

GRE Mathematics Practice Test 96

Advanced Challenge

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

The most demanding form in this five-test cycle. Emphasis on connected ideas, non-routine reasoning, and careful conceptual distinctions. This is cycle 19 of 19, position 1 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Find the curvature of the helix $r(t) = (20 \cos t, 20 \sin t, 6t)$.

- (A) $\frac{5}{218}$
- (B) $\frac{5}{109}$
- (C) $\frac{10}{109}$
- (D) $-\frac{5}{109}$
- (E) $\frac{114}{109}$

2. Fix a nonzero vector $u \in \mathbb{R}^{98}$. What is the dimension of the space of skew-symmetric 98×98 matrices A satisfying $Au = 0$?

- (A) 4657
- (B) 4654
- (C) 4655
- (D) 4658
- (E) 4656

3. Let $f(x, y) = e^{20x+6y}$. Find $\partial^6 f / \partial x^5 \partial y$ at $(0, 0)$.

- (A) $\frac{1}{6} (6 \cdot 20^5)$
- (B) $6 \cdot 20^5$
- (C) $\frac{81}{10000} (6 \cdot 20^5)$
- (D) 46656
- (E) $\frac{10}{3} (6 \cdot 20^5)$

Questions 4–6

4. X has density cx^{38} on $0 < x < 1$ and zero elsewhere. Find $E[X^2]$.

- (A) $\frac{39}{41}$
- (B) $\frac{40}{41}$
- (C) $\frac{39}{82}$
- (D) $\frac{13}{41}$
- (E) $\frac{2}{41}$

5. Let $f(x, y) = x^2 + 40xy + (406)y^2$. What is the product of the eigenvalues of the Hessian of f ?

- (A) 1624
- (B) 12
- (C) 6
- (D) 24
- (E) 406

6. Let $A = \begin{bmatrix} 1 & 92 & 0 \\ 0 & 1 & 14 \\ 0 & 0 & 1 \end{bmatrix}$. What is the $(1, 3)$ entry of A^6 ?

- (A) 19319
- (B) 19318
- (C) 19320
- (D) 19322
- (E) 19321

Questions 7–9

7. If $y'' - 40y' + 400y = 6e^{20x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) $-3e^{20}$
- (B) $\frac{3e^{20}}{2}$
- (C) $6e^{20}$
- (D) $3e^{20}$
- (E) $1 + 3e^{20}$

8. Independent exponential waiting times X, Y have rates 38 and 40. Find $E[\min(X, Y)]$.

- (A) $\frac{77}{78}$
- (B) $\frac{79}{156}$
- (C) $\frac{1}{78}$
- (D) $\frac{1}{156}$
- (E) $\frac{1}{234}$

9. If $y' + 20y = 6x^6e^{-20x}$ and $y(0) = 2$, find $y(1)$.

- (A) $\frac{10}{7e^{20}}$
- (B) $\frac{20}{7e^{20}} + 1$
- (C) $-\frac{20}{7e^{20}}$
- (D) $\frac{20}{7e^{20}}$
- (E) $\frac{40}{7e^{20}}$

Questions 10–12

10. An invertible 98×98 matrix A has determinant 2. Find $\det(\text{adj } A)$.

- (A) $2^{97} \cdot 5$
- (B) $2^{97} \cdot 3$
- (C) 2^{97}
- (D) 2^{98}
- (E) 2^{96}

11. Let C be $x^2/36 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C (-\frac{20y^3}{3}) dx + (\frac{x^3}{3}) dy$.

- (A) 42π
- (B) 84π
- (C) 168π
- (D) -84π
- (E) $1 + 84\pi$

12. $X_1, \dots, X_{38}$ are independent with common mean and variance 40. With $\bar{X}$ their sample mean, find $E[\sum_i (X_i - \bar{X})^2]$.

- (A) 1973
- (B) 494
- (C) 987
- (D) 1480
- (E) 2466

Questions 13–15

13. Find the coefficient of x^6 in $\cos(20x)/(1 - 5x^2)$.

- (A) $\frac{543875}{9}$
- (B) -43510000
- (C) $-\frac{543884}{9}$
- (D) $-\frac{543875}{9}$
- (E) $-\frac{800000}{9}$

14. On P_{99} , the real polynomials of degree at most 99, define $T(p) = x^2p'' - 34xp' + 306p$. Find its rank.

- (A) 96
- (B) 98
- (C) 97
- (D) 99
- (E) 100

15. Find the upward flux of $\mathbf{F} = (x, y, z)$ through $z = 20x^2 + 6y^2$, $0 \leq x, y \leq 1$.

- (A) $-\frac{13}{3}$
- (B) $\frac{26}{3}$
- (C) $-\frac{26}{3}$
- (D) $-\frac{52}{3}$
- (E) $-\frac{23}{3}$

Questions 16–18

16. A permutation of 40 objects is chosen uniformly. Let Z be the number of fixed points. Find $E[Z^2]$.

- (A) 3
- (B) 4
- (C) 0
- (D) 1
- (E) 2

17. Evaluate $\int_0^1 x^{20}(1-x)^5 dx$.

- (A) $21 \left(\frac{20! 5!}{26!} \right)$
- (B) $\frac{32890}{3} \left(\frac{20! 5!}{26!} \right)$
- (C) $26 \left(\frac{20! 5!}{26!} \right)$
- (D) $\frac{1}{6} \left(\frac{20! 5!}{26!} \right)$
- (E) $\frac{20! 5!}{26!}$

18. Subspaces U, V of $\mathbb{R}^{100}$ have dimensions 4 and 5. What is the smallest possible value of $\dim(U \cap V)$?

- (A) 0
- (B) 4
- (C) 2
- (D) 3
- (E) 1

Questions 19–21

19. Find the coefficient of x^6 in the Taylor series of $(1 - 20x)^{-2}$ at zero.

- (A) $2(7 \cdot 20^6)$
- (B) $\frac{1}{20}(7 \cdot 20^6)$
- (C) $7 \cdot 20^6$
- (D) $\frac{6}{7}(7 \cdot 20^6)$
- (E) $\frac{1}{7}(7 \cdot 20^6)$

20. A coin success probability p has uniform prior on $[0, 1]$. After observing 38 successes and 2 failures in independent tosses conditional on p , what is the predictive probability of success on the next toss?

- (A) $\frac{1}{14}$
- (B) $\frac{27}{28}$
- (C) $\frac{13}{14}$
- (D) $\frac{13}{42}$
- (E) $\frac{13}{28}$

21. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(20x) - \sum_{j=0}^4 (-1)^j (20x)^{2j+1} / (2j+1)!}{x^{11}}$.

- (A) $\frac{(-1)^5 20^{11}}{11!}$
- (B) $\frac{1}{20} \left(\frac{(-1)^5 20^{11}}{11!} \right)$
- (C) $20 \left(\frac{(-1)^5 20^{11}}{11!} \right)$
- (D) $-\frac{(-1)^5 20^{11}}{11!}$
- (E) $11 \left(\frac{(-1)^5 20^{11}}{11!} \right)$

Questions 22–24

22. Let V consist of real polynomials of degree at most 9 that have both value and first derivative zero at 92, 93, and 95. Find $\dim V$.

- (A) 2
- (B) 4
- (C) 5
- (D) 3
- (E) 6

23. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(20j)!}{(j!)^{20}} x^{6j}$?

- (A) $20^{-\frac{3}{10}}$
- (B) 20^{-20}
- (C) $20^{-\frac{10}{3}}$
- (D) $20^{\frac{10}{3}}$
- (E) $\frac{1}{20}$

24. For which real a does $f_m(x) = m^a x^{33}(1-x)^m$ converge uniformly to zero on $[0, 1]$?

- (A) All real a
- (B) $a < 33$
- (C) $a > 33$
- (D) $a \leq 33$
- (E) $a < 0$

Questions 25–27

25. For $x > 0$, $x^2y'' + (-45)xy' + 520y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

- (A) $-\frac{(1 - e^6)e^{20}}{6}$
(B) $-\frac{(1 - e^6)e^{20}}{12}$
(C) $-\frac{e^{20}}{6} + 1 + \frac{e^{26}}{6}$
(D) $-\frac{(1 - e^6)e^{20}}{3}$
(E) $\frac{(1 - e^6)e^{20}}{6}$

26. The roots r, s of $x^2 - 106x + 1288 = 0$ are counted with multiplicity. Find $r^4 + s^4$.

- (A) $106^4 + 2(1288^2)$
(B) $106^4 - 4(1288)(106^2) + 1288^2$
(C) $106^4 - 4(1288)(106^2) + 2(1288^2)$
(D) $106^4 - 2(1288)(106^2) + 2(1288^2)$
(E) $106^4 - 4(1288)(106^2)$

27. For $D = \{(x, y) : x^2 + y^2 \leq 20\}$, evaluate $\iint_D (x^2 + y^2)^5 dA$.

- (A) 64000000π
(B) $\frac{64000000\pi}{3}$
(C) $\frac{64000000\pi}{11}$
(D) $\frac{32000000\pi}{3}$
(E) $\frac{1600000\pi}{3}$

Questions 28–30

28. For which real p does $\sum_{k=2}^{\infty} 1/[k(\log k)^{33p}]$ converge?

- (A) $p < 1/33$
- (B) $p > 1/33$
- (C) All real p
- (D) $p \geq 1/33$
- (E) $p > 0$

29. Under $x = u + 20v$, $y = u - 6v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) 120
- (B) 676
- (C) 14
- (D) $\frac{1}{26}$
- (E) 26

30. For real t , let $M(t) = \begin{pmatrix} t-92 & 14 & 0 \\ 0 & t-92 & 1 \\ 0 & 0 & t-92 \end{pmatrix}$. At $t = 92$, what is $\dim \ker(M(t)^2) - \dim \ker M(t)$?

- (A) 4
- (B) 1
- (C) 2
- (D) 3
- (E) 0

Questions 31–33

31. Evaluate $\int_0^{\pi/2} \sin^{13} t \cos^{41} t \, dt$.

- (A) $2 \left(\frac{6! 20!}{2 \cdot 27!} \right)$
- (B) $\frac{1}{54}$
- (C) $\frac{1}{2} \left(\frac{6! 20!}{2 \cdot 27!} \right)$
- (D) $27 \left(\frac{6! 20!}{2 \cdot 27!} \right)$
- (E) $\frac{6! 20!}{2 \cdot 27!}$

32. How many accumulation points in $\mathbb{R}$ does $S = \{1/k : k \geq 1\} \cup \{33 + 1/k : k \geq 1\}$ have?

- (A) 3
- (B) 4
- (C) 2
- (D) 1
- (E) 0

33. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 20z^2 = x + 6y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.

- (A) 240
- (B) -40
- (C) -120
- (D) -6
- (E) -240

Questions 34–36

34. Let $u = (19, 6)^T$ and $v = (1, 0)^T$. Find the squared distance from $(19, 0)^T$ to $\text{span}(u)$.

- (A) $\frac{12996}{397}$
- (B) $\frac{26389}{794}$
- (C) $\frac{25595}{794}$
- (D) $\frac{13393}{397}$
- (E) $\frac{12599}{397}$

35. Find the total outward flux of $\mathbf{F} = (20x^6y, y^6z, xz^6)$ through the boundary of $[0, 1]^3$.

- (A) -11
- (B) 11
- (C) $\frac{11}{2}$
- (D) 12
- (E) 22

36. How many connected components does $\mathbb{R} \setminus \{1, 2, \dots, 33\}$ have?

- (A) 56
- (B) 23
- (C) 12
- (D) 45
- (E) 34

Questions 37–39

37. Evaluate $\lim_{x \rightarrow 0} \frac{(1 + 20x)^{1/(5x)} - e^{20/5}}{x}$.

- (A) $-40e^4$
- (B) $-2e^4$
- (C) e^4
- (D) $40e^4$
- (E) $-80e^4$

38. An idempotent 98×98 real matrix P has rank 5. What is $\det(I + P)$?

- (A) 30
- (B) 33
- (C) 34
- (D) 32
- (E) 31

39. The region $0 \leq x \leq 1$, $0 \leq y \leq 20x^6$ is revolved about the line $x = -2$. Find its volume.

- (A) $\frac{230\pi}{7}$
- (B) $\frac{115\pi}{14}$
- (C) $-\frac{115\pi}{7}$
- (D) $1 + \frac{115\pi}{7}$
- (E) $\frac{115\pi}{7}$

Questions 40–42

40. Let $\zeta = e^{2\pi i/25}$. Find $\sum_{k=0}^{24} \zeta^k$.

- (A) 2
- (B) 4
- (C) 0
- (D) 3
- (E) 1

41. Let $F(x) = \int_0^x (x-t)^5 e^{20t} dt$. Find $F^{(6)}(0)$.

- (A) 0
- (B) 120
- (C) 720
- (D) $\frac{1}{120}$
- (E) 2400

42. In the ring $\mathbb{Z}/576\mathbb{Z}$, how many elements satisfy $x^2 = 1$?

- (A) 8
- (B) 7
- (C) 6
- (D) 9
- (E) 10

Questions 43–45

43. Find the directional derivative of $f(x, y, z) = x^2y + 6xz^2 + 20y^2z$ at $(1, 1, 1)$ in the direction $(1, 2, 0)$.

- (A) $9\sqrt{5}$
- (B) $1 + 18\sqrt{5}$
- (C) $18\sqrt{5}$
- (D) $-18\sqrt{5}$
- (E) $36\sqrt{5}$

44. An analytic function has real part $u(x, y) = x^2 - y^2 + 23x$ and satisfies $f(0) = 0$. Find $\text{Im } f(1 + i)$.

- (A) 33
- (B) 17
- (C) 41
- (D) 25
- (E) 9

45. For $f(x) = x^6 e^{20x}$, find $f^{(8)}(0)$.

- (A) $\frac{8! 20^2}{2}$
- (B) $\frac{1}{56} \left(\frac{8! 20^2}{2} \right)$
- (C) $\frac{1}{10} \left(\frac{8! 20^2}{2} \right)$
- (D) $\frac{1}{20160} \left(\frac{8! 20^2}{2} \right)$
- (E) $2 \left(\frac{8! 20^2}{2} \right)$

Questions 46–48

46. A permutation consists of disjoint cycles of lengths 4 and 9, together with a disjoint transposition when the sum of these two lengths is odd. It fixes all other points. What is its order?

- (A) 36
- (B) 34
- (C) 35
- (D) 38
- (E) 37

47. Evaluate $\lim_{x \rightarrow 0} \frac{e^{20x} - \sum_{j=0}^5 (20x)^j / j!}{x^6}$.

- (A) $6 \left(\frac{20^6}{6!} \right)$
- (B) $-\frac{20^6}{6!}$
- (C) $720 \left(\frac{20^6}{6!} \right)$
- (D) $\frac{20^6}{6!}$
- (E) $\frac{1}{20} \left(\frac{20^6}{6!} \right)$

48. For Newton iteration on $f(x) = x^2 - 1444$ near the root $r = 38$, find $\lim_{k \rightarrow \infty} (x_{k+1} - r) / (x_k - r)^2$ for nonterminating convergent iterates.

- (A) $\frac{75}{76}$
- (B) $\frac{1}{152}$
- (C) $\frac{1}{228}$
- (D) $\frac{77}{152}$
- (E) $\frac{1}{76}$

Questions 49–51

49. For $\mathbf{F} = (20x^6y, y^6z, xz^6)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) 439
- (B) 219
- (C) -438
- (D) 876
- (E) 438

50. In $R = \mathbb{F}_{11}[x]/(x^{20})$, how many elements are units?

- (A) $11^{20} - 11^{19}$
- (B) $11^{20} - 11^0$
- (C) $11^{22} - 11^{21}$
- (D) $11^{21} - 11^{20}$
- (E) $11^{21} - 11^{19}$

51. What is the maximum of x^6y^{20} over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\left(\frac{6}{26}\right)^6 \left(\frac{20}{26}\right)^{20}$
- (B) $\frac{1}{2^{26}}$
- (C) $\left(\frac{6}{26}\right)^{20} \left(\frac{20}{26}\right)^6$
- (D) $26 \left(\left(\frac{6}{26}\right)^6 \left(\frac{20}{26}\right)^{20} \right)$
- (E) $\frac{30}{169}$

Questions 52–54

52. Bisection starts with an interval of length 2^{38} . After 40 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{1}{8}$
- (B) $\frac{1}{16}$
- (C) $\frac{1}{24}$
- (D) $\frac{7}{8}$
- (E) $\frac{9}{16}$

53. Evaluate $\int_0^1 \int_x^1 20y^6 \, dy \, dx$.

- (A) $\frac{5}{14}$
- (B) $\frac{10}{3}$
- (C) $\frac{20}{7}$
- (D) 10
- (E) $\frac{5}{2}$

54. Integers x satisfy $x \equiv 4 \pmod{5}$ and $x \equiv 6 \pmod{8}$. Find the sum of all such x with $0 \leq x < 80$.

- (A) 70
- (B) 68
- (C) 67
- (D) 69
- (E) 66

Questions 55–57

55. For the curve $x = t^6$, $y = t^7 + 20t$, find d^2y/dx^2 at $t = 1$.

- (A) $-\frac{10}{3}$
- (B) $\frac{107}{36}$
- (C) $-\frac{31}{2}$
- (D) $-\frac{31}{12}$
- (E) $\frac{9}{2}$

56. What is the least positive multiplicative inverse of 2 modulo 77?

- (A) 65
- (B) 13
- (C) 52
- (D) 39
- (E) 26

57. Let $f(x) = \log(1 + 20x)$. Find $f^{(6)}(0)$.

- (A) $-(-1)^5 5! 20^6$
- (B) $(-1)^5 5! 20^6$
- (C) $6 \left((-1)^5 5! 20^6 \right)$
- (D) $\frac{1}{20} \left((-1)^5 5! 20^6 \right)$
- (E) $\frac{1}{6} \left((-1)^5 5! 20^6 \right)$

Questions 58–60

58. The symmetric group S_{98} acts on the two-element subsets of $\{1, \dots, 98\}$. What is the index of the setwise stabilizer of $\{1, 2\}$?

- (A) 4752
- (B) 4755
- (C) 4751
- (D) 4754
- (E) 4753

59. The solution of $y' = 20x^5y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\frac{2^{\frac{2}{3}} \sqrt[6]{3} \cdot 5^{\frac{5}{6}}}{10}$
- (B) $\frac{3}{10}$
- (C) $\frac{\sqrt[6]{10} \cdot 3^{\frac{5}{6}}}{3}$
- (D) $\frac{2^{\frac{2}{3}} \cdot 5^{\frac{5}{6}}}{10}$
- (E) $\frac{10^{\frac{5}{6}} \sqrt[6]{3}}{10}$

60. How many truth assignments to $p, q, r_1, \dots, r_{36}$ make $p \Rightarrow q$ false?

- (A) 2^{36}
- (B) 2^{37}
- (C) 2^{39}
- (D) 2^{35}
- (E) 2^{38}

Questions 61–63

61. Find the arc length of $r(t) = (20(t - t^3/3), 20t^2)$ for $0 \leq t \leq 1/6$.

- (A) $\frac{545}{324}$
- (B) $\frac{162}{707}$
- (C) $\frac{162}{545}$
- (D) $-\frac{545}{162}$
- (E) $\frac{545}{81}$

62. In $\mathbb{Z}/3054\mathbb{Z}$, how many residues are not idempotent ($x^2 \neq x$)?

- (A) 3045
- (B) 3048
- (C) 3044
- (D) 3046
- (E) 3047

63. Two solutions of $y'' + 20x^6y' + 6y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $\frac{2}{e^{\frac{20}{7}}}$
- (B) $-\frac{1}{e^{\frac{20}{7}}}$
- (C) $\frac{1}{2e^{\frac{20}{7}}}$
- (D) $e^{-\frac{20}{7}}$
- (E) $e^{-\frac{20}{7}} + 1$

Questions 64–66

64. Find the coefficient of x^3 in $(1+x)^{38}(1-x)^{38}(1+2x)^2$.

- (A) -202
- (B) -252
- (C) -152
- (D) -52
- (E) -102

65. Evaluate $\int_C (6x^5 + 20y) dx + (20x + 6y^5) dy$, where C is $r(t) = (t, 2t^2)$, $0 \leq t \leq 1$.

- (A) $\frac{105}{2}$
- (B) 105
- (C) 106
- (D) 210
- (E) -105

66. In the additive group $\mathbb{Z}/120\mathbb{Z}$, find the index of the subgroup generated by $[2]$.

- (A) 1
- (B) 3
- (C) 0
- (D) 4
- (E) 2

Answer Key

Practice Test 96 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	B	23	C	45	A
2	E	24	B	46	A
3	B	25	A	47	D
4	A	26	C	48	E
5	D	27	D	49	E
6	C	28	B	50	A
7	D	29	E	51	A
8	C	30	B	52	A
9	D	31	E	53	E
10	C	32	C	54	B
11	B	33	E	55	D
12	D	34	A	56	D
13	D	35	B	57	B
14	B	36	E	58	E
15	C	37	A	59	E
16	E	38	D	60	A
17	E	39	E	61	C
18	A	40	C	62	D
19	C	41	B	63	D
20	C	42	A	64	C
21	A	43	C	65	B
22	B	44	D	66	E

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