

GRE Mathematics Practice Test 90

Standard-Level Target

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A mixed standard-level target, intended to approximate public GRE Mathematics practice in scope and overall demand. This is cycle 17 of 19, position 5 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. For $D = \{(x, y) : x^2 + y^2 \leq 18\}$, evaluate $\iint_D (x^2 + y^2)^1 dA$.
- (A) 324π
(B) 81π
(C) 108π
(D) 162π
(E) 9π
2. A sequence satisfies $u_0 = 86$ and $u_{k+1} = 1u_k + 10$ for all $k \geq 0$. Find u_4 .
- (A) 128
(B) 126
(C) 127
(D) 124
(E) 125
3. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(18x) - \sum_{j=0}^0 (-1)^j (18x)^{2j+1} / (2j+1)!}{x^3}$.
- (A) $-\frac{(-1)^1 18^3}{3!}$
(B) $\frac{1}{18} \left(\frac{(-1)^1 18^3}{3!} \right)$
(C) $18 \left(\frac{(-1)^1 18^3}{3!} \right)$
(D) $3 \left(\frac{(-1)^1 18^3}{3!} \right)$
(E) $\frac{(-1)^1 18^3}{3!}$

Questions 4–6

4. $X_1, \dots, X_{37}$ are independent with common mean and variance 39. With $\bar{X}$ their sample mean, find $E[\sum_i (X_i - \bar{X})^2]$.

- (A) 936
- (B) 468
- (C) 2340
- (D) 1404
- (E) 1872

5. Evaluate $\int_C (2x + 18y) dx + (18x + 2y) dy$, where C is the line segment from $(0, 0)$ to $(6, 1)$.

- (A) $\frac{145}{2}$
- (B) 290
- (C) -145
- (D) 146
- (E) 145

6. A linear map $T : \mathbb{R}^{93} \rightarrow \mathbb{R}^{95}$ has rank 3. What is its nullity?

- (A) 91
- (B) 90
- (C) 88
- (D) 92
- (E) 89

Questions 7–9

7. For $x > 0$, $x^2 y'' + (1 - 20)xy' + 36y = 0$, $y(1) = 0$, $y'(1) = 18$. Find $y(e)$.
- (A) $\frac{9(1 - e^{16})e^2}{8}$
(B) $-e^2 + e^{18}$
(C) $-\frac{9(1 - e^{16})e^2}{8}$
(D) $e^2 + e^{18}$
(E) $\frac{e^{18}}{20}$
8. A permutation of 39 objects is chosen uniformly. Let Z be the number of fixed points. Find $E[Z]$.
- (A) 3
(B) 2
(C) 0
(D) 4
(E) 1
9. Let C be the unit circle $x^2 + y^2 = 1$, $z = 2$, counterclockwise viewed from above. Evaluate $\int_C -18y(x^2 + y^2) dx + (18x(x^2 + y^2)) dy$.
- (A) -36π
(B) 36π
(C) 72π
(D) 18π
(E) $1 + 36\pi$

Questions 10–12

10. What is the dimension of the real vector space of 92×92 symmetric matrices?

- (A) 4278
- (B) 4277
- (C) 4279
- (D) 4280
- (E) 4276

11. Let $f(x) = \log(1 + 18x)$. Find $f^{(2)}(0)$.

- (A) $\frac{1}{2} \left((-1)^1 1! 18^2 \right)$
- (B) $-(-1)^1 1! 18^2$
- (C) $(-1)^1 1! 18^2$
- (D) $2 \left((-1)^1 1! 18^2 \right)$
- (E) $\frac{1}{18} \left((-1)^1 1! 18^2 \right)$

12. A coin success probability p has uniform prior on $[0, 1]$. After observing 37 successes and 2 failures in independent tosses conditional on p , what is the predictive probability of success on the next toss?

- (A) $\frac{38}{41}$
- (B) $\frac{38}{123}$
- (C) $\frac{19}{41}$
- (D) $\frac{79}{82}$
- (E) $\frac{3}{41}$

Questions 13–15

13. Let $F(x) = \int_0^x (x-t)^1 e^{18t} dt$. Find $F^{(2)}(0)$.
- (A) 18
(B) $\frac{1}{2}$
(C) 1
(D) 0
(E) 2
14. Let $A = \begin{bmatrix} 1 & 86 & 0 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{bmatrix}$. What is the $(1, 2)$ entry of A^3 ?
- (A) 257
(B) 258
(C) 259
(D) 260
(E) 256
15. Find the curvature at $t = 0$ of $r(t) = (2t, 18t^2)$.
- (A) -9
(B) 10
(C) 9
(D) $\frac{9}{2}$
(E) 18

Questions 16–18

16. A random variable has mean 0 and variance 37. What upper bound does the Chebyshev inequality give for $P(|X| \geq 74)$?

- (A) $\frac{1}{296}$
- (B) $\frac{147}{148}$
- (C) $\frac{1}{444}$
- (D) $\frac{149}{296}$
- (E) $\frac{1}{148}$

17. If $y'' + 4y = 0$, $y(0) = 18$, and $y'(0) = 6$, find $y(\pi/4)$.

- (A) -3
- (B) 6
- (C) 3
- (D) 4
- (E) $\frac{3}{2}$

18. An invertible 92×92 matrix A has determinant 2. Find $\det(A^T A)$.

- (A) 5
- (B) 4
- (C) 6
- (D) 3
- (E) 2

Questions 19–21

19. Find the coefficient of x^4 in $\cos(18x)/(1 - 1x^2)$.

- (A) -4213
- (B) 4374
- (C) 4213
- (D) 4212
- (E) 101112

20. A box is chosen uniformly from two boxes. Box A contains 42 red and 44 blue balls; box B contains 44 red and 42 blue balls. One ball is drawn and is red. What is the conditional probability that A was chosen?

- (A) $\frac{32}{43}$
- (B) $\frac{7}{43}$
- (C) $\frac{21}{86}$
- (D) $\frac{22}{43}$
- (E) $\frac{21}{43}$

21. If $y' = 18y$, $y(0) = 1$, what is the coefficient of x^2 in the Maclaurin series of y ?

- (A) 162
- (B) 81
- (C) 163
- (D) 324
- (E) -162

Questions 22–24

22. On the space P_{93} of real polynomials of degree at most 93, define $T(p) = p'$. What is the rank of T ?

- (A) 94
- (B) 91
- (C) 95
- (D) 93
- (E) 92

23. Evaluate $\iiint_B 18(x^2 + y^2 + z^2)^1 dV$, where B is the unit ball.

- (A) $\frac{36\pi}{5}$
- (B) $\frac{144\pi}{5}$
- (C) $\frac{72\pi}{5}$
- (D) $-\frac{72\pi}{5}$
- (E) $1 + \frac{72\pi}{5}$

24. Which subset of $\mathbb{R}$ is compact, with 32 fixed as displayed?

- (A) $\mathbb{Q} \cap [0, 32]$
- (B) $\{0\} \cup \{32/k : k \geq 1\}$
- (C) $(0, 32)$
- (D) $\{32/k : k \geq 1\}$
- (E) $32\mathbb{Z}$

Questions 25–27

25. For $\mathbf{F} = (18x^2, y^2, z^2)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) 21
- (B) 42
- (C) -42
- (D) 84
- (E) 43

26. Subspaces U, V of $\mathbb{R}^{94}$ have dimensions 3 and 4. What is the largest possible value of $\dim(U \cap V)$?

- (A) 1
- (B) 3
- (C) 4
- (D) 5
- (E) 2

27. Evaluate $\int_0^1 x^{18}(1-x)^1 dx$.

- (A) $\frac{18! 1!}{20!}$
- (B) $\frac{1}{2} \left(\frac{18! 1!}{20!} \right)$
- (C) $10 \left(\frac{18! 1!}{20!} \right)$
- (D) $20 \left(\frac{18! 1!}{20!} \right)$
- (E) $19 \left(\frac{18! 1!}{20!} \right)$

Questions 28–30

28. For $a_k = (-1)^k + 32/k$, find $\limsup_{k \rightarrow \infty} a_k - \liminf_{k \rightarrow \infty} a_k$.

- (A) 1
- (B) 3
- (C) 0
- (D) 2
- (E) 4

29. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^{18} y^1 dV$.

- (A) $\frac{18! 1!}{22!}$
- (B) $6 \left(\frac{18! 1!}{22!} \right)$
- (C) $\frac{1}{6} \left(\frac{18! 1!}{22!} \right)$
- (D) $\frac{1}{38}$
- (E) $22 \left(\frac{18! 1!}{22!} \right)$

30. Let V consist of real polynomials of degree at most 9 that vanish at 86, 87, and 89. Find $\dim V$.

- (A) 7
- (B) 8
- (C) 5
- (D) 9
- (E) 6

Questions 31–33

31. Find the coefficient of x^2 in $e^{1x} \log(1 + 18x)$.

- (A) -288
- (B) 144
- (C) -144
- (D) -126
- (E) -162

32. For which real a is $f(x) = x^a$ in $L^{32}(0, 1)$?

- (A) $a > -1/32$
- (B) $a > 0$
- (C) $a < 1/32$
- (D) All real a
- (E) $a \geq -1/32$

33. Evaluate $\int_0^\infty x^2 e^{-18x} dx$.

- (A) $\frac{1}{18} \left(\frac{2!}{18^3} \right)$
- (B) $5832 \left(\frac{2!}{18^3} \right)$
- (C) $\frac{2!}{18^3}$
- (D) $\frac{1}{2} \left(\frac{2!}{18^3} \right)$
- (E) $18 \left(\frac{2!}{18^3} \right)$

Questions 34–36

34. The roots r, s of $x^2 - 96x + 860 = 0$ are counted with multiplicity. Find $r^2 + s^2$.

- (A) 7497
- (B) 7498
- (C) 7496
- (D) 7494
- (E) 7495

35. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(18+2j)(20+2j)}$.

- (A) $\frac{1}{2}$
- (B) $\frac{1}{18}$
- (C) $\frac{1}{20}$
- (D) $\frac{1}{36}$
- (E) $\frac{1}{9}$

36. A map $T : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $|T(x) - T(y)| \leq |x - y|/32$. If $T(x_*) = x_*$ and $|x_0 - x_*| \leq 1$, what is the smallest guaranteed upper bound for $|T^3(x_0) - x_*|$?

- (A) $\frac{1}{98304}$
- (B) $\frac{1}{65536}$
- (C) $\frac{1}{32768}$
- (D) $\frac{32768}{32769}$
- (E) $\frac{65536}{32767}$

Questions 37–39

37. Evaluate $\int_0^1 \int_0^x x^{18} y^2 \, dy \, dx$.

- (A) $\frac{1}{66}$
- (B) $\frac{1}{57}$
- (C) $\frac{1}{63}$
- (D) $\frac{1}{22}$
- (E) $\frac{1}{88}$

38. For real t , let $M(t) = \begin{pmatrix} t - 86 & 10 & 0 \\ 0 & t - 86 & 1 \\ 0 & 0 & t - 86 \end{pmatrix}$. At $t = 86$, what is $\text{rank } M(t)$?

- (A) 0
- (B) 4
- (C) 1
- (D) 2
- (E) 3

39. For $f(x, y) = x^2 + 18xy + 2y^2$, what is its maximum directional derivative at $(1, -1)$?

- (A) 30
- (B) $2\sqrt{113}$
- (C) 452
- (D) -16
- (E) 14

Questions 40–42

40. The circle $|z| = 2$ is traversed counterclockwise. What is $\frac{1}{2\pi i} \int_{|z|=2} \frac{e^z}{z^{28}} dz$?

- (A) $1/28!$
- (B) $1/54!$
- (C) $1/27!$
- (D) $27!$
- (E) $1/26!$

41. Evaluate $\int_0^1 \frac{x^1}{1 + 18x^2} dx$.

- (A) $\frac{\log(19)}{2}$
- (B) $\log(19)$
- (C) $\frac{\log(19)}{18}$
- (D) $-\frac{\log(19)}{36}$
- (E) $\frac{\log(19)}{36}$

42. Let $u = (17, 2)^T$ and $v = (1, 0)^T$. Find the squared norm of the orthogonal projection of v onto $\text{span}(u)$.

- (A) $\frac{871}{586}$
- (B) $\frac{582}{293}$
- (C) $\frac{1457}{586}$
- (D) $\frac{285}{586}$
- (E) $\frac{289}{293}$

Questions 43–45

43. The solution of $y' = 18x^2y^2$ has $y(0) = 1/19$. Find $y(1)$.

- (A) $\frac{1}{13}$
- (B) $\frac{2}{13}$
- (C) $\frac{1}{26}$
- (D) $\frac{14}{13}$
- (E) $-\frac{1}{13}$

44. Find the residue of $z^{27}/(z-1)^2$ at $z = 1$.

- (A) 36
- (B) 9
- (C) 45
- (D) 18
- (E) 27

45. Let $f(x, y) = x^2 + 36xy + (326)y^2$. What is the product of the eigenvalues of the Hessian of f ?

- (A) 4
- (B) 2
- (C) 1304
- (D) 8
- (E) 326

Questions 46–48

46. An idempotent 92×92 real matrix P has rank 2. What is $\text{tr } P$?

- (A) 0
- (B) 1
- (C) 3
- (D) 4
- (E) 2

47. Let C be $x^2/4 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C (-y) dx + (18x) dy$.

- (A) 19π
- (B) 38π
- (C) 76π
- (D) $1 + 38\pi$
- (E) -38π

48. What is the least positive multiplicative inverse of 2 modulo 75?

- (A) 26
- (B) 14
- (C) 62
- (D) 50
- (E) 38

Questions 49–51

49. Evaluate $\int_0^{\pi/2} \sin^5 t \cos^{37} t \, dt$.

- (A) $2 \left(\frac{2! 18!}{2 \cdot 21!} \right)$
- (B) $190 \left(\frac{2! 18!}{2 \cdot 21!} \right)$
- (C) $\frac{2! 18!}{2 \cdot 21!}$
- (D) $21 \left(\frac{2! 18!}{2 \cdot 21!} \right)$
- (E) $\frac{1}{2} \left(\frac{2! 18!}{2 \cdot 21!} \right)$

50. In $R = \mathbb{F}_2[x]/(x^{18})$, how many elements are there?

- (A) 2^{19}
- (B) 2^{17}
- (C) 2^{21}
- (D) 2^{20}
- (E) 2^{18}

51. For $x > 0$, $xy' = 18y$ and $y(1) = 1$. Find $y(e^2)$.

- (A) e^{36}
- (B) $1 + e^{36}$
- (C) $2e^{36}$
- (D) $-e^{36}$
- (E) $\frac{e^{36}}{2}$

Questions 52–54

52. How many truth assignments to $p, q, r_1, \dots, r_{35}$ make $p \Rightarrow q$ false?

- (A) 2^{36}
- (B) 2^{37}
- (C) 2^{34}
- (D) 2^{35}
- (E) 2^{38}

53. The function $y(x)$ satisfies $y^3 + 18y = 1x$ and $y(0) = 0$. Find $y'(0)$.

- (A) 18
- (B) 0
- (C) $-\frac{1}{18}$
- (D) $\frac{1}{21}$
- (E) $\frac{1}{18}$

54. Integers x satisfy $x \equiv 1 \pmod{5}$ and $x \equiv 4 \pmod{8}$. Find the least nonnegative solution.

- (A) 37
- (B) 36
- (C) 38
- (D) 34
- (E) 35

Questions 55–57

55. The region $0 \leq x \leq 1$, $0 \leq y \leq 18x^2$ is revolved about the x -axis. Find its volume.

- (A) $-\frac{324\pi}{5}$
- (B) $\frac{648\pi}{5}$
- (C) $\frac{162\pi}{5}$
- (D) $\frac{324\pi}{5}$
- (E) $1 + \frac{324\pi}{5}$

56. What is the coefficient of x^2 in $(1 + 2x)^{37}$?

- (A) 1776
- (B) 888
- (C) 3552
- (D) 2664
- (E) 4440

57. Find the total outward flux of $\mathbf{F} = (18x^2, y^2, z^2)$ through the boundary of $[0, 1]^3$.

- (A) 20
- (B) 10
- (C) -20
- (D) 21
- (E) 40

Questions 58–60

58. The symmetric group S_{92} acts on $\{1, \dots, 92\}$. What is the index of the stabilizer of 1?

- (A) 92
- (B) 91
- (C) 94
- (D) 90
- (E) 93

59. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 18z^2 = x + 2y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.

- (A) -36
- (B) 72
- (C) -144
- (D) -72
- (E) -2

60. How many edges does the complete bipartite graph $K_{37,39}$ have?

- (A) 2405
- (B) 481
- (C) 1924
- (D) 962
- (E) 1443

Questions 61–63

61. Find the arc length of $r(t) = (18t, 2t)$ for $0 \leq t \leq 1$.

- (A) $-2\sqrt{82}$
- (B) $1 + 2\sqrt{82}$
- (C) $2\sqrt{82}$
- (D) $4\sqrt{82}$
- (E) $\sqrt{82}$

62. In $\mathbb{Z}/2802\mathbb{Z}$, how many residues are idempotent ($x^2 = x$)?

- (A) 10
- (B) 8
- (C) 9
- (D) 7
- (E) 6

63. For $y' = 18y(1 - y/2)$, what is the derivative of the right-hand side with respect to y at the positive equilibrium?

- (A) -18
- (B) -17
- (C) -9
- (D) -36
- (E) 18

Questions 64–66

64. How many functions from a set of 42 elements onto a set of 2 elements exist?

- (A) $2^{42} - 3$
- (B) $2^{42} - 2$
- (C) $2^{42} - 1$
- (D) 2^{41}
- (E) 2^{42}

65. Evaluate $\lim_{x \rightarrow 0} (1 + 18x)^{1/(1x)}$.

- (A) $e^{\frac{1}{18}}$
- (B) e^{19}
- (C) 18
- (D) e^{18}
- (E) e^{324}

66. In the additive group $\mathbb{Z}/120\mathbb{Z}$, find the order of $[3]$.

- (A) 41
- (B) 39
- (C) 38
- (D) 40
- (E) 42

Answer Key

Practice Test 90 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	D	23	C	45	D
2	B	24	B	46	E
3	E	25	B	47	B
4	D	26	B	48	E
5	E	27	A	49	C
6	B	28	D	50	E
7	C	29	A	51	A
8	E	30	A	52	D
9	B	31	C	53	E
10	A	32	A	54	B
11	C	33	C	55	D
12	A	34	C	56	D
13	C	35	D	57	A
14	B	36	C	58	A
15	C	37	A	59	D
16	E	38	D	60	E
17	C	39	B	61	C
18	B	40	C	62	B
19	C	41	E	63	A
20	E	42	E	64	B
21	A	43	A	65	D
22	D	44	E	66	D

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