

GRE Mathematics Practice Test 93

Upper-Intermediate Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

Intermediate reinforcement, combining standard applications with selected more demanding arguments. This is cycle 18 of 19, position 3 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Find the coefficient of x^4 in $e^{3x} \log(1 + 19x)$.

- (A) -634752
- (B) -26448
- (C) -26429
- (D) 26448
- (E) $-\frac{130321}{4}$

2. Linear maps $T : \mathbb{R}^{96} \rightarrow \mathbb{R}^{96}$ and $U : \mathbb{R}^{96} \rightarrow \mathbb{R}^{96}$ have ranks 5 and 3. What is the largest possible rank of UT ?

- (A) 4
- (B) 2
- (C) 1
- (D) 5
- (E) 3

3. Evaluate $\int_0^{\log 2} e^x (e^x + 19)^4 dx$.

- (A) $\frac{21^5 - 20^5}{5}$
- (B) $\frac{21^5}{5}$
- (C) $21^4 - 20^4$
- (D) $-\frac{21^5 - 20^5}{5}$
- (E) $5 \left(\frac{21^5 - 20^5}{5} \right)$

Questions 4–6

4. Let $X_1, \dots, X_{40}$ be independent uniform random variables on $[0, 1]$. What is $E[(\max_i X_i)^2]$?
- (A) $\frac{20}{21}$
(B) $\frac{20}{63}$
(C) $\frac{10}{21}$
(D) $\frac{41}{42}$
(E) $\frac{1}{21}$
5. Evaluate $\int_C (4x^3 + 19y) dx + (19x + 4y^3) dy$, where C is $r(t) = (t, 4t^2)$, $0 \leq t \leq 1$.
- (A) 333
(B) 666
(C) 334
(D) $\frac{333}{2}$
(E) -333
6. Fix a nonzero vector $u \in \mathbb{R}^{95}$. What is the dimension of the space of skew-symmetric 95×95 matrices A satisfying $Au = 0$?
- (A) 4369
(B) 4373
(C) 4371
(D) 4370
(E) 4372

Questions 7–9

7. Let $f(x, y) = x^2 + 38xy + (365)y^2$. What is the product of the eigenvalues of the Hessian of f ?
- (A) 365
(B) 1460
(C) 8
(D) 16
(E) 4
8. Independent trials have success probability $1/40$. Let T count trials up to and including the first success. Find $E[T]$.
- (A) 14
(B) 66
(C) 53
(D) 40
(E) 27
9. Find the upward flux of $\mathbf{F} = (x, y, z)$ through $z = 19x^2 + 4y^2$, $0 \leq x, y \leq 1$.
- (A) $-\frac{23}{6}$
(B) $-\frac{23}{3}$
(C) $-\frac{20}{3}$
(D) $\frac{23}{3}$
(E) $-\frac{46}{3}$

Questions 10–12

10. Let $A = \begin{bmatrix} 1 & 89 & 0 \\ 0 & 1 & 14 \\ 0 & 0 & 1 \end{bmatrix}$. What is the $(1, 3)$ entry of A^2 ?

- (A) 1247
- (B) 1248
- (C) 1245
- (D) 1246
- (E) 1244

11. For $\mathbf{F} = (19x^4y, y^4z, xz^4)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) -188
- (B) 188
- (C) 189
- (D) 376
- (E) 94

12. A uniformly random subset of 40 objects is selected from a collection with 41 red and 42 blue objects. If X is the number of red objects selected, find $E[X]$.

- (A) $\frac{1723}{83}$
- (B) $\frac{1806}{83}$
- (C) $\frac{3280}{83}$
- (D) $\frac{1640}{83}$
- (E) $\frac{820}{83}$

Questions 13–15

13. Evaluate $\int_0^{\pi/2} \sin^9 t \cos^{39} t \, dt$.

- (A) $8855 \left(\frac{4! 19!}{2 \cdot 24!} \right)$
- (B) $\frac{4! 19!}{2 \cdot 24!}$
- (C) $\frac{1}{2} \left(\frac{4! 19!}{2 \cdot 24!} \right)$
- (D) $24 \left(\frac{4! 19!}{2 \cdot 24!} \right)$
- (E) $2 \left(\frac{4! 19!}{2 \cdot 24!} \right)$

14. An invertible 95×95 matrix A has determinant 2. Find $\det(\text{adj } A)$.

- (A) $2^{94} \cdot 3$
- (B) $2^{94} \cdot 5$
- (C) 2^{95}
- (D) 2^{94}
- (E) 2^{93}

15. If $y' + 19y = 4x^4 e^{-19x}$ and $y(0) = 4$, find $y(1)$.

- (A) $\frac{48}{5e^{19}}$
- (B) $\frac{24}{5e^{19}} + 1$
- (C) $\frac{12}{5e^{19}}$
- (D) $-\frac{24}{5e^{19}}$
- (E) $\frac{24}{5e^{19}}$

Questions 16–18

16. X, Y are independent Poisson variables with means 40 and 41. Given $X + Y = 42$, find $\text{Var}(X \mid X + Y)$.

- (A) $\frac{27334}{2187}$
- (B) $\frac{45920}{2187}$
- (C) $\frac{11480}{2187}$
- (D) $\frac{22960}{2187}$
- (E) $\frac{25147}{2187}$

17. Let $F(x) = \int_0^x (x - t)^3 e^{19t} dt$. Find $F^{(4)}(0)$.

- (A) 114
- (B) $\frac{1}{6}$
- (C) 6
- (D) 0
- (E) 24

18. On P_{96} , the real polynomials of degree at most 96, define $T(p) = x^2 p'' - 240p$. Find its rank.

- (A) 94
- (B) 95
- (C) 96
- (D) 98
- (E) 97

Questions 19–21

19. Evaluate $\lim_{x \rightarrow 0} \frac{(1 + 19x)^{1/(3x)} - e^{19/3}}{x}$.

- (A) $e^{\frac{19}{3}}$
- (B) $-\frac{361e^{\frac{19}{3}}}{6}$
- (C) $-\frac{361e^{\frac{19}{3}}}{3}$
- (D) $-\frac{19e^{\frac{19}{3}}}{6}$
- (E) $\frac{361e^{\frac{19}{3}}}{6}$

20. X, Y are independent, with variances 40 and 41. Find $\text{Cov}(X + 2Y, 3X - Y)$.

- (A) 38
- (B) 62
- (C) 50
- (D) 14
- (E) 26

21. Let C be $x^2/16 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C (-\frac{19y^3}{3}) dx + (\frac{x^3}{3}) dy$.

- (A) 35π
- (B) $\frac{35\pi}{2}$
- (C) $1 + 35\pi$
- (D) -35π
- (E) 70π

Questions 22–24

22. Subspaces U, V of $\mathbb{R}^{97}$ have dimensions 5 and 6. What is the smallest possible value of $\dim(U \cap V)$?

- (A) 0
- (B) 4
- (C) 1
- (D) 2
- (E) 3

23. Find the total outward flux of $\mathbf{F} = (19x^4y, y^4z, xz^4)$ through the boundary of $[0, 1]^3$.

- (A) $\frac{21}{2}$
- (B) $\frac{23}{2}$
- (C) $\frac{21}{4}$
- (D) 21
- (E) $-\frac{21}{2}$

24. A continuous $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(q) = q^{30}$ for every rational q . Find $f(2)$.

- (A) 2^{32}
- (B) 2^{29}
- (C) 2^{33}
- (D) 2^{30}
- (E) 2^{31}

Questions 25–27

25. Find the coefficient of x^4 in the Taylor series of $(1 - 19x)^{-2}$ at zero.

- (A) $\frac{1}{5} (5 \cdot 19^4)$
- (B) $\frac{1}{19} (5 \cdot 19^4)$
- (C) $5 \cdot 19^4$
- (D) $\frac{4}{5} (5 \cdot 19^4)$
- (E) $2 (5 \cdot 19^4)$

26. Let V consist of real polynomials of degree at most 12 that vanish at 89, 90, and 92. Find $\dim V$.

- (A) 11
- (B) 10
- (C) 8
- (D) 9
- (E) 12

27. Evaluate $\int_0^\infty x^4 e^{-19x} dx$.

- (A) $19 \left(\frac{4!}{19^5} \right)$
- (B) $\frac{1}{19} \left(\frac{4!}{19^5} \right)$
- (C) 24
- (D) $\frac{1}{4} \left(\frac{4!}{19^5} \right)$
- (E) $\frac{4!}{19^5}$

Questions 28–30

28. Find the radius of convergence of $\sum_{k=1}^{\infty} k^{30} x^k / 30^k$.

- (A) 10
- (B) 30
- (C) 20
- (D) 50
- (E) 40

29. If $y'' + 16y = 19$ and $y(0) = y'(0) = 0$, find $y(\pi/4)$.

- (A) $\frac{19}{8}$
- (B) $\frac{27}{8}$
- (C) $-\frac{19}{8}$
- (D) $\frac{19}{16}$
- (E) $\frac{19}{4}$

30. The roots r, s of $x^2 - 103x + 1246 = 0$ are counted with multiplicity. Find $r^3 + s^3$.

- (A) 707713
- (B) 707712
- (C) 707714
- (D) 707711
- (E) 707715

Questions 31–33

31. Let $f(x, y) = e^{19x+4y}$. Find $\partial^4 f / \partial x^3 \partial y$ at $(0, 0)$.

- (A) $\frac{16}{361} (4 \cdot 19^3)$
- (B) $\frac{64}{6859} (4 \cdot 19^3)$
- (C) $\frac{19}{4} (4 \cdot 19^3)$
- (D) $\frac{1}{4} (4 \cdot 19^3)$
- (E) $4 \cdot 19^3$

32. On $[0, 1]$, f is 30 on rationals and $1/30$ on irrationals. Find its upper Darboux integral minus its lower Darboux integral.

- (A) $\frac{929}{30}$
- (B) $\frac{959}{30}$
- (C) $\frac{899}{30}$
- (D) $\frac{899}{60}$
- (E) $\frac{899}{15}$

33. The local inverse $y(x)$ of $x = 19y + y^3$ satisfies $y(0) = 0$. Find $y^{(3)}(0)$.

- (A) $\frac{1}{6} \left(-\frac{6}{19^4} \right)$
- (B) $19 \left(-\frac{6}{19^4} \right)$
- (C) $\frac{6}{19^4}$
- (D) $-\frac{6}{19^4}$
- (E) $\frac{1}{19} \left(-\frac{6}{19^4} \right)$

Questions 34–36

34. For real t , let $M(t) = \begin{pmatrix} t-89 & 14 & 0 \\ 0 & t-89 & 1 \\ 0 & 0 & t-89 \end{pmatrix}$. At $t = 89$, what is $\dim \ker(M(t)^2) - \dim \ker M(t)$?

- (A) 4
- (B) 1
- (C) 0
- (D) 3
- (E) 2

35. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{19t} \cos(4t)$, and $y = e^{19t} \sin(4t)$, find d^2f/dt^2 at $t = 1$.

- (A) 1444
- (B) 754
- (C) 38
- (D) 8
- (E) 0

36. A set with 30 elements has the discrete topology. How many of its subsets are simultaneously open and closed?

- (A) 2^{33}
- (B) 2^{31}
- (C) 2^{30}
- (D) 2^{29}
- (E) 2^{32}

Questions 37–39

37. Let $f(x) = \log(1 + 19x)$. Find $f^{(4)}(0)$.

- (A) $\frac{1}{19} \left((-1)^3 3! 19^4 \right)$
- (B) $4 \left((-1)^3 3! 19^4 \right)$
- (C) $(-1)^3 3! 19^4$
- (D) $\frac{1}{4} \left((-1)^3 3! 19^4 \right)$
- (E) $-(-1)^3 3! 19^4$

38. Let $u = (18, 4)^T$ and $v = (1, 0)^T$. Find the squared distance from $(18, 0)^T$ to $\text{span}(u)$.

- (A) $\frac{2677}{170}$
- (B) $\frac{1381}{85}$
- (C) $\frac{1211}{85}$
- (D) $\frac{1296}{85}$
- (E) $\frac{2507}{170}$

39. If $y' = 38xy$ and $y(0) = 1$, find the coefficient of x^8 in the Maclaurin series of y .

- (A) $\frac{130321}{12}$
- (B) $\frac{130321}{24}$
- (C) $\frac{130345}{24}$
- (D) $-\frac{130321}{24}$
- (E) $\frac{130321}{48}$

Questions 40–42

40. How many zeros, counted with multiplicity, does $z^{25} + 28z + 1$ have in $|z| < 1$?

- (A) 2
- (B) 0
- (C) 3
- (D) 4
- (E) 1

41. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^{19} y^3 dV$.

- (A) $25 \left(\frac{19! 3!}{25!} \right)$
- (B) $6 \left(\frac{19! 3!}{25!} \right)$
- (C) $\frac{1}{80}$
- (D) $\frac{1}{6} \left(\frac{19! 3!}{25!} \right)$
- (E) $\frac{19! 3!}{25!}$

42. The field K has 5^{19} elements. Find the largest possible multiplicative order of a nonzero element of K .

- (A) $5^{18} - 1$
- (B) 5^{19}
- (C) $5^{19} - 1$
- (D) $5^{19} + 1$
- (E) $5^{20} - 1$

Questions 43–45

43. Let C be the unit circle $x^2 + y^2 = 1$, $z = 4$, counterclockwise viewed from above. Evaluate $\int_C -19y(x^2 + y^2)^3 dx + (19x(x^2 + y^2)^3) dy + 4z dz$.

- (A) 38π
- (B) 19π
- (C) -38π
- (D) 76π
- (E) $1 + 38\pi$

44. A meromorphic function has 27 zeros and 2 poles inside a positively oriented simple closed contour, counted with multiplicity, and no zeros or poles on it. Find $\frac{1}{2\pi i} \int f'(z)/f(z) dz$.

- (A) 33
- (B) 41
- (C) 17
- (D) 25
- (E) 9

45. Evaluate $\int_0^1 \int_0^x x^{19} y^4 dy dx$.

- (A) $\frac{1}{125}$
- (B) $\frac{1}{120}$
- (C) $\frac{1}{25}$
- (D) $\frac{1}{100}$
- (E) $\frac{1}{150}$

Questions 46–48

46. Let $G = \mathbb{Z}/564\mathbb{Z} \times \mathbb{Z}/380\mathbb{Z}$. Find the order of G/H , where $H = \{g \in G : 4g = 0\}$.

- (A) 13393
- (B) 13397
- (C) 13394
- (D) 13396
- (E) 13395

47. Solve $y' + 19y = x^4 e^{-19x}$, $y(0) = 0$, and find $y(1)$.

- (A) $\frac{1}{4e^{19}}$
- (B) $\frac{e^{19}}{5}$
- (C) $\frac{1}{5e^{19}}$
- (D) $\frac{19}{5e^{19}}$
- (E) e^{-19}

48. How many nonnegative integer triples sum to 80, if each coordinate is at most 40?

- (A) 1148
- (B) 1435
- (C) 574
- (D) 861
- (E) 287

Questions 49–51

49. Let $\mathbf{F} = (19y^4, z^4, x^4)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (1, 1, 4)/\sqrt{18}$.

- (A) $-26\sqrt{2}$
- (B) $-52\sqrt{2}$
- (C) $-104\sqrt{2}$
- (D) $52\sqrt{2}$
- (E) $1 - 52\sqrt{2}$

50. How many group homomorphisms $\mathbb{Z}/564\mathbb{Z} \rightarrow \mathbb{Z}/380\mathbb{Z}$ are there?

- (A) 4
- (B) 2
- (C) 3
- (D) 5
- (E) 6

51. If $y'' - 38y' + 361y = 4e^{19x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) $4e^{19}$
- (B) $2e^{19}$
- (C) $-2e^{19}$
- (D) $1 + 2e^{19}$
- (E) e^{19}

Questions 52–54

52. A tree has 44 vertices. What is the sum of its vertex degrees?

- (A) 30
- (B) 114
- (C) 58
- (D) 142
- (E) 86

53. The solution of $y' = 19x^3y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\frac{4}{19}$
- (B) $\frac{19^{\frac{3}{4}}\sqrt[4]{2}}{19}$
- (C) $\frac{19^{\frac{3}{4}}}{19}$
- (D) $\frac{\sqrt[4]{19}\sqrt{2}}{2}$
- (E) $\frac{19^{\frac{3}{4}}\sqrt{2}}{19}$

54. How many 1-dimensional subspaces does $\mathbb{F}_{61}^5$ have?

- (A) $\frac{61^8 - 1}{61 - 1}$
- (B) $\frac{61^6 - 1}{61 - 1}$
- (C) $\frac{61^4 - 1}{61 - 1}$
- (D) $\frac{61^7 - 1}{61 - 1}$
- (E) $\frac{61^5 - 1}{61 - 1}$

Questions 55–57

55. What is the maximum of x^4y^{19} over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\left(\frac{4}{23}\right)^{19} \left(\frac{19}{23}\right)^4$
- (B) $23 \left(\left(\frac{4}{23}\right)^4 \left(\frac{19}{23}\right)^{19} \right)$
- (C) $\frac{76}{529}$
- (D) $\frac{1}{2^{23}}$
- (E) $\left(\frac{4}{23}\right)^4 \left(\frac{19}{23}\right)^{19}$

56. A connected planar graph has 43 vertices and 81 edges. How many faces, including the unbounded face, are in a planar embedding?

- (A) 66
- (B) 53
- (C) 40
- (D) 27
- (E) 14

57. Find the curvature of the helix $r(t) = (19 \cos t, 19 \sin t, 4t)$.

- (A) $\frac{19}{754}$
- (B) $\frac{396}{377}$
- (C) $\frac{19}{377}$
- (D) $\frac{38}{377}$
- (E) $-\frac{19}{377}$

Questions 58–60

58. How many monic irreducible polynomials of degree 3 are there over $\mathbb{F}_{61}$?

- (A) 75640
- (B) 75642
- (C) 75641
- (D) 75639
- (E) 75638

59. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-19)^j}{j^1 (\log j)^m}$ converge absolutely at $x = 18$? Give the least such m , or 0 if none exists.

- (A) 0
- (B) 2
- (C) 1
- (D) 4
- (E) 3

60. How many integers in $\{1, \dots, 240\}$ are divisible by 2 or 3?

- (A) 213
- (B) 160
- (C) 107
- (D) 54
- (E) 266

Questions 61–63

61. Find the arc length of $r(t) = (19(t - t^3/3), 19t^2)$ for $0 \leq t \leq 1/4$.

- (A) $\frac{1123}{192}$
- (B) $-\frac{931}{192}$
- (C) $\frac{931}{192}$
- (D) $\frac{931}{384}$
- (E) $\frac{931}{96}$

62. How many elements z of $\mathbb{F}_{5^{19}}$ satisfy $z^{25} = z$?

- (A) 7
- (B) 4
- (C) 6
- (D) 5
- (E) 3

63. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 19x$ and has density $\rho(x, y) = x^2y$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{5}{3}$
- (B) $\frac{11}{6}$
- (C) $\frac{5}{12}$
- (D) $\frac{5}{6}$
- (E) $-\frac{5}{6}$

Questions 64–66

64. Let $F_0 = 0$, $F_1 = 1$, and $F_{k+2} = F_{k+1} + F_k$. How many binary strings of length 40 have no adjacent 1s?

- (A) F_{41}
- (B) F_{43}
- (C) F_{42}
- (D) F_{40}
- (E) F_{44}

65. For $x > 0$, $x^2y'' + (-41)xy' + 437y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

- (A) $-\frac{(1 - e^4)e^{19}}{8}$
- (B) $\frac{(1 - e^4)e^{19}}{4}$
- (C) $-\frac{e^{19}}{4} + 1 + \frac{e^{23}}{4}$
- (D) $-\frac{(1 - e^4)e^{19}}{4}$
- (E) $-\frac{(1 - e^4)e^{19}}{2}$

66. How many generators does the cyclic group of order 13845841 have?

- (A) $61^6(61 - 1)$
- (B) $61^2(61 - 1)$
- (C) $61^5(61 - 1)$
- (D) $61^4(61 - 1)$
- (E) $61^3(61 - 1)$

Answer Key

Practice Test 93 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	B	23	A	45	A
2	E	24	D	46	E
3	A	25	C	47	C
4	A	26	B	48	D
5	A	27	E	49	B
6	C	28	B	50	A
7	D	29	A	51	B
8	D	30	A	52	E
9	B	31	E	53	E
10	D	32	C	54	E
11	B	33	D	55	E
12	D	34	B	56	C
13	B	35	E	57	C
14	D	36	C	58	A
15	E	37	C	59	B
16	D	38	D	60	B
17	C	39	B	61	C
18	C	40	E	62	D
19	B	41	E	63	D
20	A	42	C	64	C
21	A	43	A	65	D
22	A	44	D	66	E

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