

GRE Mathematics Practice Test 98

Upper-Intermediate Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

Intermediate reinforcement, combining standard applications with selected more demanding arguments. This is cycle 19 of 19, position 3 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

zarimion.com/gre

100 practice tests, complete answer keys, and future updates.

Please retain the source attribution when sharing this material.



This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Let C be the unit circle $x^2 + y^2 = 1$, $z = 4$, counterclockwise viewed from above. Evaluate $\int_C -20y(x^2 + y^2)^3 dx + (20x(x^2 + y^2)^3) dy + 4z dz$.

- (A) 80π
- (B) 20π
- (C) 40π
- (D) $1 + 40\pi$
- (E) -40π

2. For $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} 94 \\ 15 \\ 109 \end{bmatrix}$, let x minimize $\|Ax - b\|^2$. Find x_2 .

- (A) 7
- (B) $\frac{15}{2}$
- (C) 8
- (D) $\frac{17}{2}$
- (E) $\frac{13}{2}$

3. Find the coefficient of x^4 in the Taylor series of $(1 - 20x)^{-2}$ at zero.

- (A) $\frac{1}{20} (5 \cdot 20^4)$
- (B) $2 (5 \cdot 20^4)$
- (C) $\frac{1}{5} (5 \cdot 20^4)$
- (D) $\frac{4}{5} (5 \cdot 20^4)$
- (E) $5 \cdot 20^4$

Questions 4–6

4. X has density cx^{40} on $0 < x < 1$ and zero elsewhere. Find the normalizing constant c .

- (A) 41
- (B) 15
- (C) 28
- (D) 67
- (E) 54

5. Evaluate $\lim_{x \rightarrow 0} \frac{(1 + 20x)^{1/(3x)} - e^{20/3}}{x}$.

- (A) $e^{\frac{20}{3}}$
- (B) $-\frac{10e^{\frac{20}{3}}}{3}$
- (C) $-\frac{200e^{\frac{20}{3}}}{3}$
- (D) $-\frac{400e^{\frac{20}{3}}}{3}$
- (E) $\frac{200e^{\frac{20}{3}}}{3}$

6. What is the coefficient of x^7 in $(x + 19)^6(1 - x)^3$?

- (A) -5077
- (B) -5075
- (C) -5074
- (D) -5078
- (E) -5076

Questions 7–9

7. Under $x = u + 20v$, $y = u - 4v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) 24
- (B) 16
- (C) $\frac{1}{24}$
- (D) 576
- (E) 80

8. Independent exponential waiting times X, Y have rates 40 and 42. Find $P(X < Y)$.

- (A) $\frac{10}{41}$
- (B) $\frac{21}{41}$
- (C) $\frac{20}{123}$
- (D) $\frac{61}{82}$
- (E) $\frac{20}{41}$

9. Find the arc length of $r(t) = (20(t - t^3/3), 20t^2)$ for $0 \leq t \leq 1/4$.

- (A) $\frac{245}{96}$
- (B) $\frac{245}{48}$
- (C) $-\frac{245}{48}$
- (D) $\frac{245}{24}$
- (E) $\frac{293}{48}$

Questions 10–12

10. A matrix A satisfies $(A - 95I)(A - 97I) = 0$, and its minimal polynomial has degree two. If $A^4 = uA + vI$, find u .

- (A) $2^8 \cdot 3 \cdot 13 \cdot 709$
- (B) $2^6 \cdot 3 \cdot 13 \cdot 709$
- (C) $2^7 \cdot 3^2 \cdot 13 \cdot 709$
- (D) $2^7 \cdot 3 \cdot 5 \cdot 13 \cdot 709$
- (E) $2^7 \cdot 3 \cdot 13 \cdot 709$

11. For $f(x, y) = x^2 + 20xy + 4y^2$, what is its maximum directional derivative at $(1, -1)$?

- (A) -18
- (B) 468
- (C) 30
- (D) $6\sqrt{13}$
- (E) 12

12. $X_1, \dots, X_{40}$ are independent with common mean and variance 42. With $\bar{X}$ their sample mean, find $E[\sum_i (X_i - \bar{X})^2]$.

- (A) 546
- (B) 2730
- (C) 1092
- (D) 2184
- (E) 1638

Questions 13–15

13. What is the maximum of x^4y^{20} over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\frac{1}{2^{24}}$
- (B) $\left(\frac{4}{24}\right)^{20} \left(\frac{20}{24}\right)^4$
- (C) $\frac{5}{36}$
- (D) $24 \left(\left(\frac{4}{24}\right)^4 \left(\frac{20}{24}\right)^{20} \right)$
- (E) $\left(\frac{4}{24}\right)^4 \left(\frac{20}{24}\right)^{20}$

14. Let $V = \begin{bmatrix} 1 & 94 & 8836 \\ 1 & 96 & 9216 \\ 1 & 99 & 9801 \end{bmatrix}$. Find $\det(V^T V)$.

- (A) 902
- (B) 900
- (C) 901
- (D) 899
- (E) 898

15. Evaluate $\int_0^1 x^{20}(1-x)^3 dx$.

- (A) $21 \left(\frac{20! 3!}{24!} \right)$
- (B) $24 \left(\frac{20! 3!}{24!} \right)$
- (C) $506 \left(\frac{20! 3!}{24!} \right)$
- (D) $\frac{20! 3!}{24!}$
- (E) $\frac{1}{4} \left(\frac{20! 3!}{24!} \right)$

Questions 16–18

16. A permutation of 42 objects is chosen uniformly. Let Z be the number of fixed points. Find $E[Z^2]$.

- (A) 2
- (B) 0
- (C) 3
- (D) 4
- (E) 1

17. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(20x) - \sum_{j=0}^2 (-1)^j (20x)^{2j+1} / (2j+1)!}{x^7}$.

- (A) $7 \left(\frac{(-1)^3 20^7}{7!} \right)$
- (B) $20 \left(\frac{(-1)^3 20^7}{7!} \right)$
- (C) $\frac{1}{20} \left(\frac{(-1)^3 20^7}{7!} \right)$
- (D) $-\frac{(-1)^3 20^7}{7!}$
- (E) $\frac{(-1)^3 20^7}{7!}$

18. Let $u = (1, 2, \dots, 100)^T$, $v = (1, 1, \dots, 1)^T$, and $B = I + uv^T$. Find $\det B$.

- (A) 5052
- (B) 5053
- (C) 5050
- (D) 5049
- (E) 5051

Questions 19–21

19. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-20)^j}{j^1 (\log j)^m}$ converge absolutely at $x = 19$? Give the least such m , or 0 if none exists.

- (A) 4
- (B) 1
- (C) 2
- (D) 3
- (E) 0

20. A coin success probability p has uniform prior on $[0, 1]$. After observing 40 successes and 2 failures in independent tosses conditional on p , what is the predictive probability of success on the next toss?

- (A) $\frac{41}{44}$
- (B) $\frac{85}{88}$
- (C) $\frac{41}{132}$
- (D) $\frac{41}{88}$
- (E) $\frac{3}{44}$

21. Evaluate $\iiint_{x^2+y^2+z^2 \leq 20} (x^2 + y^2 + z^2)^3 dV$.

- (A) $\frac{9}{140} \left(\frac{4\pi 20^{\frac{9}{2}}}{9} \right)$
- (B) $\frac{1}{2} \left(\frac{4\pi 20^{\frac{9}{2}}}{9} \right)$
- (C) $\frac{4\pi 20^{\frac{9}{2}}}{9}$
- (D) $\frac{1}{3} \left(\frac{4\pi 20^{\frac{9}{2}}}{9} \right)$
- (E) $9 \left(\frac{4\pi 20^{\frac{9}{2}}}{9} \right)$

Questions 22–24

22. Let $D = \text{diag}(\underbrace{1, \dots, 1}_{18}, \underbrace{2, \dots, 2}_{82})$. Find the dimension of the space of real matrices X with $XD = DX$.

- (A) 7050
- (B) 7048
- (C) 7049
- (D) 7046
- (E) 7047

23. If $y' + 20y = 4x^4e^{-20x}$ and $y(0) = 4$, find $y(1)$.

- (A) $\frac{24}{5e^{20}}$
- (B) $-\frac{24}{5e^{20}}$
- (C) $\frac{24}{5e^{20}} + 1$
- (D) $\frac{12}{5e^{20}}$
- (E) $\frac{48}{5e^{20}}$

24. For which real a does $f_m(x) = m^a x^{35}(1-x)^m$ converge uniformly to zero on $[0, 1]$?

- (A) $a < 35$
- (B) $a \leq 35$
- (C) $a > 35$
- (D) All real a
- (E) $a < 0$

Questions 25–27

25. Evaluate $\iiint_B 20z^2(x^2 + y^2 + z^2)^4 dV$, where B is the unit ball.

- (A) $\frac{40\pi}{39}$
- (B) $\frac{80\pi}{39}$
- (C) $\frac{160\pi}{39}$
- (D) $1 + \frac{80\pi}{39}$
- (E) $-\frac{80\pi}{39}$

26. When $x^3 + 1$ is divided by $(x - 95)(x - 96)$, the remainder is $ux + v$. Find u .

- (A) 27359
- (B) 27362
- (C) 27360
- (D) 27363
- (E) 27361

27. For the curve $x = t^4$, $y = t^5 + 20t$, find d^2y/dx^2 at $t = 1$.

- (A) $-\frac{55}{16}$
- (B) -5
- (C) $-\frac{55}{4}$
- (D) $\frac{25}{4}$
- (E) $\frac{65}{16}$

Questions 28–30

28. For which real p does $\sum_{k=2}^{\infty} 1/[k(\log k)^{35p}]$ converge?

- (A) All real p
- (B) $p > 1/35$
- (C) $p > 0$
- (D) $p < 1/35$
- (E) $p \geq 1/35$

29. Find the upward flux of $\mathbf{F} = (x, y, z)$ through $z = 20x^2 + 4y^2$, $0 \leq x, y \leq 1$.

- (A) 8
- (B) -8
- (C) -16
- (D) -7
- (E) -4

30. Let $P = \text{diag}(\underbrace{1, \dots, 1}_1, \underbrace{0, \dots, 0}_{100})$. What is the dimension of the space of 101×101 real matrices X satisfying $PX = X$ and $XP = 0$?

- (A) 102
- (B) 98
- (C) 101
- (D) 100
- (E) 99

Questions 31–33

31. For $x > 0$, $x^2 y'' + (1 - 24)xy' + 80y = 0$, $y(1) = 0$, $y'(1) = 20$. Find $y(e)$.

- (A) $e^4 + e^{20}$
- (B) $\frac{e^{20}}{24}$
- (C) $\frac{5(1 - e^{16})e^4}{4}$
- (D) $-e^4 + e^{20}$
- (E) $-\frac{5(1 - e^{16})e^4}{4}$

32. How many accumulation points in $\mathbb{R}$ does $S = \{1/k : k \geq 1\} \cup \{35 + 1/k : k \geq 1\}$ have?

- (A) 1
- (B) 0
- (C) 3
- (D) 2
- (E) 4

33. Let $f(x, y) = e^{20x+4y}$. Find $\partial^4 f / \partial x^3 \partial y$ at $(0, 0)$.

- (A) $4 \cdot 20^3$
- (B) $5(4 \cdot 20^3)$
- (C) $\frac{1}{25}(4 \cdot 20^3)$
- (D) $\frac{1}{4}(4 \cdot 20^3)$
- (E) $\frac{1}{125}(4 \cdot 20^3)$

Questions 34–36

34. A 3×3 matrix A has characteristic polynomial $(t - 94)(t + 1)(t - 2)$. What is $\text{tr}(A^3)$?

- (A) 830592
- (B) 830590
- (C) 830589
- (D) 830591
- (E) 830593

35. Find the coefficient of x^4 in $e^{3x} \log(1 + 20x)$.

- (A) -787440
- (B) -32810
- (C) 32810
- (D) -32790
- (E) -40000

36. How many connected components does $\mathbb{R} \setminus \{1, 2, \dots, 35\}$ have?

- (A) 12
- (B) 60
- (C) 36
- (D) 24
- (E) 48

Questions 37–39

37. For $\mathbf{F} = (20x^4y, y^4z, xz^4)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) 197
- (B) 392
- (C) 196
- (D) -196
- (E) 98

38. Let N be the 100×100 matrix with ones immediately above the diagonal and zeros elsewhere. Find $\dim(\ker N^{34} / \ker N^{17})$.

- (A) 16
- (B) 18
- (C) 17
- (D) 15
- (E) 19

39. Evaluate $\int_0^1 \int_x^1 20y^4 \, dy \, dx$.

- (A) 5
- (B) 4
- (C) 10
- (D) $\frac{2}{3}$
- (E) $\frac{10}{3}$

Questions 40–42

40. Let $\zeta = e^{2\pi i/27}$. Find $\sum_{k=0}^{26} \zeta^k$.
- (A) 0
(B) 2
(C) 4
(D) 1
(E) 3
41. Solve $y' + 20y = x^4 e^{-20x}$, $y(0) = 0$, and find $y(1)$.
- (A) $\frac{e^{20}}{5}$
(B) $\frac{1}{4e^{20}}$
(C) e^{-20}
(D) $\frac{4}{e^{20}}$
(E) $\frac{1}{5e^{20}}$
42. How many 1-dimensional subspaces does $\mathbb{F}_{67}^5$ have?
- (A) $\frac{67^7 - 1}{67 - 1}$
(B) $\frac{67^4 - 1}{67 - 1}$
(C) $\frac{67^6 - 1}{67 - 1}$
(D) $\frac{67^5 - 1}{67 - 1}$
(E) $\frac{67^8 - 1}{67 - 1}$

Questions 43–45

43. Evaluate $\int_C (4x^3 + 20y) dx + (20x + 4y^3) dy$, where C is $r(t) = (t, 4t^2)$, $0 \leq t \leq 1$.

- (A) 674
- (B) 338
- (C) $\frac{337}{2}$
- (D) 337
- (E) -337

44. An analytic function has real part $u(x, y) = x^2 - y^2 + 25x$ and satisfies $f(0) = 0$. Find $\text{Im } f(1 + i)$.

- (A) 9
- (B) 45
- (C) 27
- (D) 18
- (E) 36

45. For $x > 0$, $x^2 y'' + (-43)xy' + 480y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

- (A) $-\frac{(1 - e^4) e^{20}}{4}$
- (B) $\frac{(1 - e^4) e^{20}}{4}$
- (C) $-\frac{(1 - e^4) e^{20}}{2}$
- (D) $-\frac{e^{20}}{4} + 1 + \frac{e^{24}}{4}$
- (E) $-\frac{(1 - e^4) e^{20}}{8}$

Questions 46–48

46. How many monic irreducible polynomials of degree 3 are there over $\mathbb{F}_{67}$?

- (A) 100234
- (B) 100232
- (C) 100231
- (D) 100233
- (E) 100230

47. Evaluate $\int_0^1 \int_0^x x^{20} y^4 dy dx$.

- (A) $\frac{1}{125}$
- (B) $\frac{1}{105}$
- (C) $\frac{1}{26}$
- (D) $\frac{1}{130}$
- (E) $\frac{1}{156}$

48. Apply one Newton iteration to solve $x^2 - 40 = 0$, starting at $x_0 = 1$. What is x_1 ?

- (A) $\frac{45}{2}$
- (B) $\frac{41}{2}$
- (C) $\frac{43}{2}$
- (D) $\frac{41}{4}$
- (E) 41

Questions 49–51

49. The local inverse $y(x)$ of $x = 20y + y^3$ satisfies $y(0) = 0$. Find $y^{(3)}(0)$.

- (A) $\frac{6}{20^4}$
- (B) $-\frac{6}{20^4}$
- (C) $20\left(-\frac{6}{20^4}\right)$
- (D) $\frac{1}{6}\left(-\frac{6}{20^4}\right)$
- (E) $\frac{1}{20}\left(-\frac{6}{20^4}\right)$

50. How many elements z of $\mathbb{F}_{5^{20}}$ satisfy $z^{25} = z$?

- (A) 25
- (B) 24
- (C) 26
- (D) 23
- (E) 27

51. Find the curvature of the helix $r(t) = (20 \cos t, 20 \sin t, 4t)$.

- (A) $\frac{5}{104}$
- (B) $\frac{5}{208}$
- (C) $\frac{109}{104}$
- (D) $-\frac{5}{104}$
- (E) $\frac{5}{52}$

Questions 52–54

52. Bisection starts with an interval of length 2^{40} . After 42 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{1}{16}$
- (B) $\frac{1}{24}$
- (C) $\frac{1}{8}$
- (D) $\frac{7}{8}$
- (E) $\frac{9}{16}$

53. Evaluate $\int_0^\infty x^4 e^{-20x} dx$.

- (A) $\frac{1}{4} \left(\frac{4!}{20^5} \right)$
- (B) $\frac{1}{20} \left(\frac{4!}{20^5} \right)$
- (C) 24
- (D) $\frac{4!}{20^5}$
- (E) $20 \left(\frac{4!}{20^5} \right)$

54. How many generators does the cyclic group of order 20151121 have?

- (A) $67^5(67 - 1)$
- (B) $67^2(67 - 1)$
- (C) $67^6(67 - 1)$
- (D) $67^3(67 - 1)$
- (E) $67^4(67 - 1)$

Questions 55–57

55. Evaluate $\int_0^1 \frac{x^3}{1+20x^4} dx$.

- (A) $\frac{\log(21)}{80}$
- (B) $-\frac{\log(21)}{80}$
- (C) $\frac{\log(21)}{20}$
- (D) $\frac{\log(21)}{4}$
- (E) $\log(21)$

56. What is the least positive multiplicative inverse of 2 modulo 81?

- (A) 15
- (B) 28
- (C) 41
- (D) 54
- (E) 67

57. Evaluate $\int_0^{\pi/2} \sin^9 t \cos^{41} t dt$.

- (A) $10626 \left(\frac{4! 20!}{2 \cdot 25!} \right)$
- (B) $25 \left(\frac{4! 20!}{2 \cdot 25!} \right)$
- (C) $2 \left(\frac{4! 20!}{2 \cdot 25!} \right)$
- (D) $\frac{4! 20!}{2 \cdot 25!}$
- (E) $\frac{1}{2} \left(\frac{4! 20!}{2 \cdot 25!} \right)$

Questions 58–60

58. Let $R = (\mathbb{Z}/138\mathbb{Z})/([23])$, where $([23])$ is the ideal generated by $[23]$. How many units does R have?

- (A) 20
- (B) 21
- (C) 23
- (D) 22
- (E) 24

59. Let $\mathbf{F} = (20y^4, z^4, x^4)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (1, 1, 4)/\sqrt{18}$.

- (A) $-\frac{82\sqrt{2}}{3}$
- (B) $1 - \frac{164\sqrt{2}}{3}$
- (C) $-\frac{328\sqrt{2}}{3}$
- (D) $\frac{164\sqrt{2}}{3}$
- (E) $-\frac{164\sqrt{2}}{3}$

60. How many truth assignments to $p, q, r_1, \dots, r_{38}$ make $p \Rightarrow q$ false?

- (A) 2^{40}
- (B) 2^{37}
- (C) 2^{41}
- (D) 2^{38}
- (E) 2^{39}

Questions 61–63

61. Evaluate $\int_0^{\log 2} e^x(e^x + 20)^4 dx$.

- (A) $22^4 - 21^4$
- (B) $-\frac{22^5 - 21^5}{5}$
- (C) $\frac{22^5 - 21^5}{5}$
- (D) $\frac{22^5}{5}$
- (E) $5 \left(\frac{22^5 - 21^5}{5} \right)$

62. What is the largest integer k such that 3^k divides $103!$?

- (A) 51
- (B) 47
- (C) 50
- (D) 48
- (E) 49

63. Evaluate $\int_0^1 x^{19}(\log x)^3 dx$.

- (A) $\frac{1}{4} \left(\frac{(-1)^3 3!}{20^4} \right)$
- (B) $\frac{(-1)^3 3!}{20^4}$
- (C) $-\frac{(-1)^3 3!}{20^4}$
- (D) $20 \left(\frac{(-1)^3 3!}{20^4} \right)$
- (E) $\frac{4000}{3} \left(\frac{(-1)^3 3!}{20^4} \right)$

Questions 64–66

64. What is the coefficient of x^2 in $(1 + 2x)^{40}$?

- (A) 4160
- (B) 3120
- (C) 2080
- (D) 5200
- (E) 1040

65. The solution of $y' = 20x^3y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\frac{10^{\frac{3}{4}}}{10}$
- (B) $\frac{\sqrt{2} \cdot 5^{\frac{3}{4}}}{10}$
- (C) $\frac{5^{\frac{3}{4}}}{5}$
- (D) $\frac{1}{5}$
- (E) $\sqrt[4]{5}$

66. In the ring $\mathbb{Z}/588\mathbb{Z}$, how many elements satisfy $x^2 = 1$?

- (A) 7
- (B) 6
- (C) 8
- (D) 10
- (E) 9

Answer Key

Practice Test 98 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	C	23	A	45	A
2	B	24	A	46	B
3	E	25	B	47	D
4	A	26	E	48	B
5	C	27	A	49	B
6	E	28	B	50	A
7	A	29	B	51	A
8	E	30	D	52	C
9	B	31	E	53	D
10	E	32	D	54	D
11	D	33	A	55	A
12	E	34	D	56	C
13	E	35	B	57	D
14	B	36	C	58	D
15	D	37	C	59	E
16	A	38	C	60	D
17	E	39	E	61	C
18	E	40	A	62	E
19	C	41	E	63	B
20	A	42	D	64	B
21	C	43	D	65	C
22	B	44	C	66	C

zarimion.com/gre

100 practice tests, complete answer keys, and future updates.
Please retain the source attribution when sharing this material.

