

GRE Mathematics Practice Test 99

Bridge to Standard

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A bridge toward standard practice, with routine multi-step questions and a smaller number of challenging concepts. This is cycle 19 of 19, position 4 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 20x$ and has density $\rho(x, y) = x$. Find its centroid coordinate $\bar{x}$.

- (A) $-\frac{3}{4}$
- (B) $\frac{3}{2}$
- (C) $\frac{3}{8}$
- (D) $\frac{7}{4}$
- (E) $\frac{3}{4}$

2. Let D be a diagonal 101×101 real matrix with distinct diagonal entries. What is the dimension of the space of matrices X satisfying $XD = DX$?

- (A) 99
- (B) 100
- (C) 102
- (D) 103
- (E) 101

3. Find the upward flux of $\mathbf{F} = (0, 0, z)$ through $z = 20x^2 + 3y^2$, $0 \leq x, y \leq 1$.

- (A) $\frac{46}{3}$
- (B) $-\frac{23}{3}$
- (C) $\frac{23}{6}$
- (D) $\frac{23}{3}$
- (E) $\frac{26}{3}$

Questions 4–6

4. X has density cx^{41} on $0 < x < 1$ and zero elsewhere. Find the normalizing constant c .

- (A) 28
- (B) 56
- (C) 42
- (D) 14
- (E) 70

5. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(20+3j)(23+3j)}$.

- (A) $\frac{3}{20}$
- (B) $\frac{1}{20}$
- (C) $\frac{1}{60}$
- (D) $\frac{1}{23}$
- (E) $\frac{1}{3}$

6. What is the remainder when $x^2 + 1$ is divided by $x - 96$?

- (A) 9216
- (B) 9217
- (C) 9218
- (D) 9219
- (E) 9215

Questions 7–9

7. Evaluate $\lim_{x \rightarrow 0} \frac{e^{20x} - \sum_{j=0}^2 (20x)^j / j!}{x^3}$.

- (A) $6 \left(\frac{20^3}{3!} \right)$
- (B) $\frac{1}{20} \left(\frac{20^3}{3!} \right)$
- (C) $\frac{20^3}{3!}$
- (D) $3 \left(\frac{20^3}{3!} \right)$
- (E) $-\frac{20^3}{3!}$

8. Independent exponential waiting times X, Y have rates 41 and 43. Find $P(X < Y)$.

- (A) $\frac{41}{168}$
- (B) $\frac{125}{168}$
- (C) $\frac{41}{84}$
- (D) $\frac{43}{84}$
- (E) $\frac{41}{252}$

9. For $x > 0$, $xy' = 20y$ and $y(1) = 1$. Find $y(e^3)$.

- (A) $2e^{60}$
- (B) $1 + e^{60}$
- (C) $\frac{e^{60}}{2}$
- (D) e^{60}
- (E) $-e^{60}$

Questions 10–12

10. Let $P = \text{diag}(\underbrace{1, \dots, 1}_2, \underbrace{0, \dots, 0}_{100})$. What is the dimension of the space of 102×102 real matrices X satisfying $PX = X$?

- (A) 203
- (B) 204
- (C) 202
- (D) 206
- (E) 205

11. For $f(x, y) = x^2 + 20xy + 3y^2$, what is its maximum directional derivative at $(1, -1)$?

- (A) -18
- (B) 32
- (C) 14
- (D) 520
- (E) $2\sqrt{130}$

12. $X_1, \dots, X_{41}$ are independent with common mean and variance 43. With $\bar{X}$ their sample mean, find $E[\sum_i (X_i - \bar{X})^2]$.

- (A) 2293
- (B) 2866
- (C) 574
- (D) 1147
- (E) 1720

Questions 13–15

13. For $D = \{(x, y) : x^2 + y^2 \leq 20\}$, evaluate $\iint_D (x^2 + y^2)^2 dA$.

- (A) $\frac{8000\pi}{3}$
- (B) 8000π
- (C) $\frac{400\pi}{3}$
- (D) $\frac{16000\pi}{3}$
- (E) 1600π

14. A 3×3 matrix A has characteristic polynomial $(t - 95)(t + 1)(t - 2)$. What is $\text{tr}(A^2)$?

- (A) 9028
- (B) 9030
- (C) 9032
- (D) 9029
- (E) 9031

15. Let C be $x^2/9 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C (-y) dx + (20x) dy$.

- (A) 63π
- (B) 126π
- (C) $1 + 63\pi$
- (D) $\frac{63\pi}{2}$
- (E) -63π

Questions 16–18

16. A permutation of 43 objects is chosen uniformly. Let Z be the number of fixed points. Find $E[Z]$.

- (A) 3
- (B) 2
- (C) 4
- (D) 1
- (E) 0

17. Evaluate $\int_0^1 \int_x^1 20y^3 \, dy \, dx$.

- (A) 4
- (B) 10
- (C) 1
- (D) 5
- (E) $\frac{20}{3}$

18. Let N be the 101×101 matrix with ones immediately above the diagonal and zeros elsewhere. Find $\text{rank } N^{17}$.

- (A) 83
- (B) 84
- (C) 82
- (D) 86
- (E) 85

Questions 19–21

19. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 20z^2 = x + 3y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.
- (A) -60
(B) -40
(C) -120
(D) -3
(E) 120
20. A coin success probability p has uniform prior on $[0, 1]$. After observing 41 successes and 2 failures in independent tosses conditional on p , what is the predictive probability of success on the next toss?
- (A) $\frac{1}{15}$
(B) $\frac{14}{15}$
(C) $\frac{7}{15}$
(D) $\frac{14}{45}$
(E) $\frac{29}{30}$
21. For $y' = 20y(1 - y/3)$, what is the derivative of the right-hand side with respect to y at the positive equilibrium?
- (A) 20
(B) -20
(C) -10
(D) -19
(E) -40

Questions 22–24

22. Let A be the 101×101 matrix whose diagonal entries are 98 and whose off-diagonal entries are 1. What is its smallest eigenvalue?

- (A) 96
- (B) 97
- (C) 98
- (D) 95
- (E) 99

23. The solution of $y' = 20x^2y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\frac{\sqrt[3]{150}}{10}$
- (B) $\frac{\sqrt[3]{50}}{10}$
- (C) $\frac{\sqrt[3]{20} \cdot 3^{\frac{2}{3}}}{3}$
- (D) $\frac{3}{20}$
- (E) $\frac{\sqrt[3]{3} \cdot 5^{\frac{2}{3}}}{10}$

24. For which real a does $f_m(x) = x^{36m}/m^a$ converge uniformly to zero on $[0, 1]$?

- (A) $a = 0$
- (B) All real a
- (C) $a > 1$
- (D) $a > 0$
- (E) $a < 0$

Questions 25–27

25. Find the curvature at $t = 0$ of $r(t) = (3t, 20t^2)$.

- (A) $\frac{20}{9}$
- (B) $\frac{80}{9}$
- (C) $-\frac{40}{9}$
- (D) $\frac{40}{9}$
- (E) $\frac{49}{9}$

26. Positive real numbers x, y satisfy $\log_2 x + \log_2 y = \log_2 1235$ and $x + y = 108$. Find xy .

- (A) 1233
- (B) 1234
- (C) 1235
- (D) 1237
- (E) 1236

27. For $f(x) = x^3 e^{20x}$, find $f^{(5)}(0)$.

- (A) $\frac{5! 20^2}{2}$
- (B) $\frac{1}{20} \left(\frac{5! 20^2}{2} \right)$
- (C) $2 \left(\frac{5! 20^2}{2} \right)$
- (D) $\frac{1}{60} \left(\frac{5! 20^2}{2} \right)$
- (E) $\frac{1}{10} \left(\frac{5! 20^2}{2} \right)$

Questions 28–30

28. For which real p does $\sum_{k=2}^{\infty} 1/[k(\log k)^{36p}]$ converge?

- (A) $p > 0$
- (B) $p \geq 1/36$
- (C) $p < 1/36$
- (D) $p > 1/36$
- (E) All real p

29. Under $x = u + 20v$, $y = u - 3v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) 529
- (B) $\frac{1}{23}$
- (C) 60
- (D) 23
- (E) 17

30. A sequence satisfies $u_0 = 95$ and $u_{k+1} = 1u_k + 13$ for all $k \geq 0$. Find u_4 .

- (A) 145
- (B) 149
- (C) 148
- (D) 147
- (E) 146

Questions 31–33

31. Find the coefficient of x^6 in $\cos(20x)/(1 - 2x^2)$.

- (A) $-\frac{800000}{9}$
- (B) -54970240
- (C) $-\frac{687128}{9}$
- (D) $\frac{687128}{9}$
- (E) $-\frac{687137}{9}$

32. How many accumulation points in $\mathbb{R}$ does $S = \{1/k : k \geq 1\} \cup \{36 + 1/k : k \geq 1\}$ have?

- (A) 4
- (B) 3
- (C) 1
- (D) 0
- (E) 2

33. Let $F(x) = \int_0^x (x - t)^2 e^{20t} dt$. Find $F^{(3)}(0)$.

- (A) $\frac{1}{2}$
- (B) 2
- (C) 0
- (D) 6
- (E) 40

Questions 34–36

34. A linear map $T : \mathbb{R}^{102} \rightarrow \mathbb{R}^{104}$ has rank 3. What is its nullity?

- (A) 101
- (B) 99
- (C) 97
- (D) 98
- (E) 100

35. Evaluate $\int_0^\infty x^3 e^{-20x} dx$.

- (A) 6
- (B) $\frac{1}{20} \left(\frac{3!}{20^4} \right)$
- (C) $\frac{1}{3} \left(\frac{3!}{20^4} \right)$
- (D) $20 \left(\frac{3!}{20^4} \right)$
- (E) $\frac{3!}{20^4}$

36. How many connected components does $\mathbb{R} \setminus \{1, 2, \dots, 36\}$ have?

- (A) 49
- (B) 25
- (C) 61
- (D) 13
- (E) 37

Questions 37–39

37. Find the coefficient of x^3 in the Taylor series of $(1 - 20x)^{-2}$ at zero.

- (A) $\frac{3}{4} (4 \cdot 20^3)$
- (B) $-4 \cdot 20^3$
- (C) $4 \cdot 20^3$
- (D) $\frac{1}{20} (4 \cdot 20^3)$
- (E) $\frac{1}{4} (4 \cdot 20^3)$

38. What is the dimension of the real vector space of 101×101 symmetric matrices?

- (A) 5152
- (B) 5150
- (C) 5149
- (D) 5153
- (E) 5151

39. Evaluate $\int_C (3x^2 + 20y) dx + (20x + 3y^2) dy$, where C is the line segment from $(0, 0)$ to $(5, 1)$.

- (A) -226
- (B) 113
- (C) 452
- (D) 227
- (E) 226

Questions 40–42

40. Let $\zeta = e^{2\pi i/28}$. Find $\sum_{k=0}^{27} \zeta^k$.

- (A) 0
- (B) 3
- (C) 4
- (D) 1
- (E) 2

41. Evaluate $\int_0^1 \frac{x^2}{1+20x^3} dx$.

- (A) $-\frac{\log(21)}{60}$
- (B) $\frac{\log(21)}{60}$
- (C) $\log(21)$
- (D) $\frac{\log(21)}{3}$
- (E) $\frac{\log(21)}{20}$

42. Let $A = \begin{bmatrix} 1 & 95 & 0 \\ 0 & 1 & 13 \\ 0 & 0 & 1 \end{bmatrix}$. What is the $(1,2)$ entry of A^4 ?

- (A) 381
- (B) 379
- (C) 378
- (D) 380
- (E) 382

Questions 43–45

43. Let $\mathbf{F} = (20y^3, z^3, x^3)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (0, 0, 1)$.

- (A) -59
- (B) -120
- (C) -30
- (D) -60
- (E) 60

44. An analytic function has real part $u(x, y) = x^2 - y^2 + 26x$ and satisfies $f(0) = 0$. Find $\text{Im } f(1 + i)$.

- (A) 28
- (B) 37
- (C) 19
- (D) 46
- (E) 10

45. Evaluate $\iiint_B 20(x^2 + y^2 + z^2)^{3/2} dV$, where B is the unit ball.

- (A) $-\frac{40\pi}{3}$
- (B) $\frac{20\pi}{3}$
- (C) $1 + \frac{40\pi}{3}$
- (D) $\frac{40\pi}{3}$
- (E) $\frac{80\pi}{3}$

Questions 46–48

46. A permutation consists of disjoint cycles of lengths 7 and 3 and fixes all other points. What is its order?

- (A) 22
- (B) 21
- (C) 20
- (D) 23
- (E) 19

47. Solve $y' + 20y = x^3e^{-20x}$, $y(0) = 0$, and find $y(1)$.

- (A) $\frac{1}{4e^{20}}$
- (B) $\frac{5}{e^{20}}$
- (C) $\frac{4}{e^{20}}$
- (D) $\frac{1}{3e^{20}}$
- (E) e^{-20}

48. Apply one Newton iteration to solve $x^2 - 41 = 0$, starting at $x_0 = 1$. What is x_1 ?

- (A) 14
- (B) 35
- (C) 21
- (D) 28
- (E) 7

Questions 49–51

49. Evaluate $\int_0^1 x^{19}(\log x)^2 dx$.

- (A) $\frac{(-1)^{22!}}{20^3}$
- (B) $\frac{1}{3} \left(\frac{(-1)^{22!}}{20^3} \right)$
- (C) $200 \left(\frac{(-1)^{22!}}{20^3} \right)$
- (D) $-\frac{(-1)^{22!}}{20^3}$
- (E) $20 \left(\frac{(-1)^{22!}}{20^3} \right)$

50. In $R = \mathbb{F}_3[x]/(x^{20})$, how many elements are there?

- (A) 3^{22}
- (B) 3^{19}
- (C) 3^{21}
- (D) 3^{23}
- (E) 3^{20}

51. For $\mathbf{F} = (20x^3, y^3, z^3)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) $\frac{75}{2}$
- (B) 150
- (C) 75
- (D) -75
- (E) 76

Questions 52–54

52. Bisection starts with an interval of length 2^{41} . After 43 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{7}{8}$
- (B) $\frac{1}{24}$
- (C) $\frac{9}{16}$
- (D) $\frac{1}{16}$
- (E) $\frac{1}{8}$

53. Find the directional derivative of $f(x, y, z) = x^2 + 20y^2 + 3z^2$ at $(1, 1, 1)$ in the direction $(1, 5, 0)$.

- (A) $\frac{101\sqrt{26}}{13}$
- (B) $-\frac{101\sqrt{26}}{13}$
- (C) $\frac{202\sqrt{26}}{13}$
- (D) $\frac{101\sqrt{26}}{26}$
- (E) $1 + \frac{101\sqrt{26}}{13}$

54. Integers x satisfy $x \equiv 2 \pmod{5}$ and $x \equiv 1 \pmod{8}$. Find the least nonnegative solution.

- (A) 17
- (B) 19
- (C) 18
- (D) 15
- (E) 16

Questions 55–57

55. Find the arc length of $r(t) = (20t, 3t)$ for $0 \leq t \leq 1$.

- (A) $-\sqrt{409}$
- (B) $\sqrt{409}$
- (C) $2\sqrt{409}$
- (D) $\frac{\sqrt{409}}{2}$
- (E) $1 + \sqrt{409}$

56. What is the least positive multiplicative inverse of 2 modulo 83?

- (A) 14
- (B) 28
- (C) 42
- (D) 70
- (E) 56

57. The solution of $y' = 20x^3y^2$ has $y(0) = 1/21$. Find $y(1)$.

- (A) $\frac{1}{16}$
- (B) $-\frac{1}{16}$
- (C) $\frac{17}{16}$
- (D) $\frac{1}{8}$
- (E) $\frac{1}{32}$

Questions 58–60

58. The symmetric group S_{101} acts on $\{1, \dots, 101\}$. What is the index of the stabilizer of 1?

- (A) 102
- (B) 100
- (C) 103
- (D) 101
- (E) 99

59. Evaluate $\int_0^{\log 2} e^x(e^x + 20)^3 dx$.

- (A) $-\frac{22^4 - 21^4}{4}$
- (B) $4\left(\frac{22^4 - 21^4}{4}\right)$
- (C) $\frac{22^4 - 21^4}{4}$
- (D) $22^3 - 21^3$
- (E) $\frac{22^4}{4}$

60. How many truth assignments to $p, q, r_1, \dots, r_{39}$ make $p \Rightarrow q$ false?

- (A) 2^{41}
- (B) 2^{39}
- (C) 2^{42}
- (D) 2^{40}
- (E) 2^{38}

Questions 61–63

61. Let $F(x) = \int_0^{x^3} e^{20t} dt$. Find $F'(1)$.

- (A) $3e^{20}$
- (B) $\frac{e^{20}}{3}$
- (C) e^{20}
- (D) $60e^{20}$
- (E) e^{60}

62. In $\mathbb{Z}/3246\mathbb{Z}$, how many residues are idempotent ($x^2 = x$)?

- (A) 9
- (B) 10
- (C) 6
- (D) 7
- (E) 8

63. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-20)^j}{j^{\frac{2}{3}} (\log j)^m}$ converge absolutely at $x = 19$? Give the least such m , or 0 if none exists.

- (A) 4
- (B) 1
- (C) 3
- (D) 2
- (E) 0

Questions 64–66

64. What is the coefficient of x^2 in $(1 + 2x)^{41}$?

- (A) 4373
- (B) 5466
- (C) 3280
- (D) 2187
- (E) 1094

65. The region $0 \leq x \leq 1$, $0 \leq y \leq 20x^3$ is revolved about the x -axis. Find its volume.

- (A) $\frac{200\pi}{7}$
- (B) $1 + \frac{400\pi}{7}$
- (C) $\frac{800\pi}{7}$
- (D) $\frac{400\pi}{7}$
- (E) $-\frac{400\pi}{7}$

66. In the additive group $\mathbb{Z}/138\mathbb{Z}$, find the order of $[2]$.

- (A) 67
- (B) 69
- (C) 71
- (D) 70
- (E) 68

Answer Key

Practice Test 99 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	E	23	A	45	D
2	E	24	D	46	B
3	D	25	D	47	A
4	C	26	C	48	C
5	C	27	A	49	A
6	B	28	D	50	E
7	C	29	D	51	C
8	C	30	D	52	E
9	D	31	C	53	A
10	B	32	E	54	A
11	E	33	B	55	B
12	E	34	B	56	C
13	A	35	E	57	A
14	B	36	E	58	D
15	A	37	C	59	C
16	D	38	E	60	B
17	A	39	E	61	A
18	B	40	A	62	E
19	C	41	B	63	E
20	B	42	D	64	C
21	B	43	D	65	D
22	B	44	A	66	B

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