

GRE Mathematics Practice Test 91

Advanced Challenge

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

The most demanding form in this five-test cycle. Emphasis on connected ideas, non-routine reasoning, and careful conceptual distinctions. This is cycle 18 of 19, position 1 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Let $F(x) = \int_0^{x^6} e^{19t} dt$. Find $F'(1)$.

- (A) $\frac{e^{19}}{6}$
- (B) $114e^{19}$
- (C) $6e^{19}$
- (D) e^{19}
- (E) e^{114}

2. Let $V = \begin{bmatrix} 1 & 87 & 7569 \\ 1 & 89 & 7921 \\ 1 & 92 & 8464 \end{bmatrix}$. Find $\det(V^T V)$.

- (A) 901
- (B) 902
- (C) 899
- (D) 898
- (E) 900

3. The region $0 \leq x \leq 1$, $0 \leq y \leq 19x^6$ is revolved about the line $x = -2$. Find its volume.

- (A) $\frac{437\pi}{56}$
- (B) $\frac{437\pi}{14}$
- (C) $-\frac{437\pi}{28}$
- (D) $1 + \frac{437\pi}{28}$
- (E) $\frac{437\pi}{28}$

Questions 4–6

4. Let $X_1, \dots, X_{38}$ be independent uniform random variables on $[0, 1]$. What is $E[(\max_i X_i)^2]$?

- (A) $\frac{19}{20}$
- (B) $\frac{39}{40}$
- (C) $\frac{19}{40}$
- (D) $\frac{19}{60}$
- (E) $\frac{1}{20}$

5. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-19)^j}{j^{\frac{5}{3}} (\log j)^m}$ converge absolutely at $x = 18$? Give the least such m , or 0 if none exists.

- (A) 3
- (B) 4
- (C) 0
- (D) 1
- (E) 2

6. Let $u = (1, 2, \dots, 93)^T$, $v = (1, 1, \dots, 1)^T$, and $B = I + uv^T$. Find $v^T B^{-1}u$.

- (A) $\frac{4371}{4372}$
- (B) $\frac{2185}{4372}$
- (C) $\frac{6557}{4372}$
- (D) $\frac{8743}{4372}$
- (E) $\frac{10929}{4372}$

Questions 7–9

7. For $f(x, y) = x^2 + 19xy + 6y^2$, what is its maximum directional derivative at $(1, -1)$?
- (A) $13\sqrt{2}$
(B) 24
(C) 338
(D) 7
(E) -17
8. Independent trials have success probability $1/38$. Let T count trials up to and including the first success. Find $E[T^2]$.
- (A) 1900
(B) 950
(C) 4750
(D) 2850
(E) 3800
9. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{19t} \cos(6t)$, and $y = e^{19t} \sin(6t)$, find d^2f/dt^2 at $t = 1$.
- (A) 794
(B) 1444
(C) 12
(D) 38
(E) 0

Questions 10–12

10. Let $D = \text{diag}(\underbrace{1, \dots, 1}_{17}, \underbrace{2, \dots, 2}_{76})$. Find the dimension of the space of real matrices X with $XD = DX$ and $\text{tr } X = 0$.

- (A) 6065
- (B) 6066
- (C) 6064
- (D) 6062
- (E) 6063

11. Find the total outward flux of $\mathbf{F} = (19x^6y, y^6z, xz^6)$ through the boundary of $[0, 1]^3$.

- (A) $-\frac{21}{2}$
- (B) 21
- (C) $\frac{21}{2}$
- (D) $\frac{23}{2}$
- (E) $\frac{21}{4}$

12. A uniformly random subset of 38 objects is selected from a collection with 39 red and 40 blue objects. If X is the number of red objects selected, find $\text{Var}(X)$.

- (A) $\frac{62320}{6241}$
- (B) $\frac{15580}{6241}$
- (C) $\frac{37401}{6241}$
- (D) $\frac{31160}{6241}$
- (E) $\frac{43642}{6241}$

Questions 13–15

13. Let $f(x) = \log(1 + 19x)$. Find $f^{(6)}(0)$.

- (A) $\frac{1}{6} \left((-1)^5 5! 19^6 \right)$
- (B) $-(-1)^5 5! 19^6$
- (C) $6 \left((-1)^5 5! 19^6 \right)$
- (D) $(-1)^5 5! 19^6$
- (E) $\frac{1}{19} \left((-1)^5 5! 19^6 \right)$

14. When $x^3 + 1$ is divided by $(x - 88)(x - 89)$, the remainder is $ux + v$. Find u .

- (A) 23499
- (B) 23496
- (C) 23498
- (D) 23497
- (E) 23495

15. Evaluate $\int_0^1 x^{19}(1 - x)^5 dx$.

- (A) $8855 \left(\frac{19! 5!}{25!} \right)$
- (B) $\frac{1}{6} \left(\frac{19! 5!}{25!} \right)$
- (C) $25 \left(\frac{19! 5!}{25!} \right)$
- (D) $20 \left(\frac{19! 5!}{25!} \right)$
- (E) $\frac{19! 5!}{25!}$

Questions 16–18

16. X, Y are independent Poisson variables with means 38 and 39. Given $X + Y = 40$, find $\text{Var}(X \mid X + Y)$.

- (A) $\frac{71138}{5929}$
- (B) $\frac{59280}{5929}$
- (C) $\frac{29640}{5929}$
- (D) $\frac{65209}{5929}$
- (E) $\frac{118560}{5929}$

17. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 19x$ and has density $\rho(x, y) = x^4y$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{7}{16}$
- (B) $-\frac{7}{8}$
- (C) $\frac{15}{8}$
- (D) $\frac{7}{8}$
- (E) $\frac{7}{4}$

18. Let $P = \text{diag}(\underbrace{1, \dots, 1}_1, \underbrace{0, \dots, 0}_{93})$. What is the dimension of the space of 94×94 real matrices X satisfying $PX = X$ and $XP = 0$?

- (A) 93
- (B) 95
- (C) 91
- (D) 94
- (E) 92

Questions 19–21

19. If $y' = y(1 - y/6)$ and $y(0) = 3$, find $y(-\log 19)$.

- (A) $\frac{13}{10}$
- (B) $\frac{3}{5}$
- (C) $\frac{3}{10}$
- (D) $\frac{3}{20}$
- (E) $-\frac{3}{10}$

20. X, Y are independent, with variances 38 and 39. Find $\text{Cov}(X + 2Y, 3X - Y)$.

- (A) 60
- (B) 36
- (C) 48
- (D) 12
- (E) 24

21. Let C be $x^2/36 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C \left(-\frac{19y^3}{3}\right) dx + \left(\frac{x^3}{3}\right) dy$.

- (A) $\frac{165\pi}{2}$
- (B) $1 + \frac{165\pi}{2}$
- (C) $-\frac{165\pi}{2}$
- (D) $\frac{165\pi}{4}$
- (E) 165π

Questions 22–24

22. A 3×3 matrix A has characteristic polynomial $(t - 87)(t + 1)(t - 2)$. What is $\text{tr}(A^3 - 2A + I)$?

- (A) 658338
- (B) 658336
- (C) 658339
- (D) 658335
- (E) 658337

23. Let C be the unit circle $x^2 + y^2 = 1$, $z = 6$, counterclockwise viewed from above. Evaluate $\int_C -19y(x^2 + y^2)^5 dx + (19x(x^2 + y^2)^5) dy + 2z dz$.

- (A) -38π
- (B) 19π
- (C) $1 + 38\pi$
- (D) 76π
- (E) 38π

24. A continuous $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(q) = q^{28}$ for every rational q . Find $f(2)$.

- (A) 2^{30}
- (B) 2^{29}
- (C) 2^{31}
- (D) 2^{28}
- (E) 2^{27}

Questions 25–27

25. For $x > 0$, $x^2 y'' + (1 - 25)xy' + 114y = 0$, $y(1) = 0$, $y'(1) = 19$. Find $y(e)$.

- (A) $\frac{19(1 - e^{13})e^6}{13}$
- (B) $-\frac{19(1 - e^{13})e^6}{13}$
- (C) $-e^6 + e^{19}$
- (D) $\frac{e^{19}}{25}$
- (E) $e^6 + e^{19}$

26. Let N be the 93×93 matrix with ones immediately above the diagonal and zeros elsewhere. Find $\dim(\ker N^{32} / \ker N^{16})$.

- (A) 18
- (B) 14
- (C) 16
- (D) 15
- (E) 17

27. Find the arc length of $r(t) = (19(t - t^3/3), 19t^2)$ for $0 \leq t \leq 1/6$.

- (A) $\frac{2071}{324}$
- (B) $\frac{2071}{648}$
- (C) $-\frac{2071}{648}$
- (D) $\frac{2719}{648}$
- (E) $\frac{2071}{1296}$

Questions 28–30

28. Find the radius of convergence of $\sum_{k \geq 1} k^2 z^{2k} / 28^{2k} + \sum_{k \geq 0} z^{2k+1} / (29)^{2k+1}$.

- (A) 19
- (B) 46
- (C) 37
- (D) 10
- (E) 28

29. For $D = \{(x, y) : x^2 + y^2 \leq 19\}$, evaluate $\iint_D (x^2 + y^2)^5 dA$.

- (A) $\frac{47045881\pi}{6}$
- (B) $\frac{47045881\pi}{3}$
- (C) 47045881π
- (D) $\frac{2476099\pi}{6}$
- (E) $\frac{47045881\pi}{11}$

30. Let A be the 93×93 matrix whose diagonal entries are 90 and whose off-diagonal entries are 1. What is its largest eigenvalue?

- (A) 180
- (B) 181
- (C) 184
- (D) 182
- (E) 183

Questions 31–33

31. Find the coefficient of x^4 in $e^{5x} \log(1 + 19x)$.

- (A) -23009
- (B) -552216
- (C) $-\frac{130321}{4}$
- (D) -22990
- (E) 23009

32. On $[0, 1]$, f is 28 on rationals and $1/28$ on irrationals. Find its upper Darboux integral minus its lower Darboux integral.

- (A) $\frac{839}{28}$
- (B) $\frac{783}{14}$
- (C) $\frac{783}{28}$
- (D) $\frac{811}{28}$
- (E) $\frac{783}{56}$

33. Evaluate $\int_0^1 x^{18}(\log x)^5 dx$.

- (A) $19 \left(\frac{(-1)^5 5!}{19^6} \right)$
- (B) $-\frac{1}{19}$
- (C) $\frac{1}{6} \left(\frac{(-1)^5 5!}{19^6} \right)$
- (D) $-\frac{(-1)^5 5!}{19^6}$
- (E) $\frac{(-1)^5 5!}{19^6}$

Questions 34–36

34. Positive real numbers x, y satisfy $\log_2 x + \log_2 y = \log_2 1131$ and $x + y = 100$. Find $x^2 + y^2$.
- (A) 7737
(B) 7738
(C) 7736
(D) 7740
(E) 7739
35. If $y' = 38xy$ and $y(0) = 1$, find the coefficient of x^{12} in the Maclaurin series of y .
- (A) $\frac{47046601}{720}$
(B) $\frac{47045881}{360}$
(C) $\frac{47045881}{720}$
(D) $\frac{47045881}{1440}$
(E) $-\frac{47045881}{720}$
36. A set with 28 elements has the discrete topology. How many of its subsets are simultaneously open and closed?
- (A) 2^{30}
(B) 2^{28}
(C) 2^{27}
(D) 2^{31}
(E) 2^{29}

Questions 37–39

37. The local inverse $y(x)$ of $x = 19y + y^3$ satisfies $y(0) = 0$. Find $y^{(7)}(0)$.

- (A) $\frac{60480}{19^{10}}$
- (B) $\frac{1}{19} \left(-\frac{60480}{19^{10}} \right)$
- (C) $\frac{1}{5040} \left(-\frac{60480}{19^{10}} \right)$
- (D) $19 \left(-\frac{60480}{19^{10}} \right)$
- (E) $-\frac{60480}{19^{10}}$

38. A sequence satisfies $u_0 = 87$ and $u_{k+1} = 2u_k + 13$ for all $k \geq 0$. Find u_9 .

- (A) 51186
- (B) 51185
- (C) 51187
- (D) 51188
- (E) 51189

39. Find the directional derivative of $f(x, y, z) = x^2y + 6xz^2 + 19y^2z$ at $(1, 1, 1)$ in the direction $(1, 2, 0)$.

- (A) $1 + \frac{86\sqrt{5}}{5}$
- (B) $\frac{43\sqrt{5}}{5}$
- (C) $\frac{172\sqrt{5}}{5}$
- (D) $\frac{86\sqrt{5}}{5}$
- (E) $-\frac{86\sqrt{5}}{5}$

Questions 40–42

40. How many zeros, counted with multiplicity, does $z^{23} + 26z + 1$ have in $|z| < 1$?

- (A) 1
- (B) 4
- (C) 2
- (D) 0
- (E) 3

41. Evaluate $\int_0^1 \frac{x^5}{1 + 19x^6} dx$.

- (A) $-\frac{\log(20)}{114}$
- (B) $\frac{\log(20)}{6}$
- (C) $\frac{\log(20)}{114}$
- (D) $\log(20)$
- (E) $\frac{\log(20)}{19}$

42. How many generators does the cyclic group of order 51520374361 have?

- (A) $61^8(61 - 1)$
- (B) $61^5(61 - 1)$
- (C) $61^7(61 - 1)$
- (D) $61^4(61 - 1)$
- (E) $61^6(61 - 1)$

Questions 43–45

43. What is the maximum of x^6y^{19} over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\left(\frac{6}{25}\right)^6 \left(\frac{19}{25}\right)^{19}$
- (B) $\frac{114}{625}$
- (C) $\frac{1}{2^{25}}$
- (D) $\left(\frac{6}{25}\right)^{19} \left(\frac{19}{25}\right)^6$
- (E) $25 \left(\left(\frac{6}{25}\right)^6 \left(\frac{19}{25}\right)^{19} \right)$

44. A meromorphic function has 25 zeros and 2 poles inside a positively oriented simple closed contour, counted with multiplicity, and no zeros or poles on it. Find $\frac{1}{2\pi i} \int f'(z)/f(z) dz$.

- (A) 9
- (B) 16
- (C) 37
- (D) 23
- (E) 30

45. If $y'' - 38y' + 361y = 6e^{19x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) $3e^{19}$
- (B) $\frac{3e^{19}}{2}$
- (C) $-3e^{19}$
- (D) $1 + 3e^{19}$
- (E) $6e^{19}$

Questions 46–48

46. Let $R = (\mathbb{Z}/100\mathbb{Z})/([20])$, where $([20])$ is the ideal generated by $[20]$. How many units does R have?

- (A) 10
- (B) 6
- (C) 8
- (D) 9
- (E) 7

47. For $x > 0$, $x^2y'' + (-43)xy' + 475y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

- (A) $-\frac{(1 - e^6)e^{19}}{12}$
- (B) $-\frac{(1 - e^6)e^{19}}{6}$
- (C) $-\frac{e^{19}}{6} + 1 + \frac{e^{25}}{6}$
- (D) $-\frac{(1 - e^6)e^{19}}{3}$
- (E) $\frac{(1 - e^6)e^{19}}{6}$

48. How many nonnegative integer triples sum to 76, if each coordinate is at most 38?

- (A) 1300
- (B) 780
- (C) 520
- (D) 260
- (E) 1040

Questions 49–51

49. Let $f(x, y) = x^2 + 38xy + (367)y^2$. What is the product of the eigenvalues of the Hessian of f ?

- (A) 1468
- (B) 367
- (C) 12
- (D) 6
- (E) 24

50. What is the largest integer k such that 3^k divides $96!$?

- (A) 44
- (B) 48
- (C) 47
- (D) 45
- (E) 46

51. Evaluate $\lim_{x \rightarrow 0} \frac{(1 + 19x)^{1/(5x)} - e^{19/5}}{x}$.

- (A) $e^{\frac{19}{5}}$
- (B) $\frac{361e^{\frac{19}{5}}}{10}$
- (C) $-\frac{361e^{\frac{19}{5}}}{5}$
- (D) $-\frac{361e^{\frac{19}{5}}}{10}$
- (E) $-\frac{19e^{\frac{19}{5}}}{10}$

Questions 52–54

52. A tree has 38 vertices of degree 3; every other vertex has degree 1. How many leaves does it have?

- (A) 27
- (B) 40
- (C) 66
- (D) 14
- (E) 53

53. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(19j)!}{(j!)^{19}} x^{6j}$?

- (A) $19^{-\frac{6}{19}}$
- (B) $19^{-\frac{19}{6}}$
- (C) $\frac{1}{19}$
- (D) $19^{\frac{19}{6}}$
- (E) 19^{-19}

54. In the ring $\mathbb{Z}/546\mathbb{Z}$, how many elements satisfy $x^2 = 1$?

- (A) 7
- (B) 9
- (C) 10
- (D) 8
- (E) 6

Questions 55–57

55. Evaluate $\int_0^1 \int_x^1 19y^6 \, dy \, dx$.

- (A) $\frac{19}{8}$
- (B) $\frac{19}{7}$
- (C) $\frac{19}{56}$
- (D) $\frac{19}{6}$
- (E) $\frac{19}{2}$

56. A connected planar graph has 41 vertices and 77 edges. How many faces, including the unbounded face, are in a planar embedding?

- (A) 14
- (B) 50
- (C) 38
- (D) 26
- (E) 62

57. Find the coefficient of x^6 in $\cos(19x)/(1 - 5x^2)$.

- (A) $-\frac{30656731}{720}$
- (B) $\frac{30656731}{720}$
- (C) $-\frac{30657451}{720}$
- (D) $-\frac{47045881}{720}$
- (E) -30656731

Questions 58–60

58. A permutation consists of disjoint cycles of lengths 5 and 11, together with a disjoint transposition when the sum of these two lengths is odd. It fixes all other points. What is its order?

- (A) 57
- (B) 53
- (C) 54
- (D) 56
- (E) 55

59. Evaluate $\int_0^{\pi/2} \sin^{13} t \cos^{39} t \, dt$.

- (A) $\frac{1}{2} \left(\frac{6! 19!}{2 \cdot 26!} \right)$
- (B) $26 \left(\frac{6! 19!}{2 \cdot 26!} \right)$
- (C) $\frac{1}{52}$
- (D) $\frac{6! 19!}{2 \cdot 26!}$
- (E) $2 \left(\frac{6! 19!}{2 \cdot 26!} \right)$

60. How many integers in $\{1, \dots, 228\}$ are divisible by 2 or 3?

- (A) 202
- (B) 102
- (C) 252
- (D) 52
- (E) 152

Questions 61–63

61. Evaluate $\int_0^1 \int_0^x x^{19} y^6 dy dx$.

- (A) $\frac{1}{189}$
- (B) $\frac{1}{140}$
- (C) $\frac{1}{27}$
- (D) $\frac{1}{182}$
- (E) $\frac{1}{216}$

62. In $R = \mathbb{F}_{11}[x]/(x^{19})$, how many elements are units?

- (A) $11^{20} - 11^{18}$
- (B) $11^{19} - 11^0$
- (C) $11^{19} - 11^{18}$
- (D) $11^{21} - 11^{20}$
- (E) $11^{20} - 11^{19}$

63. For $f(x) = x^6 e^{19x}$, find $f^{(8)}(0)$.

- (A) $\frac{1}{20160} \left(\frac{8! 19^2}{2} \right)$
- (B) $\frac{1}{56} \left(\frac{8! 19^2}{2} \right)$
- (C) $\frac{8! 19^2}{2}$
- (D) $2 \left(\frac{8! 19^2}{2} \right)$
- (E) $\frac{2}{19} \left(\frac{8! 19^2}{2} \right)$

Questions 64–66

64. Let $F_0 = 0$, $F_1 = 1$, and $F_{k+2} = F_{k+1} + F_k$. How many binary strings of length 38 have no adjacent 1s?

- (A) F_{41}
- (B) F_{39}
- (C) F_{40}
- (D) F_{38}
- (E) F_{42}

65. Under $x = u + 19v$, $y = u - 6v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) 13
- (B) 25
- (C) 114
- (D) $\frac{1}{25}$
- (E) 625

66. Integers x satisfy $x \equiv 3 \pmod{7}$ and $x \equiv 4 \pmod{9}$. Find the sum of all such x with $0 \leq x < 126$.

- (A) 123
- (B) 125
- (C) 126
- (D) 127
- (E) 124

Answer Key

Practice Test 91 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	C	23	E	45	A
2	E	24	D	46	C
3	E	25	B	47	B
4	A	26	C	48	B
5	D	27	B	49	E
6	A	28	E	50	E
7	A	29	A	51	D
8	D	30	D	52	B
9	E	31	A	53	B
10	C	32	C	54	D
11	C	33	E	55	A
12	D	34	B	56	C
13	D	35	C	57	A
14	D	36	B	58	E
15	E	37	E	59	D
16	B	38	C	60	E
17	D	39	D	61	A
18	A	40	A	62	C
19	C	41	C	63	C
20	B	42	B	64	C
21	A	43	A	65	B
22	E	44	D	66	B

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