightarrow \infty} a_k$ .

- (A) 2
- (B) 1
- (C) 0
- (D) 3
- (E) 4

**29.** The solution of  $y' = 18x^2y^2$ ,  $y(0) = 1$ , has its first positive singularity at which  $x$ ?

- (A)  $\sqrt[3]{6}$
- (B)  $\frac{\sqrt[3]{12}}{6}$
- (C)  $\frac{\sqrt[3]{18}}{6}$
- (D)  $\frac{6^{\frac{2}{3}}}{6}$
- (E)  $\frac{1}{6}$

**30.** What is the dimension of the real vector space of  $91 \times 91$  symmetric matrices?

- (A) 4185
- (B) 4186
- (C) 4188
- (D) 4184
- (E) 4187

## Questions 31–33

31. Evaluate  $\int_0^1 x^{17}(\log x)^2 dx$ .

- (A)  $\frac{1}{3} \left( \frac{(-1)^2 2!}{18^3} \right)$
- (B)  $18 \left( \frac{(-1)^2 2!}{18^3} \right)$
- (C)  $162 \left( \frac{(-1)^2 2!}{18^3} \right)$
- (D)  $-\frac{(-1)^2 2!}{18^3}$
- (E)  $\frac{(-1)^2 2!}{18^3}$

32. For which real  $a$  is  $f(x) = x^a$  in  $L^{31}(0, 1)$ ?

- (A)  $a < 1/31$
- (B) All real  $a$
- (C)  $a > -1/31$
- (D)  $a > 0$
- (E)  $a \geq -1/31$

33. For the curve  $x = t^3$ ,  $y = t^4 + 18t$ , find  $d^2y/dx^2$  at  $t = 1$ .

- (A)  $-\frac{32}{3}$
- (B)  $-\frac{32}{9}$
- (C)  $\frac{40}{9}$
- (D)  $\frac{22}{3}$
- (E)  $-6$

## Questions 34–36

34. Let  $A = \begin{bmatrix} 1 & 85 & 0 \\ 0 & 1 & 12 \\ 0 & 0 & 1 \end{bmatrix}$ . What is the  $(1, 2)$  entry of  $A^2$ ?

- (A) 169
- (B) 172
- (C) 168
- (D) 170
- (E) 171

35. Evaluate  $\int_0^1 x^{18}(1-x)^2 dx$ .

- (A)  $\frac{18! 2!}{21!}$
- (B)  $70 \left( \frac{18! 2!}{21!} \right)$
- (C)  $19 \left( \frac{18! 2!}{21!} \right)$
- (D)  $21 \left( \frac{18! 2!}{21!} \right)$
- (E)  $\frac{1}{3} \left( \frac{18! 2!}{21!} \right)$

36. A map  $T : \mathbb{R} \rightarrow \mathbb{R}$  satisfies  $|T(x) - T(y)| \leq |x - y|/31$ . If  $T(x_*) = x_*$  and  $|x_0 - x_*| \leq 1$ , what is the smallest guaranteed upper bound for  $|T^3(x_0) - x_*|$ ?

- (A)  $\frac{14896}{29791}$
- (B)  $\frac{1}{59582}$
- (C)  $\frac{1}{89373}$
- (D)  $\frac{1}{29791}$
- (E)  $\frac{29790}{29791}$

**Questions 37–39**

**37.** The solution of  $y' = 18x^3y^2$  has  $y(0) = 1/19$ . Find  $y(1)$ .

- (A)  $\frac{2}{29}$
- (B)  $\frac{4}{29}$
- (C)  $-\frac{2}{29}$
- (D)  $\frac{31}{29}$
- (E)  $\frac{1}{29}$

**38.** An invertible  $91 \times 91$  matrix  $A$  has determinant 4. Find  $\det(A^T A)$ .

- (A) 17
- (B) 16
- (C) 14
- (D) 18
- (E) 15

**39.** Evaluate  $\lim_{x \rightarrow 0} (1 + 18x)^{1/(2x)}$ .

- (A)  $e^{19}$
- (B)  $e^9$
- (C)  $e^{\frac{1}{9}}$
- (D) 18
- (E)  $e^{324}$

## Questions 40–42

40. The circle  $|z| = 2$  is traversed counterclockwise. What is  $\frac{1}{2\pi i} \int_{|z|=2} \frac{e^z}{z^{27}} dz$ ?

- (A)  $26!$
- (B)  $1/27!$
- (C)  $1/25!$
- (D)  $1/26!$
- (E)  $1/52!$

41. For  $f(x) = x^3 e^{18x}$ , find  $f^{(5)}(0)$ .

- (A)  $\frac{1}{20} \left( \frac{5! 18^2}{2} \right)$
- (B)  $\frac{1}{60} \left( \frac{5! 18^2}{2} \right)$
- (C)  $\frac{5! 18^2}{2}$
- (D)  $\frac{1}{9} \left( \frac{5! 18^2}{2} \right)$
- (E)  $2 \left( \frac{5! 18^2}{2} \right)$

42. On the space  $P_{92}$  of real polynomials of degree at most 92, define  $T(p) = p'$ . What is the rank of  $T$ ?

- (A) 90
- (B) 94
- (C) 92
- (D) 93
- (E) 91

## Questions 43–45

43. Find the arc length of  $r(t) = (18t, 3t)$  for  $0 \leq t \leq 1$ .

- (A)  $3\sqrt{37}$
- (B)  $-3\sqrt{37}$
- (C)  $\frac{3\sqrt{37}}{2}$
- (D)  $6\sqrt{37}$
- (E)  $1 + 3\sqrt{37}$

44. Find the residue of  $z^{26}/(z-1)^2$  at  $z = 1$ .

- (A) 18
- (B) 26
- (C) 34
- (D) 42
- (E) 10

45. Let  $F(x) = \int_0^x (x-t)^2 e^{18t} dt$ . Find  $F^{(3)}(0)$ .

- (A) 0
- (B) 6
- (C)  $\frac{1}{2}$
- (D) 2
- (E) 36

**Questions 46–48**

**46.** How many monic irreducible polynomials of degree 2 are there over  $\mathbb{F}_{59}$ ?

- (A) 1710
- (B) 1711
- (C) 1709
- (D) 1712
- (E) 1713

**47.** Near  $(0, 0)$ ,  $z = z(x, y)$  satisfies  $z + 18z^2 = x + 3y$  and  $z(0, 0) = 0$ . Find  $z_{xy}(0, 0)$ .

- (A)  $-36$
- (B) 108
- (C)  $-54$
- (D)  $-108$
- (E)  $-3$

**48.** What is the least positive multiplicative inverse of 2 modulo 73?

- (A) 13
- (B) 61
- (C) 37
- (D) 25
- (E) 49

## Questions 49–51

49. Evaluate  $\iiint_{x^2+y^2+z^2 \leq 18} (x^2 + y^2 + z^2)^2 dV$ .

- (A)  $7 \left( \frac{4\pi 18^{\frac{7}{2}}}{7} \right)$
- (B)  $\frac{7}{90} \left( \frac{4\pi 18^{\frac{7}{2}}}{7} \right)$
- (C)  $\frac{1}{2} \left( \frac{4\pi 18^{\frac{7}{2}}}{7} \right)$
- (D)  $\frac{4\pi 18^{\frac{7}{2}}}{7}$
- (E)  $\frac{1}{3} \left( \frac{4\pi 18^{\frac{7}{2}}}{7} \right)$

50. How many elements  $z$  of  $\mathbb{F}_{318}$  satisfy  $z^3 = z$ ?

- (A) 5
- (B) 4
- (C) 1
- (D) 3
- (E) 2

51. For  $D = \{(x, y) : x^2 + y^2 \leq 18\}$ , evaluate  $\iint_D (x^2 + y^2)^2 dA$ .

- (A)  $5832\pi$
- (B)  $3888\pi$
- (C)  $108\pi$
- (D)  $\frac{5832\pi}{5}$
- (E)  $1944\pi$

**Questions 52–54**

**52.** How many truth assignments to  $p, q, r_1, \dots, r_{34}$  make  $p \Rightarrow q$  false?

- (A)  $2^{34}$
- (B)  $2^{35}$
- (C)  $2^{33}$
- (D)  $2^{36}$
- (E)  $2^{37}$

**53.** Let  $f(x) = \log(1 + 18x)$ . Find  $f^{(3)}(0)$ .

- (A)  $(-1)^2 2! 18^3$
- (B)  $\frac{1}{3} \left( (-1)^2 2! 18^3 \right)$
- (C)  $3 \left( (-1)^2 2! 18^3 \right)$
- (D)  $\frac{1}{18} \left( (-1)^2 2! 18^3 \right)$
- (E)  $-(-1)^2 2! 18^3$

**54.** How many generators does the cyclic group of order 59 have?

- (A) 59
- (B) 60
- (C) 58
- (D) 57
- (E) 56

## Questions 55–57

55. Let  $\mathbf{F} = (18y^3, z^3, x^3)$ . Find  $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (0, 0, 1)$ .

- (A) 54
- (B)  $-27$
- (C)  $-53$
- (D)  $-108$
- (E)  $-54$

56. What is the coefficient of  $x^2$  in  $(1 + 2x)^{36}$ ?

- (A) 1680
- (B) 4200
- (C) 3360
- (D) 840
- (E) 2520

57. Let  $f(x, y) = e^{18x+3y}$ . Find  $\partial^3 f / \partial x^2 \partial y$  at  $(0, 0)$ .

- (A)  $\frac{1}{6} (3 \cdot 18^2)$
- (B)  $3 \cdot 18^2$
- (C)  $\frac{1}{3} (3 \cdot 18^2)$
- (D)  $\frac{1}{36} (3 \cdot 18^2)$
- (E)  $6 (3 \cdot 18^2)$

**Questions 58–60**

**58.** Let  $R = (\mathbb{Z}/44\mathbb{Z})/([22])$ , where  $([22])$  is the ideal generated by  $[22]$ . How many elements does  $R$  have?

- (A) 20
- (B) 21
- (C) 22
- (D) 23
- (E) 24

**59.** Find the curvature at  $t = 0$  of  $r(t) = (3t, 18t^2)$ .

- (A) 4
- (B) 5
- (C) 8
- (D) 2
- (E)  $-4$

**60.** How many edges does the complete bipartite graph  $K_{36,38}$  have?

- (A) 456
- (B) 1368
- (C) 1824
- (D) 912
- (E) 2280

## Questions 61–63

61. What is the maximum of  $x^3y^{18}$  over  $x, y \geq 0$  subject to  $x + y = 1$ ?

- (A)  $\left(\frac{3}{21}\right)^3 \left(\frac{18}{21}\right)^{18}$
- (B)  $\left(\frac{3}{21}\right)^{18} \left(\frac{18}{21}\right)^3$
- (C)  $21 \left( \left(\frac{3}{21}\right)^3 \left(\frac{18}{21}\right)^{18} \right)$
- (D)  $\frac{6}{49}$
- (E)  $\frac{1}{2^{21}}$

62. What is the largest integer  $k$  such that  $2^k$  divides  $94!$ ?

- (A) 89
- (B) 91
- (C) 87
- (D) 88
- (E) 90

63. Solve  $y' + 18y = x^3e^{-18x}$ ,  $y(0) = 0$ , and find  $y(1)$ .

- (A)  $\frac{1}{4e^{18}}$
- (B)  $\frac{9}{2e^{18}}$
- (C)  $e^{-18}$
- (D)  $\frac{1}{3e^{18}}$
- (E)  $\frac{e^{18}}{4}$

**Questions 64–66**

**64.** How many functions from a set of 41 elements onto a set of 3 elements exist?

- (A)  $3^{41}$
- (B)  $3^{41} - 3(2^{41}) - 3$
- (C)  $3^{41} - 3(2^{41}) + 3$
- (D)  $3^{41} - 3(2^{41})$
- (E)  $3^{41} - 2^{41}$

**65.** Find the total outward flux of  $\mathbf{F} = (18x^3, y^3, z^3)$  through the boundary of  $[0, 1]^3$ .

- (A) 21
- (B) 20
- (C) 10
- (D) 40
- (E)  $-20$

**66.** In the ring  $\mathbb{Z}/534\mathbb{Z}$ , how many elements are units?

- (A) 174
- (B) 177
- (C) 178
- (D) 175
- (E) 176

# Answer Key

Practice Test 89 / Questions 1–66

| Question | Answer | Question | Answer | Question | Answer |
|----------|--------|----------|--------|----------|--------|
| 1        | E      | 23       | E      | 45       | D      |
| 2        | A      | 24       | C      | 46       | B      |
| 3        | E      | 25       | D      | 47       | D      |
| 4        | A      | 26       | B      | 48       | C      |
| 5        | D      | 27       | E      | 49       | D      |
| 6        | C      | 28       | A      | 50       | D      |
| 7        | B      | 29       | D      | 51       | E      |
| 8        | E      | 30       | B      | 52       | A      |
| 9        | D      | 31       | E      | 53       | A      |
| 10       | A      | 32       | C      | 54       | C      |
| 11       | E      | 33       | B      | 55       | E      |
| 12       | A      | 34       | D      | 56       | E      |
| 13       | D      | 35       | A      | 57       | B      |
| 14       | D      | 36       | D      | 58       | C      |
| 15       | C      | 37       | A      | 59       | A      |
| 16       | C      | 38       | B      | 60       | B      |
| 17       | C      | 39       | B      | 61       | A      |
| 18       | B      | 40       | D      | 62       | A      |
| 19       | C      | 41       | C      | 63       | A      |
| 20       | E      | 42       | C      | 64       | C      |
| 21       | B      | 43       | A      | 65       | B      |
| 22       | E      | 44       | B      | 66       | E      |

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