

GRE Mathematics Practice Test 97

Advanced Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A demanding practice form with frequent multi-step arguments and a mix of computational and conceptual questions. This is cycle 19 of 19, position 2 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Find the arc length of $r(t) = (20(t - t^3/3), 20t^2)$ for $0 \leq t \leq 1/5$.

- (A) $\frac{304}{75}$
- (B) $\frac{379}{75}$
- (C) $-\frac{304}{75}$
- (D) $\frac{152}{75}$
- (E) $\frac{608}{75}$

2. Let V consist of real polynomials of degree at most 8 that have both value and first derivative zero at 93, 94, and 96. Find $\dim V$.

- (A) 1
- (B) 4
- (C) 2
- (D) 5
- (E) 3

3. Evaluate $\int_0^1 x^{19}(\log x)^4 dx$.

- (A) $-\frac{(-1)^4 4!}{20^5}$
- (B) $\frac{(-1)^4 4!}{20^5}$
- (C) $20 \left(\frac{(-1)^4 4!}{20^5} \right)$
- (D) $\frac{20000}{3} \left(\frac{(-1)^4 4!}{20^5} \right)$
- (E) $\frac{1}{5} \left(\frac{(-1)^4 4!}{20^5} \right)$

Questions 4–6

4. X has density cx^{39} on $0 < x < 1$ and zero elsewhere. Find $E[X^2]$.

- (A) $\frac{41}{42}$
- (B) $\frac{10}{21}$
- (C) $\frac{20}{21}$
- (D) $\frac{20}{63}$
- (E) $\frac{1}{21}$

5. Let $f(x, y) = e^{20x+5y}$. Find $\partial^5 f / \partial x^4 \partial y$ at $(0, 0)$.

- (A) $5 \cdot 20^4$
- (B) $\frac{1}{5} (5 \cdot 20^4)$
- (C) $\frac{1}{256} (5 \cdot 20^4)$
- (D) $\frac{1}{64} (5 \cdot 20^4)$
- (E) $4 (5 \cdot 20^4)$

6. The roots r, s of $x^2 - 105x + 1116 = 0$ are counted with multiplicity. Find $r^3 + s^3$.

- (A) 806086
- (B) 806083
- (C) 806087
- (D) 806084
- (E) 806085

Questions 7–9

7. The region $0 \leq x \leq 1$, $0 \leq y \leq 20x^5$ is revolved about the line $x = -3$. Find its volume.

- (A) $\frac{90\pi}{7}$
- (B) $1 + \frac{180\pi}{7}$
- (C) $\frac{180\pi}{7}$
- (D) $\frac{360\pi}{7}$
- (E) $-\frac{180\pi}{7}$

8. Independent exponential waiting times X, Y have rates 39 and 41. Find $P(X < Y)$.

- (A) $\frac{41}{80}$
- (B) $\frac{39}{160}$
- (C) $\frac{119}{160}$
- (D) $\frac{39}{80}$
- (E) $\frac{13}{80}$

9. The local inverse $y(x)$ of $x = 20y + y^3$ satisfies $y(0) = 0$. Find $y^{(5)}(0)$.

- (A) $-\frac{360}{20^7}$
- (B) $20 \left(\frac{360}{20^7} \right)$
- (C) $\frac{1}{20} \left(\frac{360}{20^7} \right)$
- (D) $\frac{1}{120} \left(\frac{360}{20^7} \right)$
- (E) $\frac{360}{20^7}$

Questions 10–12

10. For real t , let $M(t) = \begin{pmatrix} t-93 & 12 & 0 \\ 0 & t-93 & 1 \\ 0 & 0 & t-93 \end{pmatrix}$. At $t = 93$, what is $\dim \ker(M(t)^2) - \dim \ker M(t)$?

- (A) 4
- (B) 3
- (C) 2
- (D) 1
- (E) 0

11. For $x > 0$, $x^2 y'' + (-44)xy' + 500y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

- (A) $\frac{(1 - e^5) e^{20}}{5}$
- (B) $-\frac{(1 - e^5) e^{20}}{5}$
- (C) $-\frac{2(1 - e^5) e^{20}}{5}$
- (D) $-\frac{(1 - e^5) e^{20}}{10}$
- (E) $-\frac{e^{20}}{5} + 1 + \frac{e^{25}}{5}$

12. $X_1, \dots, X_{39}$ are independent with common mean and variance 41. With $\bar{X}$ their sample mean, find $E[\sum_i (X_i - \bar{X})^2]$.

- (A) 520
- (B) 1558
- (C) 1039
- (D) 2596
- (E) 2077

Questions 13–15

13. Evaluate $\int_0^{\infty} x^5 e^{-20x} dx$.

- (A) $\frac{1}{5} \left(\frac{5!}{20^6} \right)$
- (B) $20 \left(\frac{5!}{20^6} \right)$
- (C) $\frac{5!}{20^6}$
- (D) 120
- (E) $\frac{1}{20} \left(\frac{5!}{20^6} \right)$

14. Let $u = (19, 5)^T$ and $v = (1, 0)^T$. Find the squared distance from $(19, 0)^T$ to $\text{span}(u)$.

- (A) $\frac{8639}{386}$
- (B) $\frac{4609}{193}$
- (C) $\frac{9411}{386}$
- (D) $\frac{4416}{193}$
- (E) $\frac{9025}{386}$

15. For $x > 0$, $x^2 y'' + (1 - 25)xy' + 100y = 0$, $y(1) = 0$, $y'(1) = 20$. Find $y(e)$.

- (A) $-e^5 + e^{20}$
- (B) $-\frac{4(1 - e^{15})e^5}{3}$
- (C) $\frac{e^{20}}{25}$
- (D) $e^5 + e^{20}$
- (E) $\frac{4(1 - e^{15})e^5}{3}$

Questions 16–18

16. A permutation of 41 objects is chosen uniformly. Let Z be the number of fixed points. Find $E[Z^2]$.

- (A) 0
- (B) 4
- (C) 3
- (D) 1
- (E) 2

17. The solution of $y' = 20x^5y^3$ has $y(0) = 1/21$. Find $y(1)$.

- (A) $\frac{2\sqrt{3909}}{1303}$
- (B) $\frac{\sqrt{3909}}{1303}$
- (C) $\frac{\sqrt{3909}}{2606}$
- (D) $\frac{\sqrt{3909}}{1303} + 1$
- (E) $-\frac{\sqrt{3909}}{1303}$

18. An idempotent 99×99 real matrix P has rank 1. What is $\det(I + P)$?

- (A) 0
- (B) 3
- (C) 4
- (D) 2
- (E) 1

Questions 19–21

19. Find the directional derivative of $f(x, y, z) = x^2y + 5xz^2 + 20y^2z$ at $(1, 1, 1)$ in the direction $(1, 3, 0)$.

- (A) $\frac{13\sqrt{10}}{2}$
- (B) $-13\sqrt{10}$
- (C) $1 + 13\sqrt{10}$
- (D) $13\sqrt{10}$
- (E) $26\sqrt{10}$

20. A coin success probability p has uniform prior on $[0, 1]$. After observing 39 successes and 2 failures in independent tosses conditional on p , what is the predictive probability of success on the next toss?

- (A) $\frac{83}{86}$
- (B) $\frac{20}{43}$
- (C) $\frac{40}{129}$
- (D) $\frac{40}{43}$
- (E) $\frac{3}{43}$

21. For the curve $x = t^5$, $y = t^6 + 20t$, find d^2y/dx^2 at $t = 1$.

- (A) $\frac{86}{25}$
- (B) -4
- (C) $-\frac{74}{5}$
- (D) $-\frac{74}{25}$
- (E) $\frac{26}{5}$

Questions 22–24

22. For $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} 93 \\ 12 \\ 105 \end{bmatrix}$, let x minimize $\|Ax - b\|^2$. Find the minimum squared residual.

- (A) 5046
- (B) 5045
- (C) 5047
- (D) 5044
- (E) 5048

23. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 20x$ and has density $\rho(x, y) = x^3y$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{6}{7}$
- (B) $\frac{12}{7}$
- (C) $\frac{13}{7}$
- (D) $-\frac{6}{7}$
- (E) $\frac{3}{7}$

24. For which real a does $f_m(x) = m^a x^{34}(1-x)^m$ converge uniformly to zero on $[0, 1]$?

- (A) $a \leq 34$
- (B) All real a
- (C) $a < 0$
- (D) $a > 34$
- (E) $a < 34$

Questions 25–27

25. Evaluate $\int_0^1 \int_0^x x^{20} y^5 dy dx$.

- (A) $\frac{1}{189}$
- (B) $\frac{1}{126}$
- (C) $\frac{1}{27}$
- (D) $\frac{1}{162}$
- (E) $\frac{1}{156}$

26. What is the coefficient of x^7 in $(x + 19)^7(1 - x)^3$?

- (A) -217720
- (B) -217719
- (C) -217718
- (D) -217722
- (E) -217721

27. Let $F(x) = \int_0^x (x - t)^4 e^{20t} dt$. Find $F^{(5)}(0)$.

- (A) 0
- (B) 480
- (C) $\frac{1}{24}$
- (D) 120
- (E) 24

Questions 28–30

28. For which real p does $\sum_{k=2}^{\infty} 1/[k(\log k)^{34p}]$ converge?

- (A) $p > 0$
- (B) All real p
- (C) $p \geq 1/34$
- (D) $p < 1/34$
- (E) $p > 1/34$

29. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^{20} y^4 dV$.

- (A) $\frac{20! 4!}{27!}$
- (B) $6 \left(\frac{20! 4!}{27!} \right)$
- (C) $\frac{1}{6} \left(\frac{20! 4!}{27!} \right)$
- (D) $\frac{1}{105}$
- (E) $27 \left(\frac{20! 4!}{27!} \right)$

30. A matrix A satisfies $(A - 94I)(A - 96I) = 0$, and its minimal polynomial has degree two. If $A^5 = uA + vI$, find u .

- (A) $2^4 \cdot 3 \cdot 11 \cdot 601 \cdot 3851$
- (B) $2^5 \cdot 11 \cdot 601 \cdot 3851$
- (C) $2^4 \cdot 11 \cdot 601 \cdot 3851$
- (D) $2^4 \cdot 5 \cdot 11 \cdot 601 \cdot 3851$
- (E) $2^3 \cdot 11 \cdot 601 \cdot 3851$

Questions 31–33

31. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-20)^j}{j^{\frac{4}{3}} (\log j)^m}$ converge absolutely at $x = 19$? Give the least such m , or 0 if none exists.

- (A) 2
- (B) 0
- (C) 1
- (D) 4
- (E) 3

32. How many accumulation points in $\mathbb{R}$ does $S = \{1/k : k \geq 1\} \cup \{34 + 1/k : k \geq 1\}$ have?

- (A) 2
- (B) 1
- (C) 4
- (D) 0
- (E) 3

33. Find the curvature of the helix $r(t) = (20 \cos t, 20 \sin t, 5t)$.

- (A) $-\frac{4}{85}$
- (B) $\frac{4}{85}$
- (C) $\frac{8}{85}$
- (D) $\frac{2}{85}$
- (E) $\frac{89}{85}$

Questions 34–36

34. Let $V = \begin{bmatrix} 1 & 93 & 8649 \\ 1 & 95 & 9025 \\ 1 & 98 & 9604 \end{bmatrix}$. Find $\det(V^T V)$.

- (A) 901
- (B) 899
- (C) 898
- (D) 900
- (E) 902

35. Evaluate $\iiint_B 20z^2(x^2 + y^2 + z^2)^5 dV$, where B is the unit ball.

- (A) $\frac{32\pi}{9}$
- (B) $-\frac{16\pi}{9}$
- (C) $\frac{8\pi}{9}$
- (D) $1 + \frac{16\pi}{9}$
- (E) $\frac{16\pi}{9}$

36. How many connected components does $\mathbb{R} \setminus \{1, 2, \dots, 34\}$ have?

- (A) 35
- (B) 24
- (C) 57
- (D) 46
- (E) 13

Questions 37–39

37. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{20t} \cos(5t)$, and $y = e^{20t} \sin(5t)$, find $d^2 f/dt^2$ at $t = 1$.

- (A) 0
- (B) 850
- (C) 1600
- (D) 40
- (E) 10

38. Let $u = (1, 2, \dots, 99)^T$, $v = (1, 1, \dots, 1)^T$, and $B = I + uv^T$. Find $v^T B^{-1} u$.

- (A) $\frac{4950}{4951}$
- (B) $\frac{14851}{9902}$
- (C) $\frac{24753}{9902}$
- (D) $\frac{9901}{4951}$
- (E) $\frac{4949}{9902}$

39. Solve $y' + 20y = x^5 e^{-20x}$, $y(0) = 0$, and find $y(1)$.

- (A) $\frac{e^{20}}{6}$
- (B) $\frac{10}{3e^{20}}$
- (C) $\frac{1}{6e^{20}}$
- (D) e^{-20}
- (E) $\frac{1}{5e^{20}}$

Questions 40–42

40. Let $\zeta = e^{2\pi i/26}$. Find $\sum_{k=0}^{25} \zeta^k$.

- (A) 2
- (B) 3
- (C) 1
- (D) 0
- (E) 4

41. Let $F(x) = \int_0^{x^5} e^{20t} dt$. Find $F'(1)$.

- (A) $100e^{20}$
- (B) e^{100}
- (C) $\frac{e^{20}}{5}$
- (D) $5e^{20}$
- (E) e^{20}

42. How many invertible 3×3 matrices over $\mathbb{F}_{67}$ are there whose first column is $(1, 0, 0)^T$?

- (A) $\frac{(67^3 - 67^0)(67^3 - 67^1)(67^3 - 67^2)}{67^3 - 1}$
- (B) $\frac{(67^3 - 67^0)(67^3 - 67^0)(67^3 - 67^0)}{67^3 - 1}$
- (C) $\frac{67^3 67^3 67^3}{67^3 - 1}$
- (D) $\frac{(67^3 - 67^0)(67^3 - 67^1)}{67^3 - 1}$
- (E) $\frac{(67^3 - 67^1)(67^3 - 67^2)(67^3 - 67^3)}{67^3 - 1}$

Questions 43–45

43. Evaluate $\lim_{x \rightarrow 0} \frac{(1 + 20x)^{1/(4x)} - e^{20/4}}{x}$.

- (A) $-50e^5$
- (B) $-100e^5$
- (C) $-\frac{5e^5}{2}$
- (D) $50e^5$
- (E) e^5

44. An analytic function has real part $u(x, y) = x^2 - y^2 + 24x$ and satisfies $f(0) = 0$. Find $\text{Im } f(1 + i)$.

- (A) 26
- (B) 34
- (C) 10
- (D) 18
- (E) 42

45. Evaluate $\int_C (5x^4 + 20y) dx + (20x + 5y^4) dy$, where C is $r(t) = (t, 3t^2)$, $0 \leq t \leq 1$.

- (A) -304
- (B) 304
- (C) 152
- (D) 305
- (E) 608

Questions 46–48

46. How many permutations in S_{99} consist of one 4-cycle and 95 fixed points?

- (A) $2^3 \cdot 3^3 \cdot 7^2 \cdot 11 \cdot 97$
- (B) $2^4 \cdot 3^3 \cdot 7^2 \cdot 11 \cdot 97$
- (C) $2^5 \cdot 3^3 \cdot 7^2 \cdot 11 \cdot 97$
- (D) $2^4 \cdot 3^4 \cdot 7^2 \cdot 11 \cdot 97$
- (E) $2^4 \cdot 3^3 \cdot 5 \cdot 7^2 \cdot 11 \cdot 97$

47. Find the total outward flux of $\mathbf{F} = (20x^5y, y^5z, xz^5)$ through the boundary of $[0, 1]^3$.

- (A) 22
- (B) 11
- (C) -11
- (D) $\frac{11}{2}$
- (E) 12

48. For Newton iteration on $f(x) = x^2 - 1521$ near the root $r = 39$, find $\lim_{k \rightarrow \infty} (x_{k+1} - r)/(x_k - r)^2$ for nonterminating convergent iterates.

- (A) $\frac{1}{234}$
- (B) $\frac{77}{78}$
- (C) $\frac{79}{156}$
- (D) $\frac{1}{156}$
- (E) $\frac{1}{78}$

Questions 49–51

49. Evaluate $\int_0^{\log 2} e^x(e^x + 20)^5 dx$.

- (A) $6 \left(\frac{22^6 - 21^6}{6} \right)$
- (B) $22^5 - 21^5$
- (C) $\frac{22^6}{6}$
- (D) $-\frac{22^6 - 21^6}{6}$
- (E) $\frac{22^6 - 21^6}{6}$

50. How many maximal proper subgroups does the cyclic group of order 1164 have?

- (A) 5
- (B) 4
- (C) 3
- (D) 2
- (E) 1

51. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(20 + 5j)(25 + 5j)}$.

- (A) $\frac{1}{4}$
- (B) $\frac{1}{5}$
- (C) $\frac{1}{100}$
- (D) $\frac{1}{25}$
- (E) $\frac{1}{20}$

Questions 52–54

52. Bisection starts with an interval of length 2^{39} . After 41 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{1}{16}$
- (B) $\frac{1}{24}$
- (C) $\frac{9}{16}$
- (D) $\frac{1}{8}$
- (E) $\frac{7}{8}$

53. Let $\mathbf{F} = (20y^5, z^5, x^5)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (1, 1, 3)/\sqrt{11}$.

- (A) $1 - \frac{310\sqrt{11}}{11}$
- (B) $\frac{310\sqrt{11}}{11}$
- (C) $-\frac{155\sqrt{11}}{11}$
- (D) $-\frac{620\sqrt{11}}{11}$
- (E) $-\frac{310\sqrt{11}}{11}$

54. How many residues x modulo 126 satisfy $5x \equiv 20 \pmod{126}$ and $2x \equiv 0 \pmod{126}$?

- (A) 2
- (B) 4
- (C) 0
- (D) 1
- (E) 3

Questions 55–57

55. Evaluate $\int_0^1 x^{20}(1-x)^4 dx$.

- (A) $21 \left(\frac{20! 4!}{25!} \right)$
- (B) $\frac{1}{5} \left(\frac{20! 4!}{25!} \right)$
- (C) $\frac{20! 4!}{25!}$
- (D) $25 \left(\frac{20! 4!}{25!} \right)$
- (E) $2530 \left(\frac{20! 4!}{25!} \right)$

56. What is the least positive multiplicative inverse of 2 modulo 79?

- (A) 53
- (B) 40
- (C) 27
- (D) 14
- (E) 66

57. What is the maximum of $x^5 y^{20}$ over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\frac{1}{2^{25}}$
- (B) $\left(\frac{5}{25} \right)^5 \left(\frac{20}{25} \right)^{20}$
- (C) $25 \left(\left(\frac{5}{25} \right)^5 \left(\frac{20}{25} \right)^{20} \right)$
- (D) $\left(\frac{5}{25} \right)^{20} \left(\frac{20}{25} \right)^5$
- (E) $\frac{4}{25}$

Questions 58–60

58. The field K has 7^{20} elements. Find the largest possible multiplicative order of a nonzero element of K .

- (A) 7^{20}
- (B) $7^{20} - 1$
- (C) $7^{20} + 1$
- (D) $7^{19} - 1$
- (E) $7^{21} - 1$

59. Let C be the unit circle $x^2 + y^2 = 1$, $z = 5$, counterclockwise viewed from above. Evaluate $\int_C -20y(x^2 + y^2)^4 dx + (20x(x^2 + y^2)^4) dy + 3z dz$.

- (A) -40π
- (B) $1 + 40\pi$
- (C) 20π
- (D) 40π
- (E) 80π

60. How many truth assignments to $p, q, r_1, \dots, r_{37}$ make $p \Rightarrow q$ false?

- (A) 2^{40}
- (B) 2^{37}
- (C) 2^{38}
- (D) 2^{39}
- (E) 2^{36}

Questions 61–63

61. Under $x = u + 20v$, $y = u - 5v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) 25
- (B) 100
- (C) 625
- (D) 15
- (E) $\frac{1}{25}$

62. Let $G = \mathbb{Z}/588\mathbb{Z} \times \mathbb{Z}/396\mathbb{Z}$. Find the order of G/H , where $H = \{g \in G : 12g = 0\}$.

- (A) 1619
- (B) 1616
- (C) 1617
- (D) 1618
- (E) 1615

63. Let C be $x^2/25 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C \left(-\frac{20y^3}{3}\right) dx + \left(\frac{x^3}{3}\right) dy$.

- (A) $1 + \frac{225\pi}{4}$
- (B) $-\frac{225\pi}{4}$
- (C) $\frac{225\pi}{2}$
- (D) $\frac{225\pi}{4}$
- (E) $\frac{225\pi}{8}$

Questions 64–66

64. Find the coefficient of x^3 in $(1+x)^{39}(1-x)^{39}(1+2x)^2$.

- (A) -104
- (B) -208
- (C) -156
- (D) -260
- (E) -52

65. Find the coefficient of x^6 in $\cos(20x)/(1-4x^2)$.

- (A) $\frac{588224}{9}$
- (B) -47057920
- (C) $-\frac{588233}{9}$
- (D) $-\frac{800000}{9}$
- (E) $-\frac{588224}{9}$

66. How many group homomorphisms $\mathbb{Z}/588\mathbb{Z} \rightarrow \mathbb{Z}/396\mathbb{Z}$ have image of order 12?

- (A) 2
- (B) 6
- (C) 4
- (D) 5
- (E) 3

Answer Key

Practice Test 97 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	A	23	A	45	B
2	E	24	E	46	B
3	B	25	D	47	B
4	C	26	A	48	E
5	A	27	E	49	E
6	E	28	E	50	C
7	C	29	A	51	C
8	D	30	C	52	D
9	E	31	C	53	E
10	D	32	A	54	C
11	B	33	B	55	C
12	B	34	D	56	B
13	C	35	E	57	B
14	E	36	A	58	B
15	B	37	A	59	D
16	E	38	A	60	B
17	B	39	C	61	A
18	D	40	D	62	C
19	D	41	D	63	D
20	D	42	A	64	C
21	D	43	A	65	E
22	A	44	A	66	C

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